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# On Nonlinear Control for Lane Keeping Assist System in Steer-By-Wire Road Wheeled Vehicles<sup>\*</sup>

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**Abstract:** This paper deals with the design of a lane keeping system in steer-by-wire road four-wheeled vehicles using a nonlinear controller. The controller is designed to take into account the driver's behavior and availability and to control the wheels' angle of the steer system. A high order sliding mode control is chosen as feedback controller, having time-varying upper-bound disturbance strictly depended by the road curvature and by the influence of the wind, which allows an easier tuning process of the parameters based on a physical meaning and a consequent attenuation of the chattering effect. The controller must be robust to minimize the lateral deviation and the heading errors, and it must be sensitive to assist the driver during the shared control mode, i.e. the assist system must give the authority to the driver when the driver is available.

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**Keywords:** Sliding mode control, Automotive control, Autonomous vehicles, Interactive vehicle control, Co-operative control.

## 1. INTRODUCTION

Advanced driver assistance systems (ADASs) are a hot topic nowadays in both economics and research, and they will become even more popular in the near future because the incorporation of ADASs in modern vehicles may reduce traffic accidents and make life easier for the human driver. Different types of ADAS can be found in the literature such as lane keeping assist, collision avoidance, automated trajectory planning, adaptive cruise control, etc. The work on this paper is based on the lane keeping assist, where the vehicle must follow the lane, but the human driver must supervise the road and he has the authority over the controller.

Many papers can be found in the literature dealing with lane keeping/shared lateral control and considering the human action with or without the monitoring of the human driver's status. For example, Benloucif et al. (2019), Benloucif et al. (2017) take into account the driver's steering torque in the trajectory planning level in order to adjust the system's desired trajectory in a way that better suits the driver intention. In Guo et al. (2017), the shared control problem, with or without the driver's intervention, is formulated as a constrained optimization problem which is solved online by a model predictive controller. In Nguyen et al. (2017), the road-vehicle system is transformed into a polytopic LPV form which allows to deal with a large variation range of vehicle speed. Borroni and Tanelli (2018) propose a shared-control formulation in which the assistance and the driver steering torques

are adaptively weighted to dynamically change the control authority share between driver and controller. In Dai et al. (2018), the driver-vehicle-road model to follow large-curvature path is built under the assumption that the near and far vision information of the road for guidance is considered by the human driver, and the parameters describing the driver's steering characteristics and behaviors are considered as uncertainties of the model. In Sentouh et al. (2018) a control supervisor allows a smooth transition between two local optimal-based controllers with two predefined objectives, i.e. lane keeping and conflict management between the controller and the driver. Wang et al. (2018) propose a method where different drivers' characteristics are considered and handled by a polytope and a regional pole placement is applied, then, a method to convert the multi-objective  $H_\infty$  robust control is converted into a single objective control. Boehm et al. (2016) develop a system model which considers that a driver can tighten grip on a steering wheel, contract arm muscles, and change posture to have the authority over an automation system. All the previous papers deals with the human interaction using linear or nonlinear controllers. Among of all the controllers, nonlinear SMCs are becoming very popular tools in the last years because of their excellent robustness. A type of SMCs is super-twisting, which is preferred by researchers because of its unique property that can be applied to systems having relative degree one, as in Rath et al. (2019). However, the tuning of the parameters for nonlinear high order sliding mode controls still remains a non trivial task.

In this paper the design of a co-operative nonlinear quasi continuous sliding mode control in a steer-by-wire vehicle as lane keeping assistant is described. The time-varying

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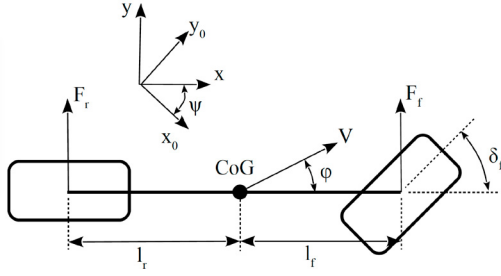


Fig. 1. Scheme of the bicycle model.

gains of the controller, which are based on the computed upper-bound disturbance and which depend by physical variables, help the tuning process. The interaction between the human driver and the controller is based on a variable parameter representing the driver's availability. The controller must be robust to minimize the lane and the heading errors, and it must be sensitive to help the driver during the shared control mode, i.e. the assist system must give the authority to the driver when the driver is available.

The paper outline is as follows. In Section 2 the vehicle's lateral dynamics and the errors' dynamics are introduced. The control system and the disturbance upper-bounds are computed in Section 3. The experimental workspace, the validation and the chattering suppression are described in Section 4. The remarks and discussions on future improvements conclude the paper in Section 5.

## 2. VEHICLE'S AND ERRORS' DYNAMICS

The lane keeping assist systems are controllers useful to keep the lane in an autonomous driving, taking into account the human driver interaction. For this reason, these systems are based on lateral dynamics of the vehicles, on lateral displacement from the center of the lane and on heading error of the vehicle.

The lateral dynamics of the vehicle can be computed from a well established bicycle model as in Fig. 1 (Rajamani (2012)), and it is given by

$$\begin{cases} m\dot{v}_y &= F_r + F_f - mv_x\dot{\psi} + F_w \\ I_z\dot{\psi} &= -l_r F_r + l_f F_f + l_w F_w \end{cases}, \quad (1)$$

where  $m$  denotes the vehicle mass,  $v_y = v_x\phi$  the lateral velocity,  $v_x$  the longitudinal driving velocity supposed to be constant,  $\phi$  the vehicle side slip angle,  $\psi$  the yaw angle,  $I_z$  the yaw moment of inertia,  $l_f$ ,  $l_r$ ,  $l_w$  respectively the distances from the center of gravity to the front, to the rear wheels and to the point where the wind disturbance is supposed to influence the vehicle,  $F_w$  the lateral aerodynamic force caused by the wind,

$$F_r = 2C_r \frac{l_r\dot{\psi} - v_y}{v_x}, \quad F_f = 2C_f \left( \delta_f - \frac{l_f\dot{\psi} + v_y}{v_x} \right),$$

are the lateral forces applied on the rear and front wheels and where  $C_r$ ,  $C_f$  are the front and rear cornering stiffness,  $\delta_f$  denotes the tire steer angle.

For lane keeping systems, additional dynamics must be introduced as in Fig. 2, and the equations are

$$\begin{cases} \dot{\psi}_l &= \dot{\psi} - \rho_r v_x \\ \dot{y}_l &= v_y + l_p\dot{\psi} + \psi_l v_x \end{cases}, \quad (2)$$

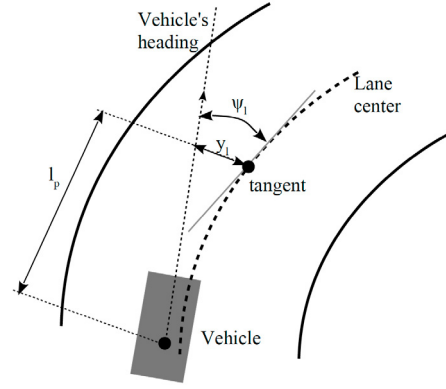


Fig. 2. Visual representation for lane and heading errors.

where  $\rho_r$  is the road curvature,  $y_l$  the lateral deviation error,  $\psi_l$  the heading error both at the look ahead distance  $l_p$  from the vehicle's center of gravity.

## 3. CONTROL SYSTEM FOR LANE KEEPING

The focus of the controller is to take into account the interaction with the human driver. The controller must be robust to minimize the lane and the heading errors, but it must be sensitive to assist the driver during the shared control mode based on the driver's availability. For this reason, the tire steer angle is divided in two parts as

$$\delta_f = (1 - \omega)\delta_{fa} + \omega\delta_{fm}, \quad (3)$$

where  $\delta_{fa}$  comes from the control system,

$$\delta_{fm} = \frac{\delta_d}{R_s}$$

comes directly from the driver,  $\delta_d$  is the driver's wheel angle, and  $\omega$  is a weighted constant reflecting the driver's availability such as  $\omega = 1$  when the driver is available and the driving is completely manual,  $\omega = 0$  when the driver is not available and the driving is completely automatic,  $0 < \omega < 1$  when it is present the shared controller proportional to the driver's availability. Using this decomposition in a steer-by-wire architecture,  $\delta_{fm}$  and  $\delta_{fa}$  are called *virtual angles*, and  $\delta_f$  is the *real angle* of the steer system.

In this subsection for synthesis of a control law, a kind of HOSMC called quasi-continuous SMC will be applied. The first step to design the controller is to compute the error, which has to be minimized, as

$$e = k_1 l_p \psi_l + k_2 y_l, \quad (4)$$

where  $k_1, k_2 > 0$  are tuned parameters, then substituting eqs. (2), (1) in its second derivative, we have

$$\ddot{e} = f + U + \Delta, \quad (5)$$

where

$$f = f(X, \delta_{f_m}) = F_r \left( \frac{k_2}{m} - \frac{l_r l_p}{I_z} (k_1 + k_2) \right) + \delta_{f_m} \omega C_f \left( \frac{l_p l_f}{I_z} (k_1 + k_2) + \frac{k_2}{m} \right), \quad (6)$$

$$\Delta = \Delta(F_w, \rho_r, \dot{\rho}_r) = F_w \left( \frac{l_p l_w}{I_z} (k_1 + k_2) + \frac{k_2}{m} \right) - \rho_r k_2 v^2 - \dot{\rho}_r k_1 v_x l_p, \quad (7)$$

$$U = U(\delta_{f_a}) = \delta_{f_a} (1 - \omega) \left( C_f \frac{l_p l_f}{I_z} (k_1 + k_2) + \frac{k_2}{m} \right), \quad (8)$$

are respectively the parts of the dynamics where known and disturbance unknown terms appear, and the control.

The control can be selected as

$$U = -f + \tilde{u}, \quad (9)$$

and substituting eq. (9) in eq. (5), the second derivative of the error becomes

$$\ddot{e} = \tilde{u} + \Delta, \quad (10)$$

where the auxiliary controller  $\tilde{u}$  must be synthesized. To design a control, which is able to compensate the disturbances, we have to evaluate the upper bounds for them. In the literature, it is often assumed that component of the disturbance admits a fixed upper bound, which means  $|\Delta| \leq \bar{\Delta}$  for some known constant  $\bar{\Delta} \geq 0$ . Unfortunately, it is a conservative hypothesis, and that is why time-varying bounds will be considered in our case for  $\Delta$ . However, we will assume boundedness of the lateral aerodynamic force by the wind  $|F_w| \leq \bar{F}_w$ , for some known  $\bar{F}_w \geq 0$ , and boundedness of road curvature  $|\rho_r| \leq \bar{\rho}_r$ ,  $|\dot{\rho}_r| \leq \bar{\dot{\rho}}_r$  for some known  $\bar{\rho}_r \geq 0$ ,  $\bar{\dot{\rho}}_r \geq 0$ . For this reason, the upper-bound of the disturbance becomes

$$|\Delta| \leq |F_w| \left( \frac{l_p l_w}{I_z} (k_1 + k_2) + \frac{k_2}{m} \right) + |\rho_r| k_2 v^2 + |\dot{\rho}_r| k_1 v_x l_p \quad (11)$$

In contrast to standard chattering reduction technique based on smoothing of the discontinuous control, the high order sliding mode approach suggests to smooth the sliding motion providing the finite-time convergence to zero for the sliding variable  $e$  together with the derivatives  $\dot{e}, \ddot{e}, \dots, e^{(r-1)}$  where  $r$  is the relative degree of  $e$  with respect to the control input ( $r = 2$  in our case). In Ding et al. (2016), it is stated that if the second order control is selected as  $\tilde{u} = -\bar{\Delta} \frac{|\dot{e}|^2 \text{sign}(e) + \alpha e}{|\dot{e}|^2 + \alpha|e|}$ , where  $\alpha > 0$  is a tuned parameter, then  $e = \dot{e} = 0$  is reached in a finite time. The proposed quasi continuous SMC can be modified to counteract the chattering avoiding the saturation functions (mostly used for first order SMCs), and using the quasi-continuous function itself as an approximation of the sign on the plane, with a modification by adding a small constant  $\beta > 0$  in the denominator, where  $\beta$  is strictly related with accuracy, as in

$$\tilde{u} = -\bar{\Delta} \frac{|\dot{e}|^2 \text{sign}(e) + \alpha e}{|\dot{e}|^2 + \alpha|e| + \beta}. \quad (12)$$

The smaller is  $\beta$  the higher is the effort on the steer system, which results in a more accentuated oscillation of the control, but with a smaller convergence error, and vice-versa. According to Ding et al. (2016), a finite-time convergence of the system can be achieved in the ideal

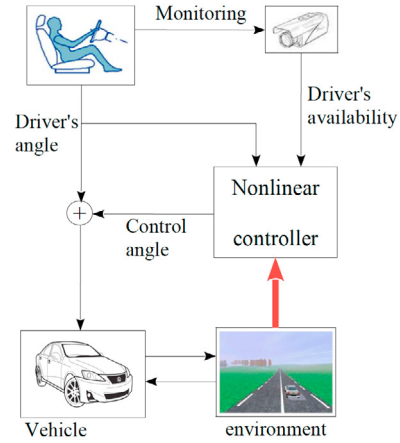


Fig. 3. Interaction between the controller, the driver and the vehicle's prototype.

case, when  $\beta = 0$  and there is no measurement or digital noises. In our case, since these restrictions are not satisfied, the convergence is assured with respect to a compact set around the desired trajectory. Then  $\beta$  is tuned accordingly to achieve a trade-off between control oscillations and convergence error.

The interaction between the controller, the human driver, and the vehicle's prototype is illustrated in Fig. 3. The peculiarities of the proposed controller are: First, the quasi-continuous SMC (Levant (2005), Ding et al. (2016)) is tested for co-operative control in autonomous vehicles with a slightly modification. Second, the upper-bounds of  $F_w, \rho_r, \dot{\rho}_r$  are considered as input to the time-varying gain. Their estimation allows a tuning process based on reasonable physical meaning, which helps the tuning process of the nonlinear controller. Third, the shared authority is assured thanks to a parameter which comes directly from the monitoring system and continuously changes over time based on the driver's availability. In other words, the controller must be sensitive to the trajectory changes based on the availability of the human driver ( $\omega$ ). Three cases are taken into account:  $\omega = 0$  when the driving is completely automatic and the driver is supposed to be not available;  $\omega = 1$  when the driving is completely manual and the driver is available;  $0 < \omega < 1$  when the driving is shared between manual and automatic. Fourth, the chattering effect reduction in the implementation of the high order sliding mode control in the vehicle's prototype. There are two possibilities to reduce the chattering of the proposed SMC: implicit discrete-time design, which is preferable but first works are for twisting and super-twisting controllers as in Brogliato et al. (2018), Huber et al. (2016); or adapt the HOSMC as an approximation of the sign on the hyper plane as described in this paper.

#### 4. EXPERIMENTAL VALIDATION

The experimental validation deals with the implementation in a vehicle's prototype running in discrete-time with sampling time of  $0.01$  s, the consequent reduction of chattering effect, and with the evaluation of practical performances of the proposed lane keeping assist system with a human driver. The controller must be robust to minimize the lateral position and the heading errors, and

it must be sensitive to help the driver during the shared mode, i.e. the assist system must give the authority to the driver when the driver is available.

The dynamic road vehicle simulator SHERPA (Simulateur Hybride d'Etude et de Recherche Pour l'Automobile) in LAMIH is used to demonstrate the effectiveness of the proposed controller. The dynamic simulator uses a modified Peugeot 206 vehicle fixed on a Stewart platform and it is structured around a SCANer network connecting 15 PC-type workstations. The SHERPA simulator is developed with RTMaps software composed by several modules which are in charge of different tasks: perception, planning, driver monitoring, human-machine interface. It is also equipped with a FaceLab driver monitoring system that indicates the driver state. The driver monitoring system continuously provides the variable  $\omega$  in accordance with the real-time availability of the driver. The tested track comes from the project CoCoVeA (Coopération Conducteur-Véhicule Automatisé), and it is composed by different road sections, which are used to test the robustness of the controller in a circuit under different road curvature conditions.

The shared authority is assured thanks to a parameter which comes directly from the monitoring system and continuously changes over time based on the driver's availability. For a better presentation of the paper, four tests, carried out in the same circuit, are illustrated: in manual mode with the support of a visual line in the center of the road's lane as reference for the driver (called *manual-line* in the graphics); in automatic mode (called *auto* in the graphics); in shared mode with  $\omega = 0.5$  (called *shared* in the graphics); in shared mode with  $\omega = 0.5$  and the visual line in the center of the road's lane (called *shared-line* in the graphics). The chosen scenarios represent the three possible cases for shared controllers: the driver is fully aware of the road ( $\omega = 1$ ); the automatic control must have the authority because the driver is not available ( $\omega = 0$ ); the controller must help the driver to follow correctly the lane ( $\omega = 0.5$ ), for example the controller must correct the trajectory when a high lateral error from the center of the lane is present. Values for the vehicle's mathematical model (simplified with respect to the real vehicle in the SHERPA simulator) and for the controller are in Table 1. The controlled system must be bound by  $|y_l| \leq 1.75 m$ ,  $|\psi_l| \leq 5 deg$ ,  $|v_y| \leq 1.5 m/s$ ,  $|\dot{v}_y| \leq 4 m/s^2$ . The particularity of the proposed controller is that the gain is time-varying and it changes based on the driving conditions and the driver's availability. In other words, the stability of the vehicle-controller-driver system is always achieved because the gain of the controller changes over time based on the necessity. The tuning process is facilitated and  $k_1$ ,  $k_2$ ,  $\alpha$  are all chosen equal to 1. The performance of the controller is affected by a correct tuning of these parameters. The road presents different sections with varying curvatures as in Fig. 4. It's worth noting that in the steer-by-wire architecture the driver's wheel is not linked mechanically to the steer system. For this reason, the angle of the driver's wheel is set to  $0 deg$  in automatic mode and it is set to  $\delta_f$  in shared-control mode, using PID controllers. The use of the PID (tuned based on the driver's preference) is twofold: to give an haptic feedback to the driver when

Table 1. Vehicle - Controller parameters

Description	Par.	Value	Unit
distance CoG , front wheels	$l_f$	1.3	$m$
distance CoG , rear wheels	$l_r$	1.6	$m$
distance CoG , wind force	$l_w$	0.4	$m$
look ahead distance	$l_p$	5	$m$
mass of vehicle	$m$	2024.86	$kg$
yaw moment of inertia	$I_z$	2800	$kg.m^2$
front wheels cornering stiffness	$C_f$	57000	$N/rad$
rear wheels cornering stiffness	$C_r$	59000	$N/rad$
long. velocity in full auto mode	$v_x$	20	$m/s$
steer gear ratio	$R_s$	16	-
sliding surface	$k_1, k_2$	1	-
quasi-continuous controller	$\alpha, \beta$	1	-

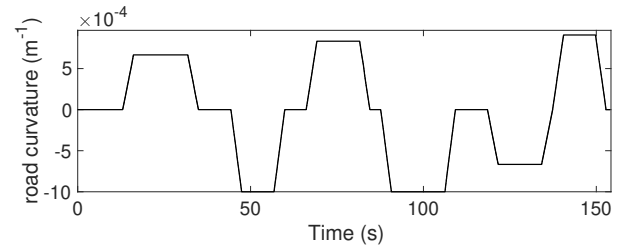
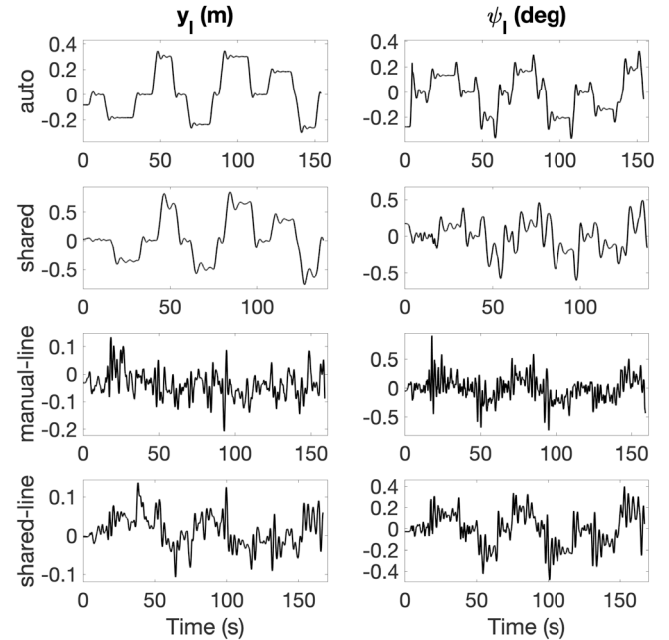


Fig. 4. Profile of the road curvature over time.

Fig. 5. Errors in manual (*manual-line*), automatic (*auto*), shared control (*shared*), and shared control with line reference (*shared-line*) modes.

the angles  $\delta_{fa}$  and  $\delta_{fm}$  have opposite sign, and to obtain a desired smooth transition from the actual to the desired driver's angle. In steer-by-wire architecture, drivers cannot feel the road information because the system does not provide the steering reactive torque reflecting the external environment. Then, the steering reactive torque to inform the driver of the road condition can be designed as in Hiraoka et al. (2008).

The errors and the angles  $\delta_{fa}$ ,  $\delta_{fm}$ , and  $\delta_f$  in the four modes are illustrated and compared in Figures 5, 6.

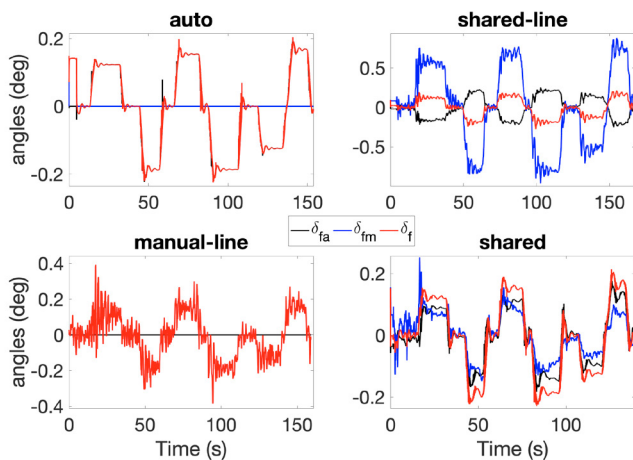


Fig. 6. Angles in manual, automatic and shared control modes.

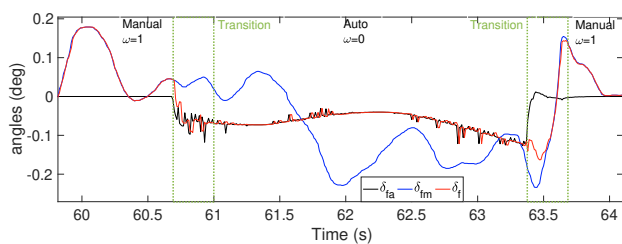


Fig. 7. Transitions between manual and automatic modes.

The *manual-line* mode presents an unpredictable behavior based on the personal driver ability to follow the line in the center of the lane, the driving presents many corrections, and the steer angle presents many solicitations. In *shared-line* mode the driver tested the shared control with the presence of the reference line in the center of the lane. The errors are similar to the manual mode, but less corrections are needed because the driver is helped by the controller. In this case, the conflict between the driver operation and the assistance can be seen, and it comes from the two different references that the driver and the controller have. The introduction of the novel parameter  $\beta$  allows the controller to follow as reference a trajectory which presents a lateral deviation from the center of the lane, while the driver follows the line which is visibly at the center of the lane. The *auto* mode presents a smoother behavior than the previous cases as expected since the vehicle follows automatically the center of the lane with some acceptable lateral deviations coming from the novel parameter  $\beta$  that it was introduced in this paper, modifying the original quasi-continuous controller proposed in Ding et al. (2016). The corresponding steer angle presents an optimal behavior. In the *shared* mode the driver tested the shared control without any auxiliary visual help to follow the center of the lane. In this scenario the driver is helped only by the haptic feedback from the controller. In other words, the driver is free to follow the lane, and the controller helps to compensate the errors and provides an haptic feedback to the driver's wheel with a small vibration proportional to the errors. This last mode has the best performance in terms of the compromise between the lateral and angle position errors and comfort

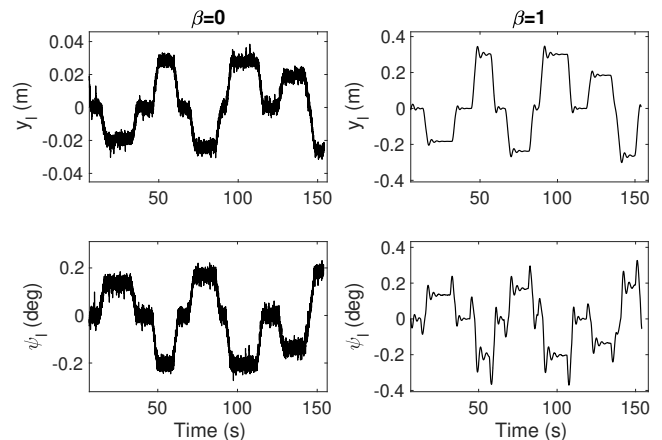


Fig. 8. Errors using different  $\beta$  in automatic mode.

of the driver, i.e. it presents less driver's wheel vibration and acceptable errors.

The shared controllers must be evaluated also during the transitions phases between manual and automatic drivings. In this paper, the transition law is governed by a smooth switch. Fig. 7 shows the performance of the proposed controller from manual to automatic and *vice versa*. The transitions are smooth in both cases to ensure the best comfort for the driver.

A big shortage of SMC is the chattering (high frequency oscillations of a discontinuous control signal in the steady-state mode caused by imprecision of values of model parameters, digital and measurement noises), which can decrease drastically the driver's comfort. In the literature the problem of chattering reduction is a well-known issue discussed in many articles. One way to reduce the chattering is to use a high order sliding mode (HOSM) control (Bernuau et al. (2014)). A kind of HOSM control called quasi-continuous SMC is chosen in this paper, which can be considered as an approximation of the sign on the plane. However, the main difficulty of the experimental validation is that the controller is designed in continuous-time domain, while the implementation is always in discrete-time. The explicit discretization made by the SCANer software does not allow to obtain a smooth sliding motion and it can destabilize the vehicle-controller system (differences between explicit and implicit discretizations in SMCs are explained in Wang et al. (2014)). For this reason, the small constant  $\beta$  is added, which affects directly the accuracy. The trade-off between the error and the chattering comes from the tuning parameter  $\beta$ , which is chosen to have small lane and heading errors and to avoid the chattering in the discretized controller. Figures 8, 9 show the influence of  $\beta$  on the errors and on the control signals in fully autonomous driving. When  $\beta = 0$  the controller presents undesired oscillations with high pitches, but the errors are minimized. In this case, the chattering produced an undesired vibration in the vehicle and it decreased the comfort for the drivers who made the tests in the experimental validation. In opposite, when  $\beta = 1$  the controller has a smooth behavior, but with higher lane errors depending linearly by the amplitude of the road curvature.

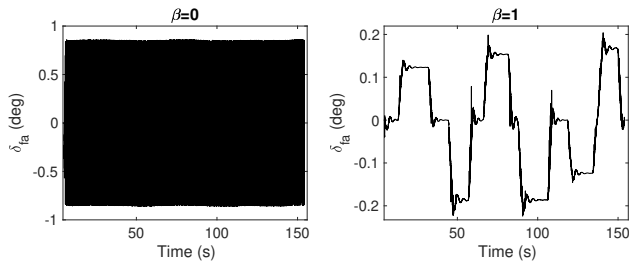


Fig. 9. Control using different  $\beta$  in automatic mode.

## 5. CONCLUSION

In this paper, a robust shared SMC approach between human driver and automated system is introduced as lane keeping assistant for road four-wheeled vehicles. The shared authority is assured thanks to a parameter which comes directly from the monitoring system and continuously changes over time based on the driver's availability. Upper-bounds of wind force and road curvatures are considered in the control design, and their estimation allows a tuning process based on reasonable physical meaning. The implementation of the SMC in the vehicle, which runs in discrete-time domain, and the consequent chattering suppression are discussed. The proposed real-time experiments are used to show the robustness of the automatic controller in a road with different curvatures, to highlight the co-operation between the driver and the controller based on the driver's availability, and to evaluate the transitions between autonomous and manual drivings. As results, the controller is robust to minimize the lateral deviation and the heading errors, it is sensitive to the trajectory's change from the human driver when needed, it allows smooth transitions between manual and automatic drivings.

As further improvement the following solutions can be considered. The controller can be designed using an implicit discretization. The switching variable  $\omega$ , which in this work considers only the driver's availability, can be improved considering the driver's comfort, the personal driver's characteristics, the lateral deviation error from the center of the lane, and the difference between the angle of the driver's wheel and angle of the steering system.

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