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# Integrated production, maintenance and quality control policy for unreliable manufacturing systems under dynamic inspection

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## A B S T R A C T

This paper proposes a new joint production, maintenance, and dynamic sampling inspection control policy, for failure-prone manufacturing systems. The integrated policy help decision-makers to find both production rate and maintenance frequency, as well as the size of the sample to be inspected. The inspection policy is based on a dynamic sampling plan, which adapts the sample size according to the system degradation and takes into account the interactions between production, maintenance, and quality control. To optimize jointly the integrated policy's control settings, a comprehensive and thorough mathematical model is developed to capture the complex dynamic and stochastic behavior of the manufacturing system. The proposed model relaxes several simplifying assumptions so that reliability and quality degradations are operation-dependent, duration and cost of inspection are non-negligible, no restriction on random distributions and failures occur at any time, as in the real manufacturing world. Besides, a new signal-based simulation model is developed and a detailed sensitivity analysis is carried out to validate analytical results. The results show the relevance of the analytical approach and help us to prove the importance of adapting the inspection effort to the inner quality in the production. An extended comparative study is provided and shows that the proposed integrated policy, based on dynamic inspection, outperforms those based on classical inspection strategies considered in the literature and practice.

### Keywords:

Dynamic inspection  
Deterioration  
Quality  
Production control  
Maintenance  
Mathematical model  
Simulation

## 1. Introduction

In an increasingly competitive environment, companies need to innovate and master their production systems to satisfy their customers. Indeed, one must reduce production stoppage due to failures or non-quality issues. To diminish these wastages, and to better adapt to customer needs and remain competitive, companies need to calibrate their production, maintenance, and quality strategies mutually: the production rate, the stock level, the maintenance frequency, and the quality controls must be adjusted, jointly. Undeniably, integrated control policies have shown better performance compared to traditional planning approaches in which production, maintenance, and quality are treated as separate issues (Colledani and Tolio, 2011); It has been shown that integrated control models lead to an increase in profits of up to 40% (Colledani and Tolio, 2012). This has created a trend on the scientists'

part during the last few decades for the search for the best-integrated control models (Colledani et al., 2014). But still few works in the literature cover the joint optimization of production, maintenance, and quality control at the same time (Colledani and Tolio, 2011), (Colledani et al., 2014), (Hadidi et al., 2012), and (Wang et al., 2019).

Implementing these models in real life is a challenge because they rely on a set of unrealistic assumptions (Van Horenbeek et al., 2013). Among these unrealistic hypotheses (Bouslah et al., 2018), we can cite (1) The assumption that the degradation of the system is time-dependent, whereas, in the real world, most machine failures and quality degradations depend on operations and machine utilization; (2) the assumption that the system cannot fail during the construction phase of the safety stock while failures can arise at any instant during the production cycle; and (3) the hypothesis that quality is fixed in advance whereas, in reality, it degrades with the system age. These assumptions,

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and others, are motivated by computational simplification considerations but often lead to poor estimates and low system performance and consequently to poor analysis and poor operations planning.

From the quality control side, several authors use static inspection plans based on inspecting all produced items (100% inspection) or inspecting a fixed proportion of all produced items (static sampling plan). But in practice, inspecting all produced items is too costly, while adopting a static sampling plan is not efficient; in fact, too much inspection than needed in the early age of the machine and not enough inspection when the machine is too old.

Indeed, making inspection strategies independent of the degradation of the system state, as it has often been considered in the literature, is costly. In reality, the quality of products is not constant and deteriorates with the operations and the age of the system. The most adequate approach is to correlate the inspection effort to the quality condition. This challenge of building an adaptive inspection plan within the design of the joint control of the three key policies is what we plan to grasp in the present research.

Therefore, in the present paper, we propose an integrated production, maintenance, and quality control policy with a dynamic sampling plan, which is calibrated to the quality needs. We take care of the impact of the age of the machine in terms of usage (number of produced items) on the reliability of the system and the quality of the final goods. The studied system is subject to an increasing stochastic failure rate that leads to more and more non-quality products. The objective is to find the optimal production and maintenance control policies and the right sampling strategy, for the inspection, that minimize the total cost over the long run.

The present paper is organized as follows. Section 2 is dedicated to the literature review; we summarize the state of the art to highlight our contributions. In Section 3, we present the system under study and introduce the different policies used in the model. Section 4 is dedicated to the mathematical formulation of problem. A detailed simulation model is developed in Section 5 to validate the proposed mathematical model. A numerical example is analyzed in Section 6. Section 7 discusses the results through an extensive sensitivity analysis. We present, in section 8, a comparative study between different control policies, adapted, from the literature and usually used in practice to highlight the effectiveness of the proposed policy. Section 9 concludes the paper.

## 2. Literature review

Integrated approaches for production maintenance and quality controls in unreliable manufacturing systems have become one of the most attractive research subjects in the last decades (Hadidi et al., 2012). Inman et al. (2003) highlighted the fact that increasing quality is mandatory for modern companies. Kim and Gershwin (2005), (Kim and Gershwin, 2008) presented an analytical model with a method for the performance analysis of production systems. They analyzed how production system design, quality, and productivity are inter-related in such systems. Mhada et al. (2011) presented analytical expressions for the optimal production threshold and the optimal cost of a production control problem of a failure-prone manufacturing system that produces a random fraction of defective items. Dhouib et al. (2012) propose a model for a manufacturing system, which may shift to an out-of-control state producing defectives; they determined the inventory level and the age to conduct preventive actions. Njike et al. (2011) used iterative feedback based on the quantity of defective products to determine the optimal maintenance and production planning since they proposed that defective products are a consequence of global manufacturing system deterioration. Integrated approaches are also studied while considering the scheduling of the jobs instead of production control, to prove that the integrated model performs better. Sinisterra and Cavalcante (2020) propose analytical and simulation models to establish the best sequence that integrates the schedule of resumable jobs with the inspection's action. The models minimize the expected total cost of maintenance and

tardiness cost. The authors do not cover the quality aspect but, highlight the benefit of integrated approaches. Also, Zheng et al. (2020) consider, jointly, optimizing the job scheduling with preventive maintenance and quality monitoring, via a control chart, to minimize the expected total cost per time unit.

In recent years, there has been particularly a surge in interest in incorporating the quality degradation aspect into integrated production and preventive maintenance (PM) models through inspection strategies. For example, Colledani and Tolio (2011) presented an analytical model for evaluating the performance of production systems monitored by statistical control charts. They consider inspection integrated to stations in the production line. Radhoui et al. (2010) proposed an integrated production and PM control policy considering a 100% inspection and assuming that feedback information from an inspection is used to improve PM planning. Rivera-Gomez et al. ((Rivera-Gómez et al., 2016), (Rivera-Gómez et al., 2018), (Rivera-Gómez et al., 2020)) used a numerical approach and simulation to estimate the optimal integrated production and PM control for single-unit production systems subject to degradation with a 100% inspection plan. Recently, Bouslah et al. (2018) developed models integrating the hedging point policy (HPP), age-based PM and static sampling plan for manufacturing systems subject to operation-dependent degradation and they solved the problem with simulation optimization techniques. Newly, and based on simulation modeling, Rivera-Gomez et al. (Rivera-Gómez et al., 2020) proposed a dynamic sampling plan for a manufacturing system composed of one production unit. They considered quality deterioration that is caused by corrective maintenance actions and neglected the deterioration effects on reliability.

From these models, we note the relevance of quality and inspection integration with maintenance and production. Moreover, we notice another limiting assumption: the inspection has a negligible duration and cost as in (Radhoui et al., 2010), (Mhada et al., 2013), and (Radhoui et al., 2009). Sahnoun et al. (2014) marginalized the impact of inspection duration but observed that significant savings are possible by using an optimized sampling plan. Recently, Bouslah et al. (2016) considered the problem of integrating the batch production strategy and quality control that is performed using a single acceptance-sampling plan by attributes. Also, they considered that maintenance is undertaken once the proportion of defectives in a rejected lot reaches a given threshold. This shows the importance of adapting the inspection effort to the inner quality in system. A static sampling plan is better than a 100% inspection which is too expensive. Even though, with a static plan we still losing money in the early age of the machine because we are doing too much inspection meanwhile the quality is good, and when the machine is aged due to the bad quality because the inspection effort is not enough to stop the bad parts from reaching the customer. Then a dynamic sampling plan, that takes into account the system degradation with age, should be better. For instance, as the quality level degrades with operations, the inspected quantity and the sampling plan should adapt to this quality level to be more efficient. As can be noted, more research is needed in this domain to fully integrate production-quality and maintenance functions in a joint control strategy. To the best of our knowledge, there is no analytical model neither simulation model that addresses dynamic inspection in the joint control of production, PM, and quality.

Table 1, highlights the main contributions in the literature related to the considered problem. The listed references, in this table, cover works that addressed deteriorating systems in the joint control context, which is the main concern of this paper. The comparison is done on the key features of the studied problem, such as the considered policies, type of degradation, and inspection strategy. We do compare also the approaches used to model and solve the problem by highlighting if the work uses a simulation and/or analytical model.

Mainly, the majority of works cover only two of the three policies. Also, few works consider the operation-dependent degradation of the system. And even fewer consider the inspection duration. And no work

**Table 1**

Our paper in comparison with the literature.

Author(s)	Problem features										Approaches	
	operation dependent degradation	Reliability degradation	Quality degradation	Production policy	Maintenance strategy	100% inspection	fixed fraction inspection	Dynamic fraction inspection	Non-negligible inspection duration	Stochastic context	Simulation approach	Analytical approach
Bouslah et al. (2016)	✓	✓	✓	✓	✓		✓		✓	✓	✓	
Bouslah et al. (2018)	✓	✓	✓	✓	✓		✓		✓	✓	✓	
Colledani and Tolio (2011)	✓		✓				✓			✓		✓
Colledani and Tolio (2012)	✓	✓	✓		✓		✓		✓	✓		✓
Dhouib et al (2012)		✓		✓	✓					✓	✓	✓
Gharbi and Kenné (2005)		✓		✓	✓					✓		✓
Kenné and Gharbi (2004)				✓	✓					✓		✓
Kim and Gershwin (2008)	✓	✓								✓	✓	✓
Kouedeu et al. (2015)		✓		✓	✓					✓		✓
Lu et al. (2016)		✓			✓					✓		✓
Mhada et al. (2011)		✓		✓						✓	✓	
Mhada et al. (2013)				✓		✓				✓	✓	✓
Njike et al. (2011)		✓		✓	✓					✓		✓
Radhoui et al. (2009)		✓	✓	✓	✓	✓				✓	✓	✓
Radhoui et al. (2010)		✓	✓	✓	✓	✓				✓	✓	✓
Rivera-Gómez et al. (2016)	✓		✓	✓	✓	✓				✓	✓	✓
Rivera-Gómez et al. (2018)	✓	✓	✓	✓	✓	✓				✓	✓	✓
Rivera-Gómez et al. (2020)			✓	✓	✓			✓		✓	✓	
Sahnoun et al. (2014)			✓				✓			✓	✓	
Schutz et al. (2013)		✓		✓	✓					✓		✓
Shrivastava et al. (2016)		✓			✓					✓		✓
Wang et al. (2019)		✓	✓	✓	✓	✓				✓		✓
Xao et al. (2019)		✓	✓	✓	✓	✓				✓		✓
<b>Proposed approach</b>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

has addressed the dynamic inspection integrated with maintenance and production controls, in a context of operation-dependent degradations that act on both reliability and quality.

In summary, from the literature review, we come with these major deficient elements:

First, one common assumption used is that the reliability degradation of machines is time-dependent ((Gharbi and Kenné, 2005), (Kenné and Gharbi, 2004), (Kouedeu et al., 2015), (Lu et al., 2016), (Schutz et al., 2013), (Xiao et al., 2019)), whereas, in real life, most machine failures are operation-dependent ((Bouslah et al., 2016), (Buzacott and Hanifin, 1978), (Colledani and Tolio, 2011), (Colledani and Tolio, 2012), (Dhouib et al., 2008), (Polotski et al., 2019), (Rivera-Gómez et al., 2018)). This assumption is prevalent in the literature because, analytically, modeling operation-dependent failures is much more complex than modeling time-dependent ones (Matta and Simone, 2016). The complexity here lies in the fact that in modeling operation-dependent failures only the time during which the machine is operational needs to be tracked. Also, it has been shown that modeling machines subject to operation-dependent failures in the same fashion as with machines subject to time-dependent failures may lead to a significant underestimation of overall production capacity (Sherwin, 2000).

Second, in lean manufacturing systems, the operating speed is aligned with the demand rate, which leads to low inventory levels. However, in degrading manufacturing systems, safety stock is generally used to protect the system against the risk of shortage when machines are not available. Nevertheless, building a safety stock requires an extra production capacity, which means accelerating the production rate and increasing the degradation intensity accordingly (Groenevelt et al., 1992). To restore and maintain the overall reliability and quality performance, corrective and planned maintenance actions are required (Ben-Daya and Duffuaa, 1995). Hence, an effective integrated operations control for degrading systems should take into consideration these complex interactions between production, inventory, reliability, maintenance, and quality, as in real life, to realize the best trade-off strategy.

Third, the degradation modeling of quality in integrated control policies is based also on many simplifying assumptions. For instance, the defective rate is considered whether constant, whether following a known probability distribution, or whether following a time-dependent deterioration model (Rosenblatt and Lee, 1986). These assumptions completely ignore the fact that quality degradation in manufacturing systems is directly and intrinsically affected by the produced quantity (the system age). For example, this has been seen in machining processes

were increasing the production rate accelerates the deterioration of cutting and drilling tools, and accordingly, impacts the quality of machined surfaces (Njike et al., 2011). In discrete manufacturing processes such as stamping or machining, during the production phase, random drifts of the tool components can considerably degrade the quality of the product (Chen and Jin, 2006).

And the major conclusions from this literature review are: (1) that the inspection policy is not well implemented: Authors consider the inspection effort (fraction controlled or sampling plan parameters) is constant even for the cases where the quality is degrading with time or system age. We claim that this is irrelevant and one should tune, wisely, the inspection to meet the needed effort that guarantee the right level of quality. (2) That while the joint control of production and maintenance is valuable, the integrated mathematical models are nonetheless too difficult to address real-life problems. Difficulties arise from the stochasticity of the problem and the complex analytical formulation of degradation phenomenon of both quality and reliability. To deal with these limitations, authors, either, use many assumptions in their mathematical models or use simulations. In the first case, the assumptions could lead to incorrect results, and no work has proven the effectiveness of these assumptions in all conditions. For the second case, the computing effort is high; in fact, simulations take a long time to complete, and many replications are needed to perform significant and extra analyses.

In the present paper, we tackle the challenge of formulating an analytical model for the integrated production, quality, and preventive maintenance problem with a dynamic inspection plan. We propose, also, to develop a dynamic sampling plan that closely shadows the system's inner quality and compute the right inspection effort to apply. The implemented approach could be used to address all inspection strategy from 100% inspection to dynamic sampling plan via static sampling; which allows us to compare our policy with other policies considered in the literature and practice.

The paper aims to overcome existing limitations in analytical approaches and proposes an exhaustive mathematical formulation that integrates the three policies relaxing the most unrealistic assumptions. The main contributions are i) Introduction of a dynamic sampling approach to quality inspection; ii) Joint control of the production, maintenance, and quality; iii) Exact approach to solve the optimal control problem through a stochastic mathematical model; iv) a simulation model to validate the mathematical results.

### 3. Problem description

The manufacturing system under study consists of two workstations; a manufacturing one that makes the parts and a sampling/inspection station that checks the quality of products. The manufacturing workstation is subject to stochastic failures and quality deterioration. The production system feeds a servicing buffer that holds final products until requested from the customer. The customer demand is qualified by a continuous and constant rate,  $d$ , and a specific level of product quality. The average fraction of bad quality products sent to the customer (the average outgoing quality, AOQ) should not exceed a critical level called the average outgoing quality limit (AOQL); see Fig. 1.

The machine failure and the product quality are operation-dependent: The aging of the machine (the number of produced items ( $a$ )) leads to an increasing failure rate and an increasing proportion of defectives produced. To dodge unexpected break-downs, an Age-Based Preventive Maintenance policy (PM) is adopted. Even though failures occur, they are instantaneously detected and are removed by corrective maintenance (CM) interventions. The production is controlled by a hedging-point policy.

Due to the high inspection cost, only a proportion of produced items are inspected based on a dynamic sampling plan: A 'dynamic fraction' ( $f$ ) of the production is inspected depending on the machine age ( $a$ ) and the detected defects are rejected from the system. As the inspection policy is

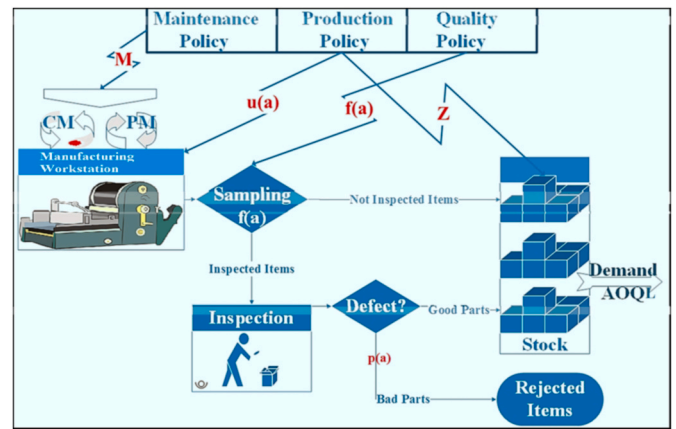


Fig. 1. Manufacturing system under study.

not 100%, the delivered products are not defect-free. The bad parts that arrive at customers have a high cost. In other words, the average outgoing quality (AOQ) has a cost. The demand rate is constant and unfulfilled sales are lost. The objective is to jointly optimize the production, maintenance, and quality/inspection control policies while meeting the average outgoing quality limit (AOQL) imposed by the customer; i.e., the model aims to jointly determine the optimal production rate, the optimal inventory level, the optimal sampling parameter and the optimal maintenance rate that minimize the total incurred cost. The incurred cost includes the inventory holding cost, the lost sales cost, the CM and PM costs, and the inspection, rejection, and bad sales costs. The optimal solution must ensure the right trade-off between production, maintenance, and quality costs while ensuring that the AOQL constraint is fully satisfied.

The problem under consideration is modeled under the following assumptions:

- After each maintenance action, PM or CM, the system is brought to the state "as good as new".
- Any inspected defective item is immediately detected and then rejected from the system.
- After each maintenance action, PM or CM, if the buffer size  $s$  is positive we wait until the buffer is empty before restarting the production. Otherwise, if the buffer is empty, the machine starts immediately.

#### 3.1. Notations

The problem under consideration is based on the following notations:

- $d$  Product demand rate;
- $a$  Age of the system (total produced items);  $a = \int u dt$ ;
- $u(a)$  The production rate of the system at age  $a$ ;
- $U_{max}$  The maximum production rate;
- $Z$  Safety stock level;
- $A_Z$  Age of the system when the safety stock level is reached;
- $s(a)$  Inventory level at the age  $a$ , of the system,  $s \in ] - \infty, Z]$ . It is also the state of the buffer. In the following, we will use, indistinctively,  $s$  or  $s(a)$  to refer to the buffer level;
- $s^+$  The surplus;  $s^+ = \max(s, 0)$ ;
- $s^-$  The lost sales;  $s^- = \max(-s, 0)$ ;
- $T$  The cycle length;
- $t_f$  Time to failure. Time from the moment the system is Up until it fails;

$A_f$  Age of the system at failure instant  $t_f$ . The failure rate is increasing with the machine age  $a$ ;

$f_f$  Probability Density Function (PDF) associated with the random variable (r.v.)  $A_f$ ;

$F_f$  Cumulative Distribution Function (CDF) associated with the r.v.  $A_f$ ;

$M_{i,f}(a)$  The  $i^{th}$  order lower partial moment of  $f_f$ ;  $M_{i,f}(a) = \int_{-\infty}^a x^i \cdot f_f(x) \cdot dx$ ;

$MUTF$  The mean usage (i.e., the mean number of produced parts) to the failure;

$t_r$  Time to repair (repair duration).  $f_r, F_r$  are, respectively, the associated PDF and CDF;

$M_{i,r}(t)$  The  $i^{th}$  order lower partial moment of  $f_r$ ;  $M_{i,r}(t) = \int_{-\infty}^t x^i \cdot f_r(x) \cdot dx$ ;

$MTTR$  The mean time to repair;

$t_m$  Time to maintain (preventive maintenance duration).  $f_m, F_m$  are, respectively, the associated PDF and CDF;

$M_{i,m}(t)$  The  $i^{th}$  order lower partial moment of  $f_m$ ;  $M_{i,m}(t) = \int_{-\infty}^t x^i \cdot f_m(x) \cdot dx$ ;

$MTTM$  The mean time to maintain;

$M$  Maintenance threshold expressed as the age of the system (the produced items before the next preventive maintenance action, once the machine is up);

$t_M$  Instant at which the preventive maintenance is due. Time at which the age of the system =  $M$ ;

$p(a)$  The proportion of defects at a specific age  $a$  of the machine. The aging of the machine leads to more defects, i.e.,  $p(a)$  is increasing with the age;

$\bar{p}$  The long-run average proportion of defects;

$q_g(a)$  The proportion of good parts at the age  $a$  of the machine;

$q_g(a) = 1 - p(a)$ ;

$Q_g(a)$  The total good parts produced up to the age  $a$  of the machine;

$Q_g(a) = \int_0^a q_g(x) \cdot dx$ ;

$T_{insp}$  Inspection duration for a produced item;

$f(a)$  The fraction of production to be inspected at machine age  $a$ ;

$\bar{f}$  The long-run average inspected fraction;

$I_s$  The inspection severity factor;

$q(a)$  The proportion of parts sent to servicing buffer at the age  $a$  of the machine;  $q(a) = 1 - p(a)f(a)$ ;

$Q(a)$  The total parts sent to the servicing buffer up to the age  $a$  of the machine;  $Q(a) = \int_0^a q(x) \cdot dx$ ;

$AOQ(a)$  Average outgoing quality at age  $a$ ; the average fraction of bad parts sent to the customer;

$AOQ_\infty$  The long-run average outgoing quality;

$AOQL$  Average outgoing quality limit (The threshold for the quality level); the maximum average fraction of bad parts accepted by the customer;

$C_h$  Unit inventory holding cost per time unit;

$C_l$  Cost of one item of lost sales due to shortage;

$C_{pm}$  Preventive maintenance cost;

$C_{cm}$  Corrective maintenance cost;

$C_{insp}$  Unit inspection cost;

$C_{rej}$  Unit cost of rejected parts;

$C_{def}$  Unit cost of a bad item sold to the customer;

During the rest of this document, and for any expression or function  $\phi$ , we use  $\phi(t)$  to state its expression that depends on the time and  $\phi(a)$  to state its expression that depends on the age of the system (number of produced items).

## 4. Mathematical formulation

In this section, we will develop the mathematical formulation of the described system under the proposed integrated production-maintenance-quality policy. We show that the system dynamics follow a renewal process. Then, we detail the considered production, maintenance, and quality control policies. Finally, we develop the mathematical model of the problem and give the formulation of the total incurred cost per unit of time. The implemented approach for the present work is as follows: i) we develop each policy and present its parameters. ii) We validate that our process is a “renewal process” and, thus, the cost function is a “renewal-reward process”. iii) This leads to the expression of the objective function as the average cost over a cycle divided by the average length of a time cycle. iv) We compute the optimal strategy. v) We develop a simulation model to validate analytical results generated by the proposed mathematical model.

### 4.1. The renewal-reward process

The state of the machine goes, episodically, from UP to DOWN and from DOWN to UP. Fig. 2 shows the machine cycle; it has three parts: i) UP of length  $t_f$  or  $M$ ; ii) DOWN of length  $t_r$  or  $t_m$ ; iii) the waiting time (Off period) if the buffer is not empty. Consequently, the system has three phases:

- Phase 1: Inventory building phase: the machine is producing at the maximum rate.
- Phase 2: Production at demand phase, once the buffer is full ( $s = Z$ ), the production rate is slowed-down to maintain the buffer level  $Z$ .
- Phase 3: Reparation phase: after a failure or a specific age (production of  $M$  items), we repair the machine, either correctively or preventively, respectively.

Fig. 3 presents the buffer state for different scenarios. In this figure, we have the evolution of buffer size for three major scenarios: i) scenario 1: the failure occurs during phase 1, the safety stock building phase. The age of the machine at failure  $A_f$  is below  $A_z$ . ii) Scenario 2: the failure occurs in phase 2. The age of the machine at failure  $A_f$  is above  $A_z$  and below  $M$ . iii) Scenario 3: the failure did not happen because the machine reaches the preventive maintenance due age,  $M$ . Each scenario can end with two possible situations depending on the maintenance action duration: 1) At the end of the maintenance the buffer is empty, the machine restarts immediately, we have lost sales, and the buffer level is set to zero; 2) At the end of the maintenance, the buffer level is positive, then we wait until the buffer is empty before restarting the machine and we do not have any lost sales. We opt for emptying the buffer before the machine starts again. Indeed, since the machine is as good as new, there is no need to build a safety stock immediately.

The buffer state has the same cycles as the machine in Fig. 2. It is characterized by the buffer level  $s(t) \in ] - \infty, Z]$ . From Figs. 2 and 3, we could notice that the buffer size, at the beginning and the end of each cycle is equal to zero, and the state of the machine is renewed at the beginning of each cycle. From these observations, and due to the independence between cycles, we could assure that the process is a renewal process.

Therefore, all KPIs (key performance indicators) will be computed over each  $Cycle_i$ , like:

- $I_i$  total inventory during  $Cycle_i$ ,
- $L_i$  lost sales during  $Cycle_i$ ,
- $Cost_i$  total cost during  $Cycle_i$ ,
- $NbPM_i$  number of preventive maintenances during  $Cycle_i$ ,
- $NbCM_i$  number of corrective maintenances during  $Cycle_i$ ,
- $NbInsp_i$  number of inspections done during  $Cycle_i$ ,
- $NbRej_i$  number of rejected parts during  $Cycle_i$ ,

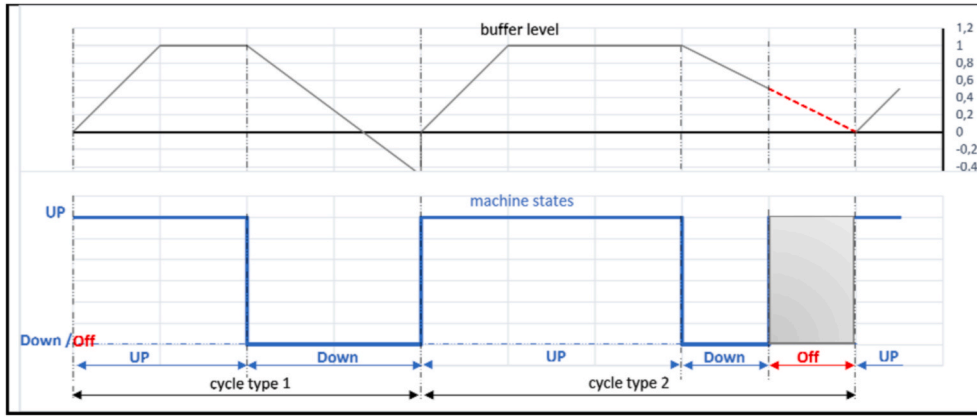


Fig. 2. Machine states & buffer level.

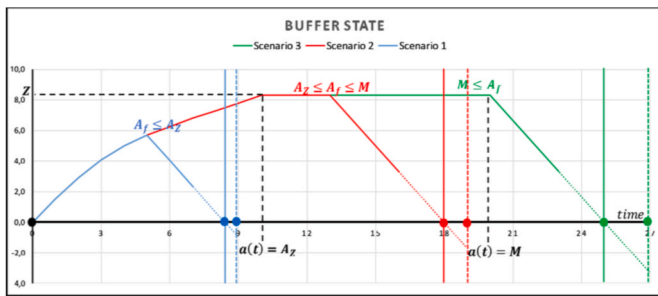


Fig. 3. Buffer state for different scenarios (the solid circles depict the end of the cycles).

- $TBSold_i$  number of defects sent to customer during  $Cycle_i$ ,
- $AOQ_i$  average outgoing quality at the end of  $Cycle_i$ ,
- $InsI_i$  average inventory in the inspection shop during  $Cycle_i$ ,
- $Avail_i$  average system availability in  $Cycle_i$ ,

All these stochastic variables are “rewards” and their cumulative, over a horizon, are “renewal-reward process”.

Consequently, to compute the average value of any KPI per unit of time over an infinite horizon, we apply “the elementary renewal theorem for renewal reward processes” (Christer, 1978), which means:

$$\lim_{t \rightarrow \infty} \frac{KPI(t)}{t} = \frac{E[KPI_i]}{E[T_i]} \quad (1)$$

In the following subsection, we will detail the production, the maintenance, and the quality control policies and then we propose the integrated analytical model.

#### 4.2. Production policy

The production rate  $u(a)$  of the manufacturing system at age  $a$ , is one of the main control parameters affecting the optimal control policy. It goes from 0 to  $U_{max}$ ; Where  $U_{max}$  is the maximum production rate. The proposed production control policy is based on hedging point policy (HPP) (Akella and Kumar, 1986). According to the HPP policy and due to the operation dependent failure mode, the production rate  $u(a)$  is adjusted depending on the number of produced items  $a$ , the demand rate  $d$ , the proportion of defects  $p$ , the inspected fraction  $f$ , and the threshold inventory level  $Z$  as explained below.

The proposed production control policy is defined by equation (2). According to this policy: i) The production rate,  $u(a)$ , is set to its maximum value,  $U_{max}$ , when the buffer level is below the threshold level  $Z$ ; ii) When the buffer is full,  $s(a) = Z$ , the production rate is set such that the buffer level holds its maximum value  $Z$ . We know that the out-rate of

the buffer is  $d$  – the demand rate – and the in-rate is  $u(a)(1 - f(a)p(a))$ , only the non-inspected parts and the good part from those inspected could arrive to buffer. In other words, the bad parts detected by inspection never reach the servicing buffer since they are rejected. Then to maintain the buffer level  $Z$ , the production rate should be adjusted to  $u(a) = d / (1 - f(a)p(a))$ ; iii) finally, when the buffer is above  $Z$  or when the machine is down, the production rate is zero.

$$u(a) = \begin{cases} U_{max} & \text{if } s < Z \text{ \& machine UP} \\ \frac{d}{1 - f(a)p(a)} & \text{if } s = Z \text{ \& machine UP} \\ 0 & \text{if } s > Z \text{ Or machine DOWN} \end{cases} \quad (2)$$

Under this production control policy, the system has three phases, as shown in Fig. 3: i) The safety stock building phase: the machine produces at a constant rate of  $U_{max}$ . The age of the system ( $A_z$ ) at the end of this phase, if no failure occurs, is a crucial element. It is characterized by equation (3). ii) The production at demand phase: The system rate is adjusted according to the policy defined by equation (2) to maintain the buffer at its maximum level  $Z$ . The age of the system at the end of this phase, if no failure occurs, is the maintenance threshold  $M$ . This second phase may not exist if the failure occurs during phase 1. iii) The last phase is the reparation phase during which the system may fail or reaches the threshold age  $M$  and requires to be maintained. The production rate during this phase is null and the buffer level is decreasing according to the demand rate,  $d$ .

$$\left( \int_0^{A_z} (1 - p(a)f(a)) da \right) - \frac{A_z}{U_{max}} d = Q(A_z) - \frac{A_z}{U_{max}} d = Z \quad (3)$$

Consequently, the production control policy is defined by the system state and the safety stock level  $Z$ .

#### 4.3. Maintenance control policy

The system is subject to random failures, and to dodge these breakdowns, we consider an age-based preventive maintenance policy (ABPM). When the system is Up when starting with a new cycle, we reset the counter for its age to zero (the system is considered as good as new) and if it survives till the threshold age  $M$ , then it is stopped to carry out preventive action. Otherwise, if a failure occurs before  $M$ , corrective action is carried out. The maintenance control policy is summarized by equation (4).

$$PM = \begin{cases} No & \text{if } a < M \\ Yes & \text{if } a \geq M \end{cases} \quad (4)$$

The system deterioration is operation-dependent. So, the threshold age  $M$  of the preventive maintenance policy is expressed in terms of the

usage of the system defined by the number of produced items.

Consequently, the preventive maintenance policy is defined by the maintenance threshold  $M$ .

#### 4.4. Quality control policy

Most of the time, manufacturing systems deteriorate with usage, the number of produced items. Also, the cumulative usage of the machine accelerates its degradation and therefore increases the rate of defective parts. This degradation phenomenon is common in many industries, generally in systems with a large number of components that stochastically deteriorate over time (Kouedeu et al., 2015). As a consequence of this dynamic degradation, the quality level is not the same during the lifespan of a machine, as shown in Fig. 4. In the early lifetime, when the manufacturing machine is new, the defects' rate is low. While at the end-of-life or before a breakdown, the number of defects is high. To account for this dynamic context of deterioration, we propose a dynamic inspection plan to control the level of quality: An inspection plan that adapts to the degradation of the system and therefore to the level of quality. Indeed, for better performance, a dynamic policy rather than a 100% or static inspection policy is more appropriate. The proposed dynamic inspection policy adapts the severity of the quality control to the system degradation level.

Fig. 4 epitomizes the real world, when the system is new, the rate of defects is small (min level) and the quality degradation slow. After a certain age (in Fig. 4 ~ 1 unit of the system age, the age here is not the time but the usage of the machine, number of produced items), the quality starts to degrade fast. However, when the system is old (~6 units of the system age), the rate of defects reaches its peak (peak level) and the degradation is slow or null. From these observations, we derived a function to model the quality degradation as realistic as possible. Equation (5) represents the percentage of defects during the production phase. This percentage of defects  $p(a)$  depends on the age of the system,  $a$ . It increases with  $a$ , and models the impact of the system age on the quality degradation.  $p_0$  is the initial quality level when the system is new and  $(p_0 + \eta)$  represents the peak of defects, which is reached when the system is old. The shape of  $p(a)$  modeled by this equation is the same as the one given in Fig. 4; the quality level degrades slowly and suddenly it drops rapidly to reach its climax.  $\lambda_q$  and  $\gamma_q$  are used to tune the shape and the scale of the derogation model.

$$p(a) = p_0 + \eta \cdot (1 - \exp(-\lambda_q \cdot a^{\gamma_q})) \quad (5)$$

The model described by equation (5) is general and common in the literature ((Bouslah et al., 2016), (Bouslah et al., 2018), (Cheng et al., 2018), (Cheng and Li, 2020), (Rivera-Gómez et al., 2018), (Rivera-Gómez et al., 2020)). All these authors use the same model for quality degradation. They even highlight this "S" shape by plotting it from simulation results. They argue also that this formulation is the most general, since, it can represent any case. Also, it has the advantage of modeling the saturation effect. Indeed, the percent of defects could not increase indefinitely, it has a natural limit of 100%. Usefully, one could

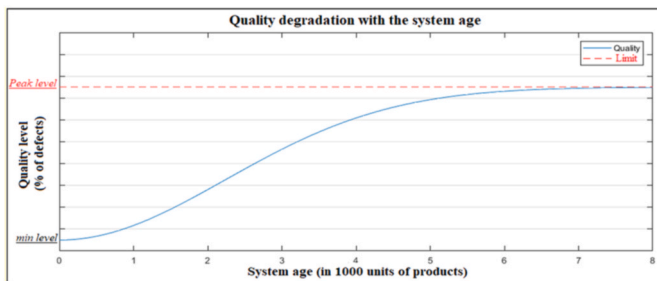


Fig. 4. Quality degradation with the system age.

model any situation; in fact, by changing the values of  $\lambda_q$  and  $\gamma_q$ , we could have a three-stage model – "S" shape, the two-stages model – exponential and polynomial, and the one-stage – linear. Also, one could adapt the initial and the peak value of defects (from 0% to 100%) by tuning  $p_0$  and  $\eta$ . Besides, this model and its parameters could be easily fitted from historical data (Bouslah et al., 2018). In section 7.3, we experiment with different quality degradation shapes and different peak values to show that our model could handle any quality degradation case. Besides, one should notice that the peak values of the quality have an impact on the feasibility of the system (to be able to meet demand and AOQL constraint). In a real system, too much defect is not representative and could lead to an infeasible system. In the reviewed papers we found that the peak level was around 10% ((Bouslah et al., 2016), (Bouslah et al., 2018), (Cheng et al., 2018), (Cheng and Li, 2020), (Rivera-Gómez et al., 2018), (Rivera-Gómez et al., 2020)). Also, even if we allow the peak level to reach 100%, the maintenance will reset the system to keep it under reasonable conditions, in our case, as in Table 7, the maximum observed peak level is.  $p_{max}^{obs} = 23\%$

Based on the quality model in equation (5), we could compute the average outgoing quality, AOQ, at any age  $a$  of the system. equation (6) expresses the AOQ as a function of the age  $a$ . At a given age,  $a$ , the average production fraction sent to customer is  $(1 - f(a)p(a))$ , and this production includes a percentage  $p(a)(1 - f(a))$  of bad parts.

$$AOQ(a) = \frac{\int_0^a (p(x) - f(x)p(x))dx}{\int_0^a (1 - f(x)p(x))dx} \quad (6)$$

As stated in the introduction, one of the objectives of this work is to find the optimal inspection policy that adapts to the level of quality. To do so, we need to design a sampling plan that adapts to the system's state and, thus, eliminate the defects in an optimal way (neither too much inspection nor not enough). The inspected fraction  $f(\cdot)$  in this sampling plan should be dynamic and adapt to the quality level. To do so, we should consider either, scenario (1), which capture the quality degradation characteristics and shape the inspection fraction in concordance with these characteristics and the machine age; Or scenario (2), which adapts the inspection effort to the quality level itself (i.e., inspected fraction as a function of the percentage of defects). The second one could have the advantage of tailoring the inspection to direct cause but has the drawback that in the real world, one could not have access to real-time system inner quality level. This real-time level could never be known before the inspection! While the first one estimates the quality level from parameters  $\lambda_q$  and  $\gamma_q$  and the machine age and it has the advantage of being easy to implement as explained in the managerial insights section (section 9). In the present study, we use scenario (1) and, in the present study. In section 6.2, we will compare the performances of the two scenarios.

Undeniably, too much inspection is costly and less inspection leads to a low average outgoing quality AOQ. Then we should target the right effort or frequency of inspection. We propose: i) To penalize the inspection effort and the AOQ in the cost function, by including in our model the cost of the inspection, the cost incurred if a part is rejected, and a high cost for any bad part sent to the customer. ii) To limit the AOQ by imposing the AOQL threshold. iii) To shape the function representing the inspected fraction according to the quality degradation. From equation (5), we build equation (7) by keeping the same shape of the quality degradation, allowing the inspection fraction to vary from 0 and go to 100%, and changing the scale factor from  $(\lambda_q)$  to  $(Is \cdot \lambda_q)$ , where  $Is$  defines the inspection severity factor. If  $Is = 0$ , there is no inspection, but if  $Is = \infty$ , all produced items are inspected (100% inspection). Fig. 5 plots the inspected fraction  $f(a)$  at age  $a$  for different values of  $Is$ . We plot, also, the quality level to show the relation between the quality degradation and the inspected fraction. The effect of the inspection severity factor  $Is$  is shown in this figure.

$$f(a) = (1 - \exp(- (Is \cdot \lambda_q) \cdot a^{\gamma_q})) \quad (7)$$



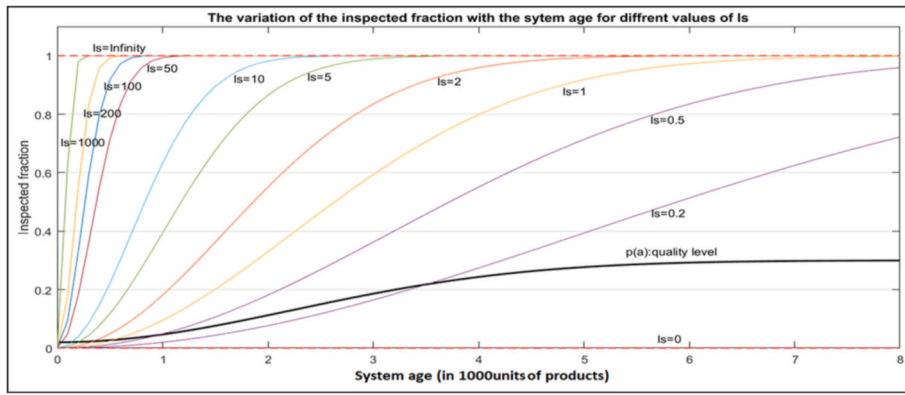


Fig. 5. Effect of the  $I_s$ , on the inspected fraction.

The expressed inspection fraction  $f$  (Eq. (7)) has many advantages: i) It allows us to implement our dynamic sampling plan. ii) It makes the model easy to optimize, there is only one parameter  $I_s$  to tune to find the right plan. iii) It allows to generalize the proposed model by implementing extreme cases: the no-inspection and the 100% inspection cases.

The AOQL constraint is mandatory because the defective products that have not been inspected will be sold to consumers. To define this constraint, we compute the long-run average proportion of defective products delivered to consumers, called the long-run Average Outgoing Quality (AOQ $_{\infty}$ ), and ensure that the AOQL limit is fully satisfied.

Consequently, the quality control policy is defined by the inspection severity factor  $I_s$ .

#### 4.5. The mathematical model

Based on the previous sub-sections, we develop now the mathematical model, starting with the cycle length, then the KPIs, and finally

expressing the objective function (the total incurred cost) and the constraints.

##### 4.5.1. The cycle length

In Fig. 6, we have an illustration of the cycle length, buffer profile, and lost sales. The cycle starts with a new system and an empty buffer and ends when the system is repaired (correctively or preventively) and the buffer is empty. In Figure (6-a), the system repair ends while the buffer is not empty ( $t_r \leq \frac{s(A_f)}{d}$  or  $t_m \leq \frac{s(M)}{d}$ ); the cycle will last until the buffer level is zero. Then the cycle length is  $t_f$  or  $t_M$  plus the time it takes to empty the buffer; the buffer level is, all the time, positive, there is no shortage and, so, no lost sales. In Figure (6-b), the system repair ends after the buffer is empty ( $t_r \geq \frac{s(A_f)}{d}$  or  $t_m \geq \frac{s(M)}{d}$ ); the production system undergoes a shortage state. Then the cycle length is equal to  $t_f$  or  $t_M$  in addition to the time required to carry out the maintenance action. In both cases, the profile of the surplus  $s^+$ , is the same.

From these observations, we compute the length of the cycle for a given  $t_r$  (or  $t_m$ ).

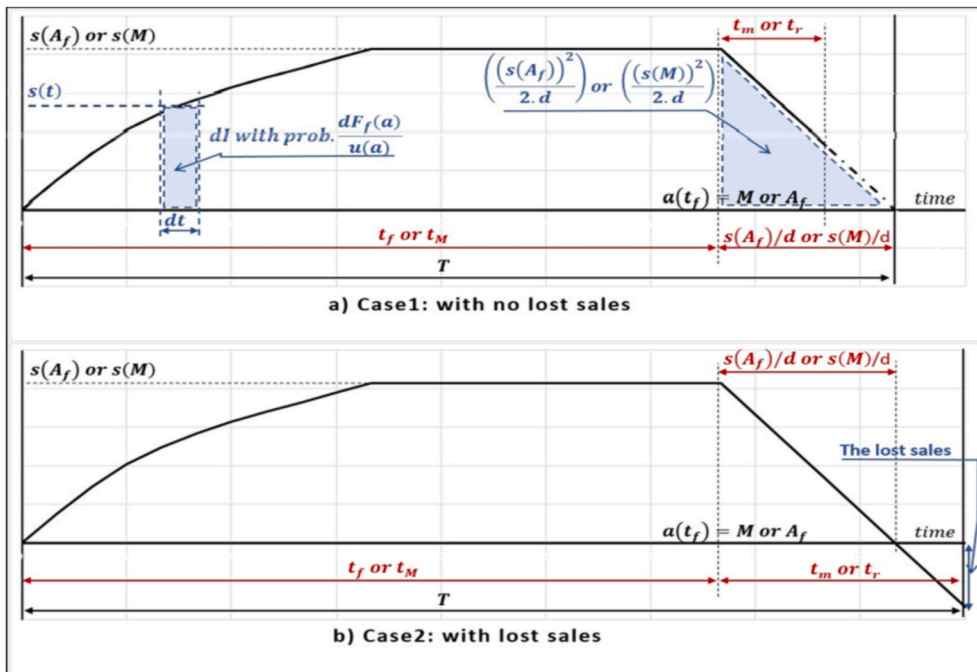


Fig. 6. Buffer profile and cycle illustration for cases (a) with no lost sales and (b) with lost sales.

$$\begin{cases} \text{case1} : T = \int_0^M \left( t_f + \frac{s(a)}{d} \right) dF_f(a) + \left( t_M + \frac{s(M)}{d} \right) (1 - F_f(M)) \\ \text{case2} : T = \int_0^M (t_f + t_r) dF_f(a) + (t_M + t_m) (1 - F_f(M)) \end{cases} \quad (8)$$

For case 1, the cycle length is also the total time required to sold or consume all produced and inspected items to the customer according to the demand rate,  $d$ . For case 2, we have to add an extra time

and the total inventory, for the part (the triangle) after failure, in Fig. 6 for case 1 and case 2,  $\Delta I$  is given by:

$$\Delta I = \frac{(s(A_f))^2}{2d} \mathbb{1}_{\{A_f < M\}} + \frac{(s(M))^2}{2d} \mathbb{1}_{\{A_f \geq M\}} \quad (13)$$

where:  $\mathbb{1}_{\{Assert\}} = \begin{cases} 1 & \text{if } Assert \text{ is true} \\ 0 & \text{otherwise} \end{cases}$

Finally, the average total inventory is the integral of equation (12) on the interval  $[0, t_f]$ , plus  $\Delta I$ , in equation (13), for a given  $t_f$  (i.e.,  $A_f$ ):

$$\begin{aligned} I = & \int_0^M \left( \left[ \int_0^{A_z} \left( Q(x) - \frac{d}{U_{max}} x \right) \frac{dx}{U_{max}} + Z \int_{A_z}^a \left( \frac{(1-f(x)p(x))}{d} \right) dx \right] \mathbb{1}_{\{a \geq A_z\}} + \left[ \int_0^a \left( Q(x) - \frac{d}{U_{max}} x \right) \frac{dx}{U_{max}} + \frac{(s(a))^2}{2d} \right] \mathbb{1}_{\{a < A_z\}} \right) dF_f(a) \\ & + \left( \left[ \int_0^{A_z} \left( Q(x) - \frac{d}{U_{max}} x \right) \frac{dx}{U_{max}} + Z \int_{A_z}^M \left( \frac{(1-f(x)p(x))}{d} \right) dx \right] \mathbb{1}_{\{M \geq A_z\}} + \left[ \int_0^M \left( Q(x) - \frac{d}{U_{max}} x \right) \frac{dx}{U_{max}} + \frac{(s(M))^2}{2d} \right] \mathbb{1}_{\{M < A_z\}} \right) (1 - F_f(M)) \end{aligned} \quad (14)$$

$((t_r - s(A_f)/d)$  or  $(t_m - s(M)/d)$ ) corresponding to the period during which the buffer is not fed (the lost sales period). The cycle length is then:

$$\begin{cases} \text{case1} : T = \frac{1}{d} \left[ \int_0^M Q(a) dF_f(a) + Q(M) (1 - F_f(M)) \right] \\ \text{case2} : T = \left[ \int_0^M \left( \frac{Q(a) - s(a)}{d} + t_r \right) dF_f(a) + \left( \frac{Q(M) - s(M)}{d} + t_m \right) (1 - F_f(M)) \right] \end{cases} \quad (9)$$

For a specific age  $a$  of the system, the probability of occurrence of case 1 is  $F_r\left(\frac{s(a)}{d}\right)$ , or  $F_m\left(\frac{s(a)}{d}\right)$  and for case 2, and any given  $t_r$ , or  $t_m$ , the probability is  $dF_r\left(t \mid t \geq \frac{s(a)}{d}\right)$ , or  $dF_m\left(t \mid t \geq \frac{s(a)}{d}\right)$ . Accordingly, the average cycle length is given by equation (10).

$$\begin{aligned} T = & \int_0^M \left( \frac{Q(a) - s(a) \left(1 - F_r\left(\frac{s(a)}{d}\right)\right)}{d} + MTTR - M_{1,r} \left(\frac{s(a)}{d}\right) \right) dF_f(a) \\ & + \left( \frac{Q(M) - s(M) \left(1 - F_m\left(\frac{s(M)}{d}\right)\right)}{d} + MTM - M_{1,m} \left(\frac{s(M)}{d}\right) \right) (1 - F_f(M)) \end{aligned} \quad (10)$$

#### 4.5.2. The total inventory and the lost sales

From the problem description and the chosen policies, the buffer level could be expressed in time as in equation (11). This level,  $s(t)$ , is a piecewise function with three pieces: Piece 1 correspond to the safety stock building phase, it is equal to the in-going quantity,  $Q(a(t))$ , minus the out-going demand. Piece 2 is equal to  $Z$ , it corresponds to the production at demand phase. Finally, piece 3 represents the phase at which the machine is not producing.

$$s(t) = \begin{cases} Q(a(t)) - d.t & \text{if } a(t) \leq \min(A_f, A_z) \\ Z & \text{if } A_z \leq a(t) \leq \min(A_f, M) \\ Z - d.t & \text{if } a(t) \geq \max(\min(A_f, M), A_z) \end{cases} \quad (11)$$

With  $A_z$  is the solution to equation (3)

Hence, from Fig. 6, the total inventory during  $dt$  for  $t \in [0, t_f]$  : is given by

$$dI = s(t).dt. \quad (12)$$

Now let's compute the lost sales. From Fig. 6, we have lost sales only in case 2, and in such case their value, for a given  $A_f$ , is:  $(d.t_r - s(A_f)) \mathbb{1}_{\{A_f < M\}} + (d.t_m - s(M)) \mathbb{1}_{\{A_f \geq M\}}$ .

Then the average lost sales are:

$$\begin{aligned} L = & \int_0^M \left( d \left( MTTR - M_{1,r} \left(\frac{s(a)}{d}\right) \right) - s(a) \left( 1 - F_r \left(\frac{s(a)}{d}\right) \right) \right) dF_f(a) \\ & + \left( d \left( MTM - M_{1,m} \left(\frac{s(M)}{d}\right) \right) - s(M) \left( 1 - F_m \left(\frac{s(M)}{d}\right) \right) \right) (1 - F_f(M)) \end{aligned} \quad (15)$$

#### 4.5.3. The other KPIs

- The number of total parts and good parts produced per cycle: The average total produced items during a given cycle is given by equation (16). It is equal to  $M$  if there was no failure during the cycle. The average total good parts are given by equation (17). It is expressed through the function  $Q_g(a) = \int_0^a (1 - p(x)) dx$  that computes the amount of good parts produced at a specific age  $a$ . The total defectives in equation (18) are obtained by the difference between total produced and total good parts.

$$TProd = M_{1,f}(M) + M (1 - F_f(M)) \quad (16)$$

$$TGProd = \int_0^M Q_g(a) f_f(a) da + Q_g(M) (1 - F_f(M)) \quad (17)$$

$$\begin{aligned} TBPProd = & M_{1,f}(M) + M (1 - F_f(M)) - \int_0^M Q_g(a) f_f(a) da \\ & - Q_g(M) (1 - F_f(M)) \end{aligned} \quad (18)$$

- The number of total parts and good parts sold per cycle: The average total products, the number of good parts, and the number of bad parts sold to customers during a given cycle are given by equations (19), (19) and (19), respectively.

$$TSold = \int_0^M Q(a) f_f(a) da + Q(M) (1 - F_f(M)) \quad (19)$$

$$TGSold = \int_0^M Q_g(a) f_f(a) da + Q_g(M) (1 - F_f(M)) \quad (20)$$

$$TBSold = \int_0^M (Q(a) - Q_g(a)) f_f(a) da + (Q(M) - Q_g(M)) (1 - F_f(M)) \quad (21)$$

- The average number of preventive and corrective maintenance per cycle: They are respectively the cumulative distribution function (Eq. (22)) and survival function (Eq. (23)) of the reliability distribution.

$$NbPM = 1 - F_f(M) \quad (22)$$

$$NbCM = F_f(M) \quad (23)$$

- The average number of inspections done per cycle:

$$NbInsp = \int_0^M \left( \int_0^a f(x) dx \right) f_f(a) da + (1 - F_f(M)) \int_0^M f(x) dx \quad (24)$$

- The average number of rejected parts per cycle:

$$NbRej = M_1 f_f(M) - M (F_f(M) - 1) - \int_0^M Q(a) f_f(a) da - Q(M) (1 - F_f(M)) \quad (25)$$

- The average inventory per cycle in the inspection shop:

$$InsI = Tins \left( \int_0^M \left( \int_0^a f(x) dx \right) f_f(a) da + (1 - F_f(M)) \int_0^M f(x) dx \right) \quad (26)$$

- The long-run average outgoing quality:

$$AOQ_\infty = \frac{\int_0^M (Q(a) - Q_g(a)) f_f(a) da + (Q(M) - Q_g(M)) (1 - F_f(M))}{\int_0^M Q(a) f_f(a) da + Q(M) (1 - F_f(M))} \quad (27)$$

- The steady-state system availability:

$$Avail = \frac{T - (MTTR \cdot F_f(M) - MTM(F_f(M) - 1))}{T} \quad (28)$$

#### 4.5.4. The total cost

Finally, the total cost per unit of time (Eq. (29)) is the sum of holding cost, lost sales cost, maintenance costs, inspection cost, rejection cost, and the cost of the defective per cycle divided by the cycle length:

$$Cost(Z, M, Is) = \frac{C_h \cdot (I + InsI) + C_l \cdot L}{T} + \frac{C_{pm} \cdot NbPM + C_{cm} \cdot NbCM}{T} + \frac{C_{insp} \cdot NbInsp + C_{rej} \cdot NbRej + C_{def} \cdot TBSold}{T} \quad (29)$$

#### 4.6. Optimization approach

The optimal strategy is defined by three decision variables: the buffer level  $Z$ , the maintenance threshold  $M$ , and the inspection severity factor  $Is$ . To find the optimal strategy, we solve the following optimization

problem:

$$\text{Minimize } Cost(Z, M, Is) \quad (30)$$

$$\text{Subject to : } A_z \leq M \Leftrightarrow Z \leq Q(M) - \frac{d \cdot M}{U_{max}} \quad (31)$$

$$AOQ_\infty \leq AOQL \quad (32)$$

$$(Z, M, Is) \in (\mathbb{R}^+)^3 \quad (33)$$

The objective function  $Cost(Z, M, Is)$  (Eq. (30)) is the total incurred cost by the studied manufacturing system and detailed in equation (29). The first constraint, equation (31), is the link between the maintenance threshold and the safety stock level. As we allow the failures to happen during the safety stock building phase, the optimal maintenance threshold could occur before the buffer is full. In other words, it could be optimal to stop the machine for preventive maintenance during the safety stock building phase. In such a case the maximum level of the safety stock will be the one reached by the maintenance due date. Hence the maximum level of the buffer  $Z$  is always below the buffer level that the machine could reach at the maintenance due date, which is equal to  $Q(M) - \frac{d \cdot M}{U_{max}}$ . We could, also, express such constraint by limiting,  $A_z$ , the age of the machine when the buffer is full (i.e., the buffer level equal  $Z$ ), to  $M$ . The second constraint, equation (32), states that the long-run average outgoing quality should be less than the limit  $AOQL$  to meet the customer expectations in terms of quality level. The third constraint, equation (33), is the type of the decision variables.

### 5. The simulation model and the validation

To validate the proposed analytical model, we compared analytical results with those generated from the simulation. In this paper, we developed a powerful, fast and precise simulation model. Unlike classical simulation models in manufacturing, which use entity-based simulation, we use a new paradigm based on signals. We model the system as signals that flow through different blocks, change their states, and mimic the changes in the real system. This approach is more suitable for a continuous simulation and allows precise results and faster execution time, which helps to do more replications (Ait El Cadi et al., 2016).

#### 5.1. Simulation model

In this section, we will present the simulation model for the studied system. The routine of building a model under Simulink consists of using blocks and connecting them; there are some blocks that one could customize with a customized code (program). The Simulink user guide explains how to build a model inside Simulink (MathWorks, 2001).

Fig. 7 illustrates the simulation model. It includes two parts:

- Part a) represents the up level of the model and has two blocks. The “Manufacturing system” block representing the model of the studied system and the KPI block computing the necessary statistics used for evaluating the system.
- Part b) represents the model beneath the “Manufacturing system” block. In this part, the numbered tags refer to the inputs (1 and 2) and the outputs (1–9) of the block in the upper level. This part summarizes the implemented logic which consists of a series of signals flowing between blocks and representing the system dynamics.

The first block “MachState” broadcast three signals. The first one provides the state of the machine (Up or Down), the second one delivers the actual age of the machine, and the third one states if the machine is under PM or CM action. This block uses the random signal generators to implement the desired distribution for the machine’s lifespan, the PM repair duration, and the CM repair duration. The threshold  $M$ , the

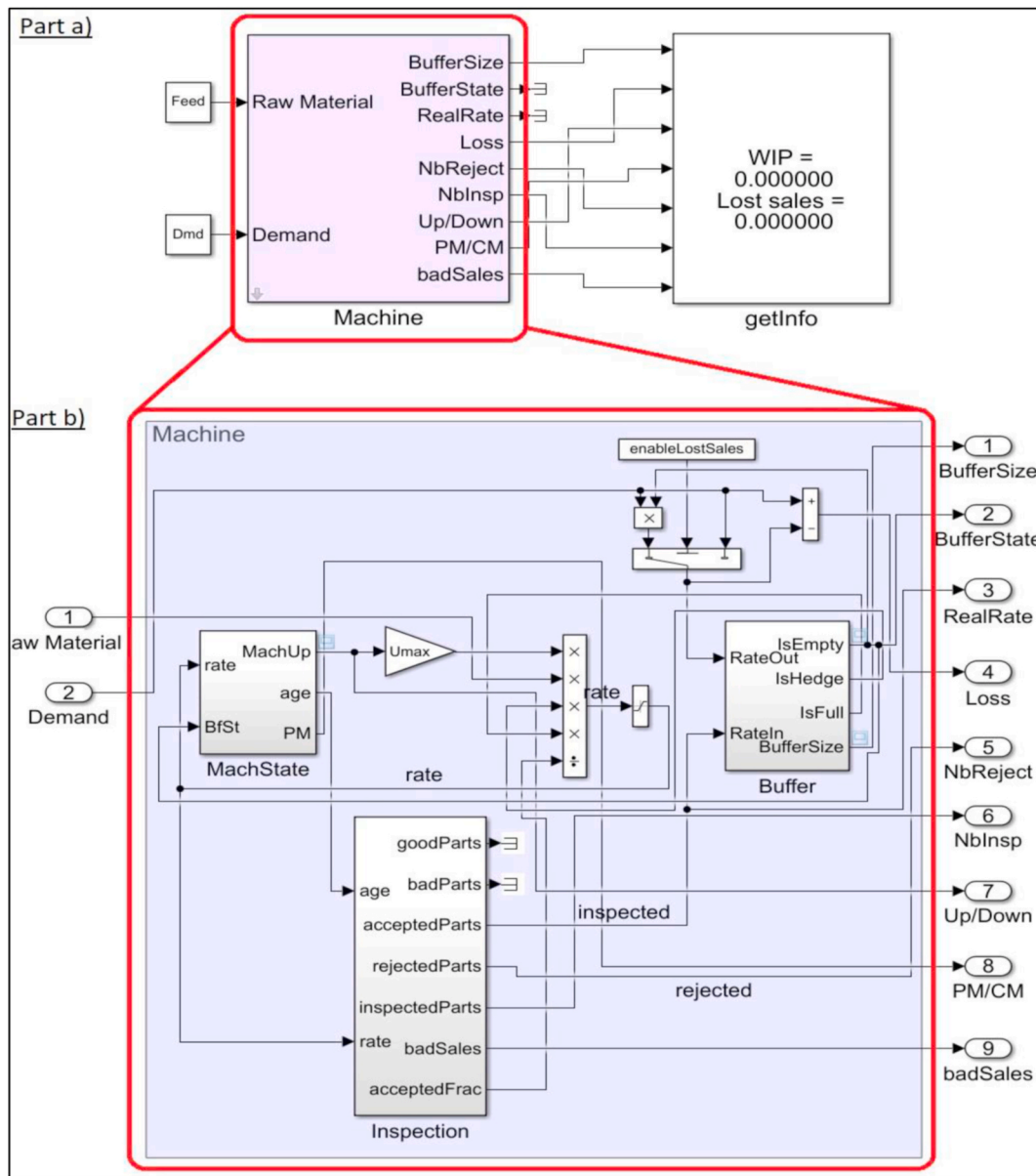


Fig. 7. The simulation model.

number of produced items before PM, indicates to machine block when the PM is due. The signal rate feeds the machine block, which allows the block to compute the age of the machine through usage. The “Buffer” block takes two feeds, RateIn and RateOut signals, and uses an “Integrator” to compute its level, its continuous state; and “Relays” that detects the (discrete) states of buffer and broadcasts them to the other blocks. The “inspection” block takes the production rate and the age of machine signals does the inspection with the right quality level that depends on the age and, then broadcast signals about the number of good parts and bad parts and some statistics about the inspection.

## 5.2. Validation

We run the simulation model in Fig. 7, with the data presented in sub-section 6.1 and with a safety buffer level of  $Z = 40$ , a maintenance threshold  $M = 400$  and an inspection severity factor  $I_s = 4.8594$ .

To validate the simulation model, we plot the main indicators of the system, like machine state and buffer level, and check their accuracy to prove that the simulation is a mimic replication of the studied system.

Fig. 8 is an example of plotted graphics that validate the simulation.

This figure shows the machine state and age, the buffer level, and the quality level change during the simulation. For example, point (1) highlights the occurrence of a preventive maintenance action: as the system age reaches the threshold of 400 items, the machine is stopped and its state is 0 (down for PM) and, at the same time, we notice that the buffer level decreases, no production, and only demand outflow is going. Point (2) shows a case in which the machine is down due to PM action and the buffer is empty, therefore there are some lost sales, the lost sales is not zero but equal to the demand rate. Point (3) displays a case of a failure: The machine is down and its age is less than the maintenance threshold and the buffer level is decreasing. Point (4) highlights how the quality level changes with the system age. We also notice that the inspected fraction is adapted to the quality level; it depends on the system age. These signals from Fig. 8, allow analyzing the evolution of the system performance indices and assessing the simulation validity. The results in this figure were obtained from a numerical instance when the control parameters are set to  $Z = 45$ ,  $M = 400$  and  $I_s = 30$ . The main conclusion from this figure is that simulation reproduces faithfully the

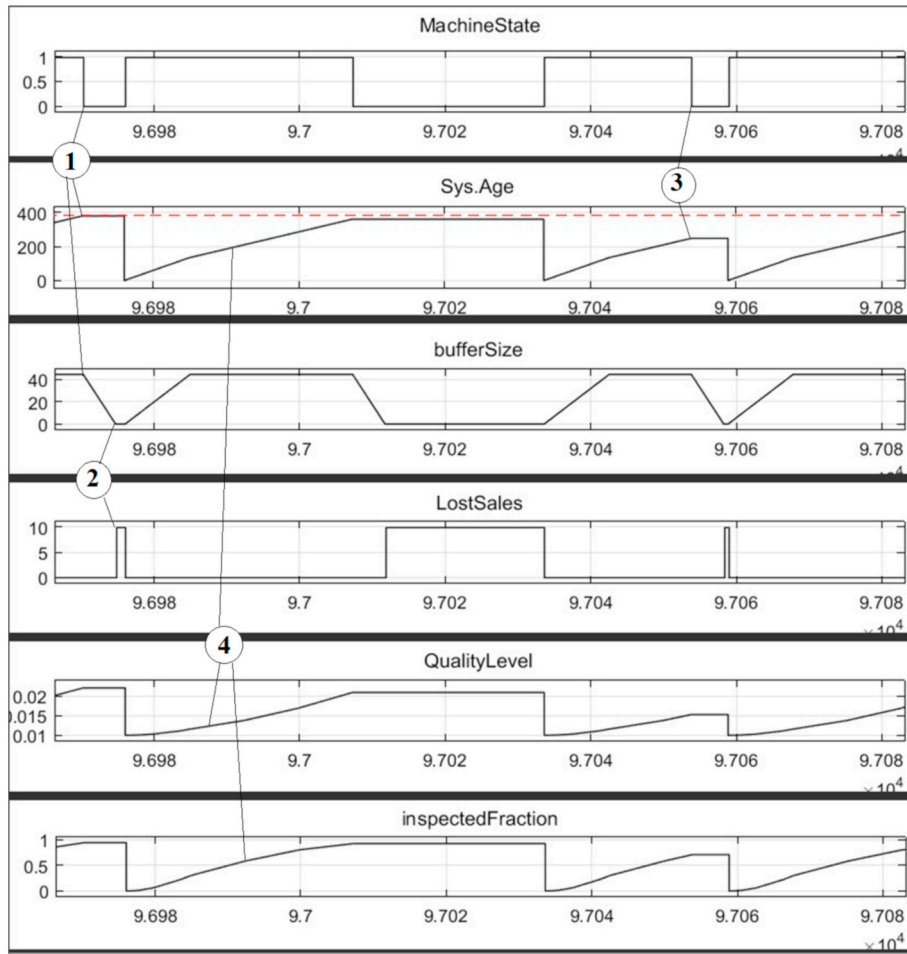


Fig. 8. Simulation Signals: Machine state and age, buffer level, lost sales, quality level and inspected fraction.

behavior of the studied manufacturing system.

## 6. Experiments and numerical results

### 6.1. Experimental data

For the sake of easiness, we will use the mean time between failure as a reference to time's unit and the demand rate for usage's unit. Then, and with no loss of generality, the demand rate is equal to ten items per time unit and the mean usage to failure (MUTF) is equal to 1000 items (i. e., mean time to failure MTF is equal to 100 units of time). These simplifications will help us to state the relations between corrective maintenance duration, preventive maintenance duration, and the time to failure from one side, and from another side the relation between the maximum capacity of the production system and the market demand. The studied case data are listed below:

- The failure density function is following a Weibull distribution

$$f_f(a) = \lambda_f \cdot \gamma_f \cdot a^{(\gamma_f - 1)} \exp(-\lambda_f \cdot a^{\gamma_f})$$

$$\text{with } \lambda_f = \frac{\pi}{4} \cdot 10^{-6} \text{ and } \gamma_f = 2.0$$

The failure rate is increasing with the machine age.

- The product quality, the proportion of defect, depends on the system age. It increases with the age following equation (5) with parameters  $\lambda_q = 2\pi \cdot 10^{-6}$ ,  $\gamma_q = 2.0$ ,  $p_0 = 0.02$ , and  $\eta = 0.28$ .

The defect proportion is chosen such that: (1) When the machine is new, the proportion is 2%; (2) When the machine is somehow aged (a usage superior to MUTF) the proportion is around 30%.

- The preventive maintenance duration has an exponential distribution with a mean of  $MTTM = 2.0$ .
- The corrective maintenance duration has an exponential distribution with a mean of  $MTTR = 4.0$ .
- The other parameters are given in Table 2.

The manufacturing machine can produce at a maximum rate  $U_{max} = 15$  which is 50% above the market demand ( $d = 10$ ). The mean time to maintain (MTTM) and the meantime to repair (MTTR) are, respectively, around 2% and 4% of the MTF (MUTF = 1000 products  $\Rightarrow$  MTF = 100 units of time; MTTM = 2; MTTR = 4).

Table 2  
Experimental parameters.

$U_{max}$	15.0	$d$	10.0	$T_{ins}$	$2.10^{-2}$	AOQL	2.5%
$C_h$	1.0	$C_l$	120.0	$C_{pm}$	2000.0	$C_{cm}$	4000.0
$C_{ins}$	12.5	$C_{rej}$	12.5	$C_{def}$	100.0		

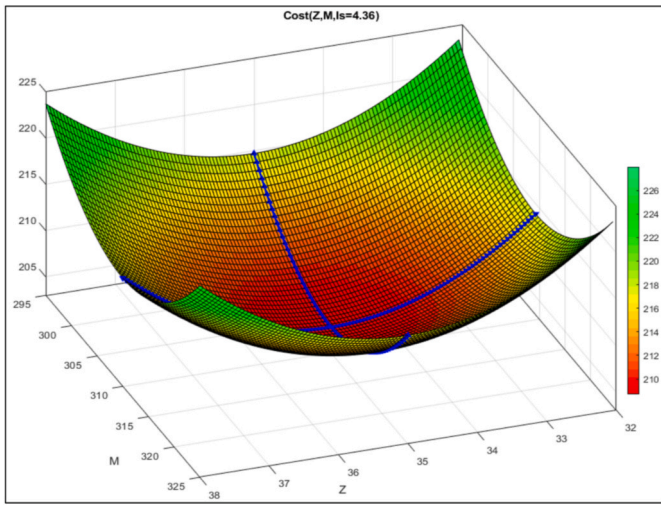


Fig. 9. Cost (Z, M, Is = 4.36).

## 6.2. Optimal control policy

We solved the optimization problem (Eqs. (30)–(33)) with MATLAB and we found the optimal cost:

$$Cost^* = 208.76$$

This optimal solution is realized with the optimal control policy given by:

$$(Z^*, M^*, Is^*) = (35.00 \quad 310.88 \quad 4.36)$$

In Fig. 9, we have the cost surface at the neighborhood of the optimal point solution  $(Z^*, M^*, Is^*)$ . The shape of the surface is convex and shows that we have an optimum point.

Table 3, presents the KPIs of the studied system at the optimal policy obtained by solving the optimization problem (Eqs. (30)–(33)). It shows the optimal buffer level ( $Z^*$ ), the optimal preventive maintenance threshold ( $M^*$ ), the optimal inspection severity factor ( $Is^*$ ), the average cycle length, the average storage (WIP), the lost sales, the number of preventive and corrective maintenance actions, the number of inspected, rejected, and defect items per time unit, and the long-run averages of outgoing quality, percentage of defects and inspected fraction. At the end of this table, we have the optimal total cost and its confidence interval at a 95% level.

The analytical results presented in Table 3 are validated through simulation. We simulated the system at the optimal point for 100 000 units of time (~25 000 cycles and 1 000 000 produced items) with ten replications, and then we computed the confidence interval at a 95%

Table 3  
KPI values at the optimal solution.

Optimal Storage level: $Z^*$	34.9997
Optimal maintenance Threshold: $M^*$	310.8796
Optimal inspection severity: $Is^*$	4.3554
Cycle length	29.4686
Average WIP	28.3536
Lost sales per unit of time	0.1525
Number of PM per unit of time	0.0315
number of CM per unit of time	0.0025
Number of Inspected items per unit of time	4.7319
Number of rejected items per unit of time	0.4411
Number of defect items per unit of time	0.2462
$AOQ_\infty$	2.50%
$\bar{p}$	6.68%
$\bar{IF}$	45.99%
Optimal total $cost^*$ per unit of time	208.7638
Confidence Interval at 95% on the total cost	[207.97, 209.18]

Table 4

Comparison of inspection models based on the quality degradation.

SCENARIOS	OPTIMAL SOLUTION			
	$Z^*$	$M^*$	$Is^*$	$Cost^*$
(1) INSPECTION LEVEL BASED THE MACHINE AGE	35.00	310.88	4.36	208.76
(2) INSPECTION LEVEL BASED DIRECTLY ON THE QUALITY LEVEL	29.73	367.26	6.34	210.08

level. The last line of Table 3 indicates the confidence interval concerning the total incurred cost and shows that the analytical value of the optimal cost is within the confidence interval.

To check that our model based on dynamic inspection fraction (Eq. (7)) reflects well the dependency with the quality degradation, we compare the results of the two scenarios discussed in section 4.4: scenario (1), our approach that captures the quality degradation characteristics and shapes the inspection fraction in concordance with these characteristics and the machine age; And scenario (2), which adapt the inspection effort directly to the quality level. As the real inner quality could not be known in advance, we use a simulation model for scenario (2), and capture in real-time the quality level,  $p$ , and use it to compute the fraction to inspect as  $f = Is \times p$ . Scenario (1) is our base case shown in Table 3. The data used to compare the two scenarios are the same as those in section 6.1.

From Table 4, we notice: we were able in the two scenarios to find the optimal solution and the corresponding cost. We find that the two solutions are different: More safety stock, More maintenance, and less inspection in scenario (1) than (2). But, scenario (1) leads to a less expensive solution than scenario (2) (208.8 vs. 210). The difference in the cost is very low (0.63%) and it could be explained by the fact that in scenario (1) the joint design of the three policies works better because it is based on the main factor that explains the degradations, the machine age; thus, the system could use adequately the maintenance and the safety level to cope with the quality degradation. In conclusion, both scenarios are valid and lead to almost the same total cost. Thus, we will continue with scenario (1) in the rest of the study because it does not require knowing the inner quality in real-time but only the parameters that summarize the quality level, parameters that could be fitted from historical data.

## 7. Sensitivity analysis

To understand how the input data affect the optimal solution and to get additional insights on the results, a set of experiments is derived from the basic case by changing the costs' inputs and the system parameters (quality and reliability). The objective behind the sensitivity analysis, besides demonstrating the quality of the proposed mathematical model and proving its effectiveness, is to investigate these effects on the optimal control settings and to study important aspects related to the interrelations between production, quality, and maintenance control settings. Tables 5–7 summarize the results for the changes in costs' inputs, in system reliability parameters, and in quality level, respectively. The three tables are organized as follows: The first column "Case" shows the case number. Column 2 (Name) and 3 (Variation) give the considered parameter and the changes in its value compared to the base case. Columns 4 to 6 exhibit the optimal values of decision variables ( $Z^*$ ,  $M^*$ ,  $Is^*$ ). Column 7 displays the value of the optimal incurred cost. Columns 8 and 9 show the quality indices which are respectively, the average inspected fraction  $\bar{IF}$ , and the average proportion of defects  $\bar{p}$  (the average inner quality). For Table 7, columns 10 and 11 give also the maximum level of quality deterioration that the system experienced ( $p_{max}^{obs}$ ), and the maximum allowed quality level ( $p_{max}$ ), respectively. The last 6 columns draw the effects of the parameters' changes on the optimal control parameters ( $Z^*$ ,  $M^*$ ,  $Is^*$ ), the optimal incurred cost, and

**Table 5**  
Sensitivity analysis, the influence of cost's elements.

CASE	COST'S ELEMENT		DECISION VARIABLES				QUALITY INDICES		CHANGES					
	Name	Variation	$Z^*$	$M^*$	$I_s^*$	$Cost^*$	$\bar{IF}$	$\bar{p}$	$Z^*$	$M^*$	$I_s^*$	$Cost^*$	$\bar{IF}$	$\bar{p}$
Base	-	-	35.00	310.88	4.36	208.764	45.99%	6.68%						
1	<i>Ch</i>	-10%	75.50	226.49	5.05	206.305	35.17%	4.71%	↑	↓	↑	↓	↓	↓
2	<i>Ch</i>	+10%	30.86	321.24	4.27	211.467	46.99%	6.94%	↓	↑	↓	↑	↑	↑
3	<i>Cl</i>	-10%	29.96	321.65	4.27	206.746	47.03%	6.95%	↓	↑	↓	↓	↑	↑
4	<i>Cl</i>	+10%	74.33	222.99	5.07	210.556	34.59%	4.64%	↑	↓	↑	↑	↓	↓
5	<i>Cpm</i>	-10%	69.89	209.66	5.15	200.809	32.24%	4.36%	↑	↓	↑	↓	↓	↓
6	<i>Cpm</i>	+10%	31.40	344.76	4.08	214.713	49.07%	7.52%	↓	↑	↓	↑	↑	↑
7	<i>Ccm</i>	-10%	34.34	316.90	4.31	207.762	46.58%	6.83%	↓	↑	↓	↓	↑	↑
8	<i>Ccm</i>	+10%	35.67	305.09	4.40	209.746	45.41%	6.54%	↑	↓	↑	↑	↓	↓
9	<i>Cins</i>	-10%	32.63	335.21	4.16	202.683	48.26%	7.28%	↓	↑	↓	↓	↑	↑
10	<i>Cins</i>	+10%	71.61	214.83	5.12	214.298	33.18%	4.46%	↑	↓	↑	↑	↓	↓
11	<i>Crej</i>	-10%	34.32	317.02	4.30	208.202	46.59%	6.83%	↓	↑	↓	↓	↑	↑
12	<i>Crej</i>	+10%	35.69	305.04	4.40	209.305	45.40%	6.54%	↑	↓	↑	↑	↓	↓
13	<i>Cdef</i>	-10%	35.10	310.65	4.36	206.302	45.97%	6.67%	↑	↓	↑	↓	↓	↓
14	<i>Cdef</i>	+10%	34.90	311.11	4.35	211.226	46.01%	6.69%	↓	↑	↓	↑	↑	↑

**Table 6**  
Sensitivity analysis, the influence of system's reliability parameters.

CASE	RELIABILITY PARAMETER		DECISION VARIABLES				QUALITY INDICES		CHANGES					
	Name	Variation	$Z^*$	$M^*$	$I_s^*$	$Cost^*$	$\bar{IF}$	$\bar{p}$	$Z^*$	$M^*$	$I_s^*$	$Cost^*$	$\bar{IF}$	$\bar{p}$
BASE	-	-	35.00	310.88	4.36	208.76	45.99%	6.68%						
15	$\lambda_f$	-20%	33.31	321.23	4.27	207.15	47.11%	6.95%	↓	↑	↓	↓	↑	↑
16	$\lambda_f$	+20%	73.45	220.36	5.09	211.46	34.07%	4.57%	↑	↓	↑	↑	↓	↓
17	$\gamma_f$	-10%	27.77	363.32	3.93	201.91	51.11%	8.09%	↓	↑	↓	↓	↑	↑
18	$\gamma_f$	+10%	69.96	209.88	5.14	221.20	31.77%	4.32%	↑	↓	↑	↑	↓	↓

**Table 7**  
Sensitivity analysis, the influence of system's quality parameters.

CASE	QUALITY PARAMETER		DECISION VARIABLES				QUALITY INDICES				CHANGES					
	Name	Variation	$Z^*$	$M^*$	$I_s^*$	$Cost^*$	$\bar{IF}$	$\bar{p}$	$p_{max}^{obs}$	$p_{max}$	$Z^*$	$M^*$	$I_s^*$	$Cost^*$	$\bar{IF}$	$\bar{p}$
BASE	-	-	35.00	310.88	4.36	208.76	45.99%	6.68%	14,7%	30%						
19	$\eta$	-50%	29.24	367.06	2.22	184.40	37.91%	5.04%	10,0%	16%	↓	↑	↓	↓	↓	↓
20	$\eta$	+50%	72.26	216.79	7.30	224.18	41.62	5.76%	12,7%	44%	↑	↓	↑	↑	↓	↓
21	$\eta$	+100%	70.55	211.64	9.30	234.97	46.12%	6.79%	15,7%	58%	↑	↓	↑	↑	↑	↑
22	$\eta$	+200%	67.00	201.01	13.01	251.86	51.43%	8.54%	20,8%	86%	↑	↓	↑	↑	↑	↑
23	$\eta$	+250%	65.32	195.97	14.79	258.98	53.16%	9.29%	23,0%	100%	↑	↓	↑	↑	↑	↑

**Table 8**  
The optimal results for different quality degradation shapes.

CASE	QUALITY PARAMETER		DECISION VARIABLES			
	Name	Variant	$Z^*$	$M^*$	$I_s^*$	$Cost^*$
BASE	Quality shape	"S" curve	35.00	310.88	4.36	208.76
24		linear	28.22	393.73	6.35	232.24
25		Exponential	28.25	388.66	5.10	221.29
26		polynomial	27.67	394.83	4.47	215.60

the quality indices.

7.1. Influence of cost parameters

The changes in the cost parameters affect the solution and the system's performance. Here is, below, for each cost parameter the changes explained and assessed. We have, in Table 5, 14 cases that go by a couple: the case with an odd (even) number corresponds to a decrease (increase) in the parameter.

- **Variation of the inventory and lost-sales costs (cases 1 to 4):** These two costs have opposites effects on the system. The effect on increasing the inventory cost *Ch* leads to the same effect of decreasing the lost-sales cost *Cl* and vice versa. We will present hereafter only the variation of *Ch*. When the inventory cost *Ch* decreases (case 1), the model reacts by increasing the safety stock level  $Z^*$  because inventory holding is less costly. As a consequence, the system produces at a higher rate implying more quality and reliability degradation. Then the system needs to conduct more maintenance actions, by decreasing  $M^*$ , and more inspection, by increasing its severity  $I_s^*$ . For case 2, where *Ch* increases, we note the opposite effects. Cases 3 and 4 produce, respectively, the opposite effects of cases 1 and 2.
- **Variation of the maintenance cost cases (5 to 8):** When the preventive maintenance cost *Cpm* decreases (case 5), it is normal to observe that  $M^*$  decreases, the system will do more preventive maintenance as it costs less. On the opposite, the safety stock level  $Z^*$  increases, this is, because doing more stops for the preventive maintenance leads to less availability and a high risk of shortage. Then to fulfill the demand as much as possible we should increase the stock. As a result, the production rate is higher and the quality degrades faster which

imposes to the system to increase the inspection severity  $I_s^*$ . For case 6, where  $C_{pm}$  increases, we note the opposite effects. The changes in the corrective maintenance cost  $C_{cm}$  (cases 7 and 8) lead, respectively to an opposite effect of preventive maintenance cost  $C_{pm}$  (cases 5 and 6).

- **Variation of the inspection, rejection, and defective costs (cases 9 to 14):** When the inspection cost  $C_{ins}$  decreases (case 9), the inspection costs less; we afford to do more inspection,  $\bar{I}$  the average inspected fraction increases. This does not mean necessary to increase  $I_s^*$ . Indeed, by inspecting more we do not need to protect the system from degradation and we notice an increase of  $M^*$ , the preventive maintenance threshold. Which leads to longer cycles and smaller  $I_s^*$  we achieve more inspections. Further, the safety stock level  $Z^*$  is reduced, because as the quality is good, we need less storage. Here also, case 10, with an increased inspection cost, has the inverse effects.

The changes in the rejection cost  $C_{rej}$  (case 11, and 12) have the same effects like the changes in the inspection cost  $C_{ins}$ , while the changes in the defectives cost  $C_{def}$  (case 13 and 14) have an opposite effect. Indeed both  $C_{rej}$  and  $C_{ins}$  are part of the inspection cost and when this cost is low the system tends to do more inspection. Further, when  $C_{def}$  decreases, bad quality has less effect on the customer so we do not need to inspect more. The inspection severity  $I_s^*$  adapts to the cycle length to achieve less inspection.

### 7.2. Influence of system reliability parameters

The changes in the system parameters affect the solution and the system performance. We have, in Table 6, four cases that go by a couple: the case with an odd (even) number corresponds to a decrease (increase) in the parameter.

The decrease of the scale parameter  $\lambda_f$  (case 15) means a decrease in the failure rate. Then, as the failure rate decreases, the system is more reliable and then reacts by increasing  $M^*$ , the preventive maintenance threshold, to do less maintenance, decreasing  $Z^*$ , the safety stock level, and decreasing the inspection severity factor  $I_s^*$ . Indeed, as the system is more reliable we need less control. The case 16, where the scale parameter increases (the failure rate increase), we have the opposite effect. The changes in the shape parameter  $\gamma_f$  (cases 17 and 18) have, respectively, the same effects as the changes in the scale parameter  $\lambda_f$ . However, the impact magnitude of the changes in the shape  $\gamma_f$  is higher than the scale  $\lambda_f$ : A change of 20% in  $\lambda_f$  (10% in  $\gamma_f$ ) leads to an average change of 1% in the total cost (5% of total cost); Because the scale  $\lambda_f$  controls the failure rate itself meanwhile the shape  $\gamma_f$  controls its derivative.

### 7.3. Influence of system quality parameters

In this subsection, we first study the effect of the quality peak level (cases 19–23) on the proposed policy, control parameters, and the total incurred cost. Then, we discuss and show how our model could handle any quality degradation shape.

- a) Variation of the inner quality peak level  $\eta$ : Table 7 summarizes the effect of the changes in the quality peak level. It shows also the maximum level of quality degradation that the system could experience  $p_{max}^{obs}$  as well as the maximum allowed one  $p_{max}$ .

First, we notice that even if we allow the system to go up to 100% percent of defects, it never reaches it, it stops at 23% maximum. This is due, first, to maintenance actions which will reset the quality and, second, it is not beneficial to produce for making scrap only.

Second, when  $\eta$  decreases (case 19), the system has fewer defects and

then fewer inspections needed, which explains the decrease in inspection severity factor  $I_s^*$ . Also, as a consequence of this high level of quality, we do not need more storage and we could permit more degradation of the system. Hence, the system opts for less maintenance, by increasing the threshold  $M^*$ , and decreases the safety stock level  $Z^*$ . For cases 20 to 23, where  $\eta$  increases, the system uses the three control parameters to cope with quality degradation. It opts for a higher inspection level by increasing  $I_s^*$ , more maintenance by decreasing the threshold  $M^*$ , and more storage by increasing  $Z^*$ . Except for the last case 23, the optimal storage level  $Z^*$  is less than the one found for case 22, because inspection severity and maintenance were enough to cope with the system quality degradation. These observations highlight that the interactions are not linear but more complex.

- b) **The inner quality shapes:** In the baseline model we use an “S” shape for the quality degradation with the age, as is the case in the majority of works in the literature. This could be the case for some manufacturing systems in which, the first stage means the system is new and under control, the second stage represents changes in the system state, which become out of control, and the third stage represents the degradation in the out-of-control state. However, this may not apply to other cases such as tools wear. So, to be impartial in our study and to show that our model could cope with all situation, we consider, as in Table 8, a linear degradation shape (case 24) which has one stage, and the cases with two stages, the exponential shape (case 25) and polynomial shape (case 26).

Fig. 10 summarizes the behavior of all the studied cases. These shapes could be fitted from historical records of the quality level. To use our approach, one could use the shape that fits at best the quality degradation of the manufacturing system under study. The objective of the experiments, shown in Table 8, is to show that the proposed integrated model based on dynamic inspection could adapt to any case. Indeed, for all the cases (24–26), we were able to find the optimal solution and the associated total incurred cost. As done to determine the reliability distribution function, the shape that most describes the system quality degradation is generated from quality historical data. However, the “S” curve could be used to emulate all these cases by tuning its parameters.

## 8. Comparative study: proposed policy vs other policies

To show and assess the contribution of the proposed policy, we compare it with policies considered in the literature and practice involving a fixed fraction, full, or no inspection, and with or without preventive maintenance interventions. The studied policies in this comparative study are:

- Policy-I: The proposed integrated policy considering a dynamic inspection.
- Policy-II: The same production and maintenance policies as Policy-I with a fixed inspected fraction ( $f^*$ ).
- Policy-III: The same production and maintenance policies as Policy-I with a 100% screening strategy for the quality inspection.
- Policy-IV: The same production and maintenance policies as Policy-I with no inspection.
- Policy-V to Policy-VIII: are respectively the same as Policies I to IV but without preventive maintenance interventions.

### 8.1. Comparison with the base case

Table 9 summarizes the eight policies and their respective optimal control parameters. The first column gives the policy name. The three following columns describe the specific production, maintenance, and quality control policies, respectively. The fifth to eight columns present



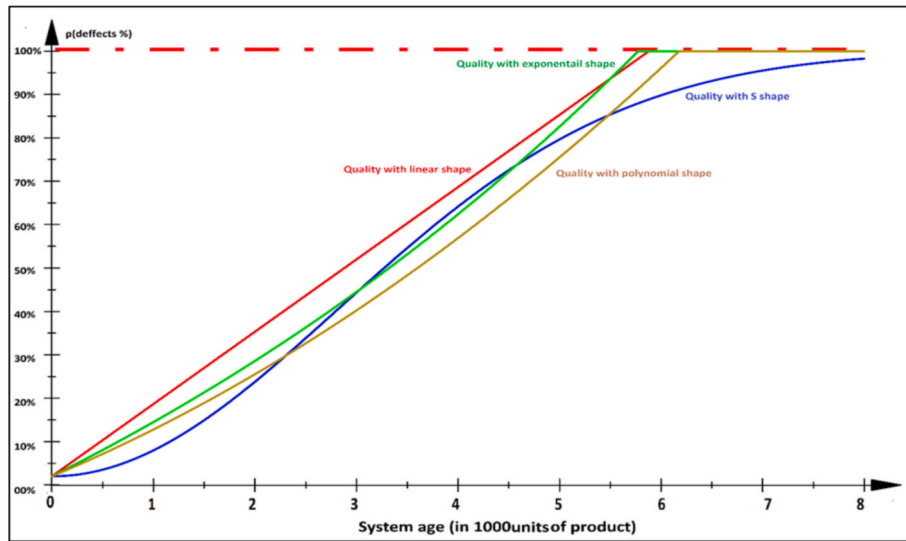


Fig. 10. Different quality degradation shapes.

Table 9  
Comparative study between different policies.

POLICIES	PRODUCTION POLICY	MAINTENANCE POLICY	QUALITY CONTROL POLICY	$Z^*$	$M^*$	$IS^*$	$F^*$ (%)	$\bar{IF}$ (%)	COST* (%)	COST RED. (%)		
PROPOSED POLICY I	HPP	ABPM	Dynamic inspection	35.0	310.9	4.355	-	46	208.76	-		
POLICY II	HPP	ABPM	Static inspection ( $f^*$ )	64.4	193.2	-	39	39	223.82	7.21		
POLICY III	HPP	ABPM	100% inspection	96.4	289.3	-	100	100	271.12	29.87		
POLICY IV	HPP	ABPM	No inspection	31.1	93.2	-	0	0	299.43	43.43		
POLICY V	HPP	No PM	Dynamic inspection	6.7	-	1.637	-	73	261.31	25.17		
POLICY VI	HPP	No PM	Static inspection ( $f^*$ )	5.4	-	-	90	90	286.11	37.05		
POLICY VII	HPP	No PM	100% inspection	6.7	-	-	100	100	286.38	37.18		
POLICY VIII	HPP	No PM	No inspection	No solution that respects the AOQL constraint								

the optimal strategy parameters: the safety stock level ( $Z^*$ ), the maintenance Threshold ( $M^*$ ), the inspection severity ( $IS^*$ ) (for the dynamic inspections) and the fixed inspected fraction ( $f^*$ ) (for the static inspections). The ninth column shows the average inspected fraction  $\bar{IF}$ . The last two columns exhibit, respectively, the total incurred minimal cost and the relative cost reduction (%) compared to the proposed policy (Policy-I).

From Table 9, we could notice that the proposed integrated policy based on dynamic inspection outperforms all other policies. The results show that the dynamic inspection (Policy-I) allows a cost reduction of 7.21%, 29.87%, and 43.43% compared to static inspection (policy-II), 100% inspection (Policy-III), and no inspection (Policy-IV), respectively. Indeed, Policies II, III, and IV do not adapt the inspection effort to the needs. The inspection effort is the same whenever the system is new or old, which does not match with the quality degradation that increases with the system age. This leads to an excessive inspection in the early age of the system and an insufficient inspection effort when the system quality is degraded. For the 100% inspection case, we waste effort and resources throughout the life of the system. However, for static plans, we waste efforts at the beginning, while the quality is good, and we do not do enough screening when the quality is bad. Regarding policy-IV that does not inspect any produced item, one can notice that the system opts for a smaller maintenance threshold. Undeniably, this was the unique way to keep the quality level under AOQL. In conclusion, preventive maintenance resets the quality degradation.

For cases without preventive maintenance (policies V to VIII), we have the same conclusion; policy-V with dynamic inspection outperforms the others. The results show that the dynamic inspection (Policy-V) allows a cost reduction of 9.49% and 9.59% compared to

static inspection (policy-VI) and 100% inspection (Policy-VII), respectively. For policy VIII, without maintenance nor inspection, we were not able to find any solution that meets the AOQL constraint. For policies V to VII, as there is no maintenance, the system decides to keep a small level of safety stock compared to policies I to IV. In this way, it reduces the production speed and then the quality degradation.

Compared to policy V, preventive maintenance integrated into the proposed policy (Policy I) has contributed to improving the overall performance of the manufacturing system by reducing the total incurred cost by 25.17%. Indeed, with no maintenance, the manufacturing system degradation is higher, and specifically the quality degradation; thus, the system produces more defects, requiring more inspections, which increase operations' costs. Effectively, and on average, Policy-V allows inspecting almost 27% more items than in Policy-I ( $\bar{IF}$  equals 73% for Policy-V versus 46% for Policy-I). The same deduction is found when comparing Policy-II to Policy-VI. This shows the importance of maintenance to protect the system against reliability and quality degradations. When there is no maintenance, the system will try to lessen the costs by finding the right trade-off between the safety stock level and the inspection level and even though the costs are too high. In other words, preventive maintenance allows resetting the system age and the quality level.

Also, considering or not preventive maintenance interventions, the proposed dynamic inspection gives good results and adapt to different situations. In fact, we notice that the gain between static and dynamic inspection is higher when there is no maintenance (9.49% without maintenance vs. 7.21% with maintenance). In conclusion, we are confident that our policy could adapt to any maintenance policy and even get the best of it.

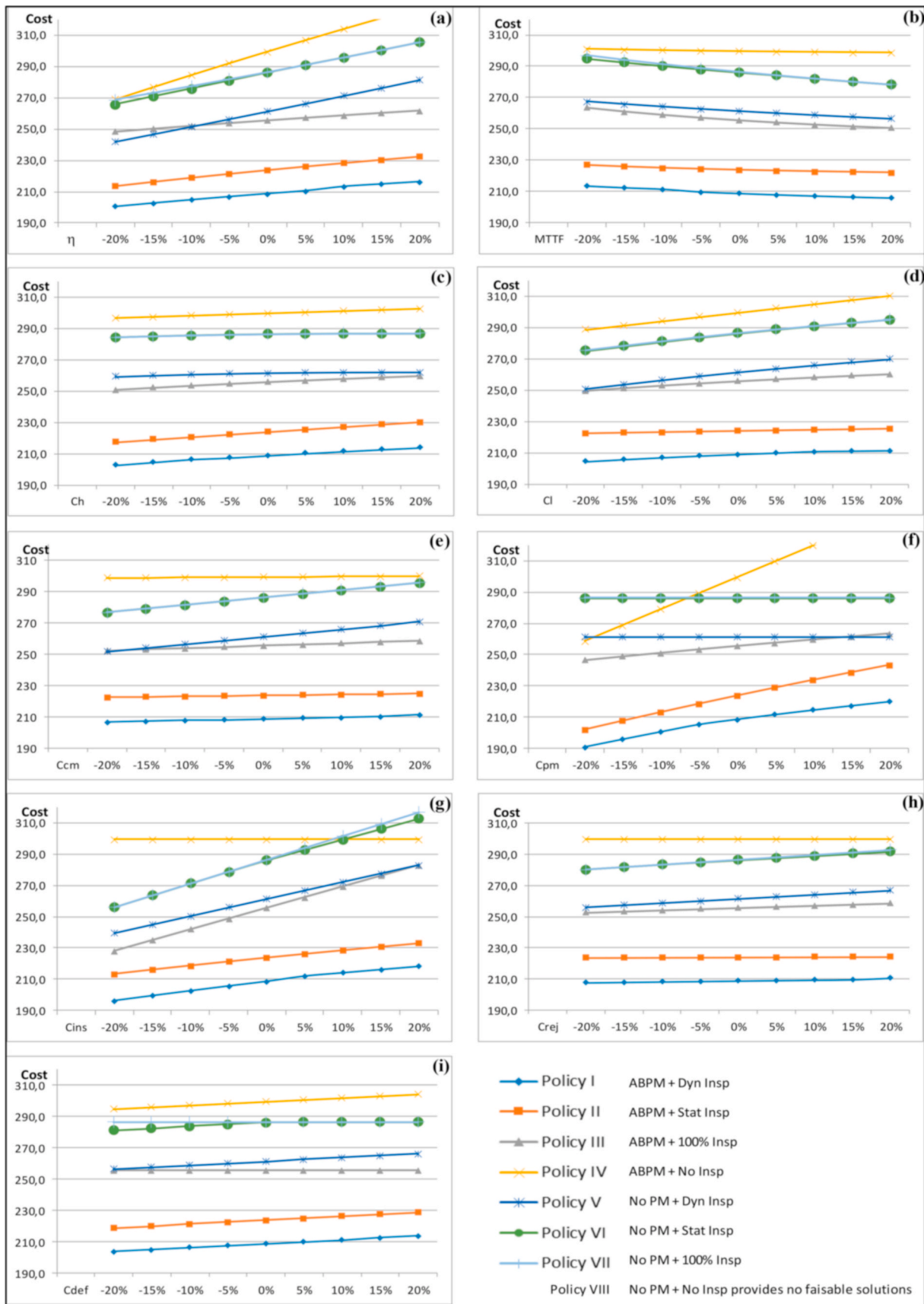


Fig. 11. Comparative analysis of the effects of system costs/parameters on the total cost for the studied policies.

8.2. Exhaustive comparative study

The objective of this subsection is to extend the comparison, of the control policies across a wide range of system parameters derived from the basic case (Table 2). We aim to highlight the evolution of the proposed control policy compared to others and to confirm the results obtained in Table 9. Fig. 11 summarizes the effect of system parameters (sub-figures a-b) and production, maintenance, and quality costs (sub-figures c-i) on the total cost incurred for each studied control policy. Policy-VIII has been discarded because it does not generate a feasible solution that respects the AOQL constraint. Indeed, for a real system subject to degradations of reliability and quality, it would be difficult - if not impossible - to comply with market requirements such, as the limit quality level, without any inspection nor maintenance.

As indicated above, Fig. 11 confirms that the proposed policy (Policy I) outperforms other policies. Indeed, our policy is more inclusive and allows the system to adjust to the requirements of quality and reliability, depending on the state of the latter.

As stated above, Policy I dominates all the others (Lowest total cost) for all scenarios. However, for other policies, the curves intersect in some cases. Also, the gap increases when the system degrades (sub-

figures a and b). Compared to policy II (static inspection), this gap increases also when the costs of PM, storage, and defects increase. We also see in this figure that the total cost, and this for all policies and all scenarios, increases when the cost parameter considered increases (sub-figures c to i) and also when the quality parameter increases (quality degrades) (sub-figure a). On the other hand, for the reliability parameter, the total cost decreases when the latter increases (because reliability improves) (sub-figure b). However, some curves are horizontal, are not affected by certain parameters. This is the case of the policy without an inspection, policy-IV, in sub-figures g and h. It is clear that the costs of inspection and rejection have no effect on this policy and thus, the curves are horizontal.

Also, for the cases of policies without preventive maintenance, policies V to VII, we can see that their curves are horizontal in sub-figure f. Effectively, the cost of preventive maintenance does not affect the total cost for these policies.

In conclusion, the dynamic inspection, embedded within the right production, maintenance, and quality policies, will allow better performance. It reduces the operational costs, permits better use of the assets, and results in a good quality of products. Also, our approach – combining the joint design of production, maintenance, and quality

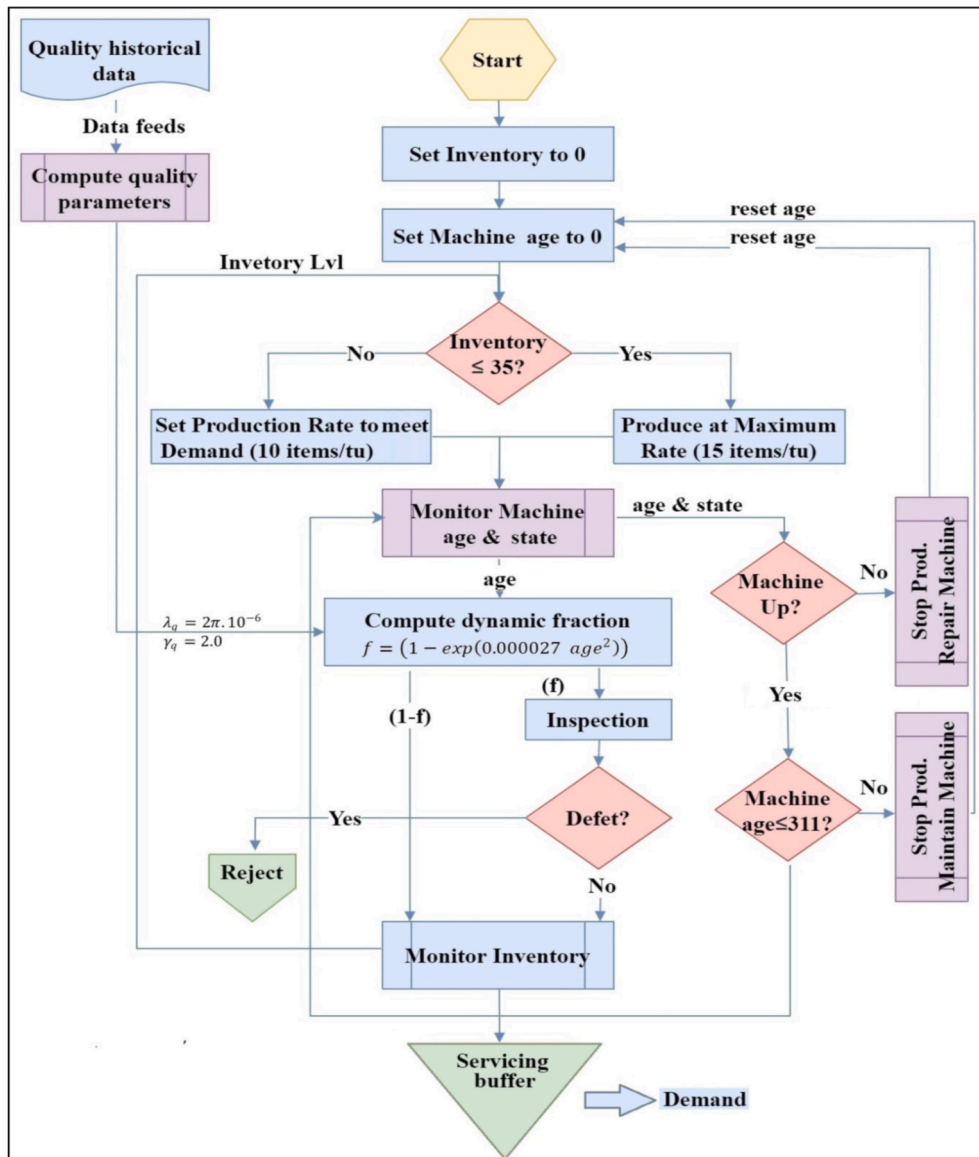


Fig. 12. Implementation flowchart of the proposed integrated policy.

policies with the right inspection strategy – allow the system to choose between maintenance or inspection or both to cope with its degradation efficiently.

## 9. Managerial insight and practical implementation

The proposed integrated production, maintenance, and quality control policy is based on the dynamic inspection principle. It adapts its effort to the system quality degradation. This is done via a simple and easy to implement formulation of the dynamic inspection fraction. This fraction is computed through the quality factors  $\lambda_q$  and  $\gamma_q$ , obtained from historical records, and the inspection severity factor  $I_s$ , optimized with our model. We establish a joint control policy that allows the manager to synchronize maintenance, production, and quality while minimizing the total incurred cost.

Fig. 12 presents a flowchart that guides the process of decision-making for the base case (Table 2). Before the implementation of the control strategy, the manager should collect historical data about the production quality to fit the model presented in equation (5). This will lead to computing the values of  $\lambda_q$  and  $\gamma_q$  and then defines the dynamic fraction to be inspected through the optimal value of  $I_s$  (Eq. (7)). Then, at the beginning, when the machine is new, the manager should reset the counter to set the age of the machine to zero. Then he has to monitor the machine state, the machine age (number of produced parts), and the inventory level. According to the optimal control parameters ( $Z^*$ ,  $M^*$ ,  $I_s^*$ ), if the machine is “Down”, it should be repaired and the manager should reset the age counter. If no failure occurs during the production of  $M^* = 311$  (the PM threshold), the manufacturing system should be stopped and a PM action is carried out. Again, the manager should reset the age counter. During the manufacturing process, a fraction  $f$  of parts is inspected and the defects are rejected. If the inventory level is less than  $Z^* = 35$  (the safety level), the manufacturing system should operate at maximum production rate  $U_{max}(=15$  items/time unit). However, if the safety stock level is reached, the production rate is adjusted to meet the demand rate  $d (=10$  items/time unit).

## 10. Conclusion

In this paper, we propose a new integrated production, maintenance, and quality control policy based on the dynamic inspection principle, which outperforms classical policies. The dynamic inspection adapts its effort to the quality degradation with the age of system. A stochastic mathematical model is developed to optimize, jointly, the production control, the inventory level, the preventive maintenance threshold, and the inspection effort to achieve the lowest operations’ cost. It is a realistic model since it relaxes several simplifying assumptions and takes into account many features of the real-life: no restriction on random distributions, allows failure’s occurrence during the safety stock building phase, duration and cost of inspection are non-negligible, and both reliability and quality are operation-dependent. We also develop a fast and precise simulation model that mimics the dynamic and stochastic behavior of the manufacturing system; unlike classical simulation approaches based on discrete event simulation software, we propose a new signal-based modeling approach using Simulink software.

Simulation experiments and detailed sensitivity analysis have been carried out, and have confirmed the validity and the robustness of the proposed mathematical model.

The results show also, that there are important interactions between production, maintenance, and quality control. Maintenance could mitigate the quality degradation and, by so, leading to less inspection effort. A good inspection strategy could allow the manager to let the system age more before the preventive intervention and without reducing the overall performance.

A comparative study has then been carried out to compare the performance of the proposed integrated policy based on dynamic inspection

to others based on classical inspection strategies. The results show clearly the contribution of preventive maintenance in reducing the total incurred cost. Indeed, the proposed policy allows a cost reduction of 25.17% compared to the same policy without PM. Also, the proposed policy allows a cost reduction of 7.21%, 29.87%, and 43.43% compared to those based on optimal static inspection, 100% inspection, and no inspection quality control strategies, respectively. An extended comparative study has confirmed the superiority of the proposed integrated policy based on dynamic inspection against those based on classical inspection strategies; this concludes that the sampling plan for inspection should not be the same at different stages of the manufacturing system life. We also provide a flowchart explaining how the manager should proceed to implement the optimal integrated policy and to adapt the inspection level according to the system degradation. The implementation of our approach could adapt to any manufacturing system. We need only, reliability and quality parameters (which could be fitted from historical data) to design the right controls with three simple main decision variables ( $Z^*$ ,  $M^*$ ,  $I_s^*$ ). Then, following the steps in the flowchart, the manager could operate the system at its optimal level by adapting the inspection to the system degradation.

In this study, we consider the case where the inspection process is perfect. Future extensions of this paper can be envisioned to integrate the imperfect inspection process. The proposed model considers an Age-Based Preventive Maintenance (ABPM). We could investigate more flexible strategies with their interaction with the dynamic inspection; for example, condition-based maintenance, when the inspection detects a certain level of quality, we call a preventive maintenance action. We are confident that our model could help in selecting the best maintenance policy, for example, a proactive one. Also, further research can be conducted to extend this model to larger manufacturing systems with multiple products and multiple machines.

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