Unknown Input Observer for Understanding Sitting Control of Persons with Spine Cord Injury

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Abstract: The present paper introduces a simple model to study sitting control for persons with complete thoracic spine cord injury. The system is obtained via Lagrangian techniques; this procedure leads to a nonlinear descriptor form, which can be written as a Takagi-Sugeno model. A first attempt to estimate the sitting control in disabled people is done via an unknown input observer. The conditions are expressed as linear matrix inequalities, which can be efficiently solved. Simulation results validate the proposed methodology as the observations are coherent with and without perturbations.

Keywords: Unknown input observer, Takagi-Sugeno models, Inverted pendulum, Spine Cord Injury, Sitting movement, Descriptor Form.

1. INTRODUCTION

Sitting stability of the upper body is critical for performing activities of daily living (e.g. in transportation), this is especially true for persons with complete thoracic spinal cord injury (SCI) who lose both muscle control and sensitivity below the injury level. Because of its unstable nature (Crisco et al. 1992), the human spine must be stabilized even without external perturbation (e.g. braking in a car). These fine adjustments are usually done via activation of the trunk and intervertebral muscles (Silfies et al. 2003) but people with complete SCI cannot rely on any muscle activity below their injury level. Instead they can use their upper limbs and head in order to maintain the stability of the upper body as they are trained in rehabilitation (Janssen-Potten et al. 2001). The stability of the upper body has been described as the combination of various subsystems (e.g. redundant voluntary joint torques, passive viscoelastic behaviour of the joint… (Panjabi 1992)). Due to this complexity, linked-segment models of the human trunk are used to systemize thinking and test hypothesis one by one which is difficult to do with human observation. The obtained mathematical model is therefore needed in order to estimate the role of all the upper body segments (trunk, head and arms) in sitting stability under external perturbation.

Models used to study sitting stability are usually based on an active torque at the lumbar joint to stabilize the upper body with head, arms and trunk represented by one rigid segment, (Cholewicki et al. 1999; Tanaka and Granata 2007; Vette et al. 2010). Not only this representation is inapplicable to people with complete thoracic SCI because of their lack of trunk muscle activity but it is also both too simplistic and erroneous: it has been shown that under perturbations a seated subject's head will move (Thrasher et al. 2010) and we can predict that the arm will do the same. A model taking into account the action of the upper limbs and head is required for this specific topic.

Therefore, in this paper, the effect of the head and upper limbs displacement on the stabilization of an individual with SCI is modelled via an H2AT configuration, which can be seen as a variation of the inverted pendulum. Such displacement is generated by a force with time varying delay. This delay is inherent in biomechanical systems and represents the time to sense a perturbation and respond with muscle activation to stabilize the system (Reeves et al. 2007).

On the other hand, modeling mechanical systems often leads to nonlinear descriptor systems with the property of the inertia matrix being invertible (Lewis et al. 2003). The use of Takagi-Sugeno (TS) models has been widely employed for the analysis and design of nonlinear systems (Tanaka and Wang 2001). Effectively, via the sector nonlinearity methodology an exact TS representation can be obtained (Ohtake et al. 2001). The extension to TS descriptor models (Taniguchi et al. 1999) have been already used in biomechanics, for example, in (Guelton et al. 2008) an unknown input observer (UIO) has been developed for estimating variables, inputs. Nevertheless, the conditions are

in terms of bilinear matrix inequalities (BMI), which are difficult to solve. Linear matrix inequality (LMI) conditions have been recently derived in (Guerra et al. 2015). Those conditions can be efficiently solved via convex optimization techniques (Boyd et al. 1994). Later, results in (Guerra et al. 2015) have been extended to an UIO (Estrada-Manzo et al. 2015). This work adopts such methodologies. Its objective is to understand the way an individual with complete thoracic SCI controls his position by recovering internal variables of force generation. This task is performed via UIO and TS descriptor models.

The SCI open-loop being unstable, a first control law – that will act as an "internal" control law – being compatible with the observed human behaviour has to be derived in order to get a stable closed-loop system. The design of this control law is based on a time-varying input control law from (Yue and Han 2005) and has been adapted for the descriptor form. This design is out of the scope of this paper. To the best of our knowledge, there are no results in the literature for TS models in descriptor form with that kind of delay to study SCI stabilization.

This paper is organized as follows: Section 2 presents the modeling of H2AT via Lagrangian techniques, Section 3 explains the way the TS descriptor model is obtained as well as the main results to derive a UIO for the observation of the stabilization force, Section 4 provides the simulation results, Section 5 discusses the obtained results and future works, and Section 6 concludes the paper.

2. PROBLEM STATEMENT

The goal is to estimate the variables that make a seated person with SCI controls his position via the top of his body without invasive measurements.

2.1 Modeling

The H2AT pendulum is an extended version of the planar inverted pendulum consisting of two rods. The first one represents the trunk as a classical inverted pendulum while the second rod represents the head and arms slides at the top of the first one. The controlling force $F(t)$ will make the upper rod slide on the lower one. Figure 1 shows the H2AT system scheme.

This model is generic and just requires a minimum of biomechanical parameters. For the simulations, we consider a $80\ \text{Kg}$ male subject. As arms and trunk mass do not change between control subject and SCI subject (Jones et al. 2003), we can use regression rules to get segment mass and length (Dumas et al. 2007): $m_1 = 16.1\ \text{Kg}$ stands for the mass of the upper segment, corresponding to the head, neck, and arms; $m_2 = 26.64\ \text{Kg}$ is the mass of the trunk; $l_0 = 477\ \text{mm}$ is the length of the trunk; and $l_c = 276.66\ \text{mm}$ is the length of the centre of mass of the trunk. A full neck flexion with both arms stretched gives a value of $x = 105.27\ \text{mm}$ whereas an extension of the neck and arms gives $x = -75.18\ \text{mm}$ (Kapandji 2005). The resulting compact set is $\Omega_X = \begin{Bmatrix} -0.075 \le x \le 0.105 \\ -0.087 \le \theta \le 0.122 \\ -0.209 \le \dot{\theta} \le 0.209 \end{Bmatrix}$ in meters, radians and rad/s.

To obtain the dynamic equations of the system, we calculate its Lagrangian $L = K - U$ where K, U are the kinetic and potential energies of the system, respectively. Thus, consider

$K = K_1 + K_2$ with $K_1 = \frac{m_1}{2}\left(l_0^2\dot{\theta}^2 + \dot{x}^2 + x^2\dot{\theta}^2 - 2l_0\dot{x}\dot{\theta}\right)$,

$K_2 = \frac{1}{2}m_1 l_C \dot{\theta}^2$; and $U = U_1 + U_2$ with

$U_1 = m_1 g\left(l_0\cos(\theta) + x\sin\sin(\theta)\right)$, $U_2 = m_2 g l_c \cos(\theta)$.

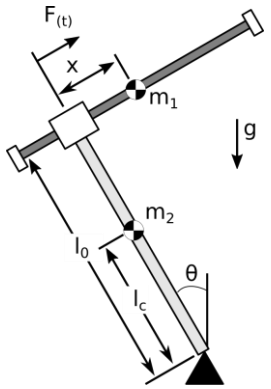

Fig. 1. H2AT Pendulum

Hence, by considering

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = F(t-\tau) \quad \text{and} \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0,$$

we obtain

$$\begin{aligned} 0 &= m_1\ddot{x}(t) - m_1 l_0\ddot{\theta}(t) - m_1 x(t)\dot{\theta}^2(t) \\ &\quad + m_1 g\sin(\theta(t)) - F(t-\tau(t)) \\ 0 &= -m_1 l_0\ddot{x}(t) + J(x(t))\ddot{\theta}(t) + 2m_1 x(t)\dot{x}(t)\dot{\theta}(t) \\ &\quad - (m_1 l_0 + m_2 l_c) g\sin(\theta(t)) + m_1 g x(t)\cos(\theta(t)), \end{aligned} \tag{1}$$

where $J(x(t)) = m_1\left(l_0^2 + x^2(t)\right) + m_2 l_c^2$. The input includes a time-varying delay $\tau(t)$ due to neural transmission and the muscle force generation and is varying according to the individual; a classical range is for example $60\text{ms} \pm 10\text{ms}$. Of course, due to the absence of control of the trunk and intervertebral muscles, the model exhibits unstable open-loop behaviour, as shown for example, in Fig. 2, using the H2AT initial parameters at $t = 0\text{s}$: $\theta = -0.2\text{rad}$, $x = 0\text{mm}$, and $F = 100N$. Because of the gravity effect, the trunk should have continued in negative values but with x increasing fast, the trunk rotates in the opposite direction and ends up falling down.

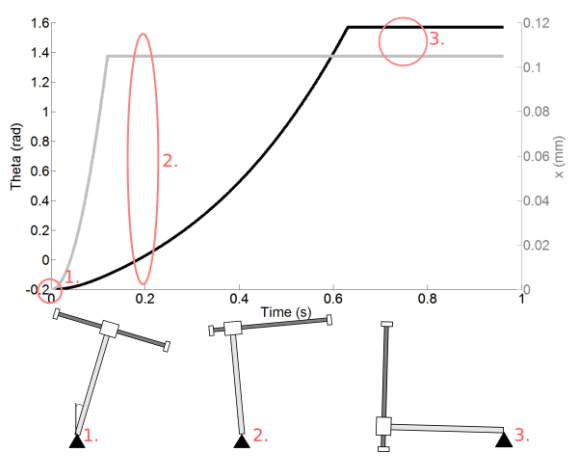

Fig. 2. Evolution in time of θ (black line) and x (gray line) with a constant input.

The goal of this study is to estimate the delayed input $F(t-\tau)$. To this end, system (1) is rewritten in a state-space form using the following state vector:

$$X(t)=\begin{bmatrix} x(t) & \dot{x}(t) & \theta(t) & \dot{\theta}(t) \end{bmatrix}^T \in \mathbb{R}^4.$$

$$\begin{aligned} E(X(t))\dot{X}(t) &= A(X(t))X(t)+BF(t-\tau) \\ y(t) &= CX(t), \end{aligned} \tag{2}$$

where $y(t)\in\mathbb{R}^2$ is the output of the system, the matrices are defined as follows:

$$A(X)=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -m_1 g z_1 & m_1 z_3 \\ 0 & 0 & 0 & 1 \\ -m_1 g z_2 & -2m_1 z_3 & (m_1 l_0 + m_2 l_c) g z_1 & 0 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

$$C=\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}^T, \quad E(X)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & m_1 & 0 & -m_1 l_0 \\ 0 & 0 & 1 & 0 \\ 0 & -m_1 l_0 & 0 & m_1(l_0^2+z_4)+m_2 l_c^2 \end{bmatrix};$$

where $z_1=\sin(\theta(t))/\theta(t)$, $z_2=\cos(\theta(t))$, $z_3=\dot{\theta}(t)$ and $z_4=x(t)^2$. Notice that the matrix $E(X)$ is regular, i.e., it is invertible $\forall X\in\mathbb{R}^4$.

Remark 1. The nonlinear model (2) exhibits an unstable behaviour; therefore it needs to be controlled prior to any estimation process. An approach based on robust control of input delayed models has been employed giving a control law of the form (Yue and Han 2005):

$$u(t)=K\left[X(t)+\int_{t-\tau_0}^{t} e^{A(t-s-\tau_0)}Bu(s)ds\right]. \tag{3}$$

The implementation was done by following (Georges et al. 2007) technique using both a state prediction approximation and the "dynamic inversion" of a fixed point problem. These issues, including design and LMI constraints, are not considered within the current paper due to space reasons.

2.2 TS descriptor form

The nonlinear descriptor model (2) can be written as an equivalent TS descriptor model form (Taniguchi et al. 1999):

$$E_v\dot{X}(t)=A_h X(t)+Bd(t), \tag{4}$$

where $E_v=\sum_{k=1}^{r_e} v_k(z)E_k$, and $A_h=\sum_{i=1}^{r_a} h_i(z)A_i$ are convex sums of matrices coming from the sector nonlinearity methodology. $d(t)$ is considered in (4) as an unknown input that has to be estimated. Thus, (4) is an exact TS representation of (2). The scalars $v_k(z)$, $k\in\{1,2,\ldots,r_e\}$ and $h_i(z)$, $i\in\{1,2,\ldots,r_a\}$ are membership functions (MFs), they hold the convex sum property in Ω_X:

$$\sum_{k=1}^{r_e} v_k(z)=1,\ 0\le v_k(z)\le 1,\ \sum_{i=1}^{r_a} h_i(z)=1,\ 0\le h_i(z)\le 1. \tag{5}$$

Once the nonlinear descriptor model (2) is rewritten as (4), the direct Lyapunov method can be applied for control/ observation purposes. In our case, the unknown input estimation approach in (Estrada-Manzo et al. 2015) is adopted in order to estimate the state $X(t)$ as well as the input $d(t)$.

First, let us construct a TS descriptor model of the form (4). From the nonlinear descriptor model (2), three nonlinear terms can be identified in matrix $A(X)$: z_1, z_2, and z_3; and one nonlinear term in $E(X)$: z_4. Of course, due to SCI limitations the variables are bounded. By considering the known bounds on these nonlinear terms and applying the sector nonlinearity approach, the following exact TS descriptor model is computed:

$$\sum_{k=1}^{2} v_k(z)E_k\dot{X}(t)=\sum_{i=1}^{8} h_i(z)\left(A_i X(t)+Bd(t)\right), \tag{6}$$

where

$$E_1=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 16.1 & 0 & -7.68 \\ 0 & 0 & 1 & 0 \\ 0 & -7.68 & 0 & 5.881 \end{bmatrix}, E_2=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 16.1 & 0 & -7.68 \\ 0 & 0 & 1 & 0 \\ 0 & -7.68 & 0 & 5.702 \end{bmatrix},$$

$$A_1=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.94 & -0.35 \\ 0 & 0 & 0 & 1 \\ -157.94 & -0.71 & 147.64 & 0 \end{bmatrix},$$

$$A_2=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.94 & -0.35 \\ 0 & 0 & 0 & 1 \\ -157.94 & 0.734 & 147.64 & 0 \end{bmatrix},$$

$$A_3=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.94 & 0.35 \\ 0 & 0 & 0 & 1 \\ -156.77 & -0.71 & 147.64 & 0 \end{bmatrix},$$

$$A_4=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.94 & -0.35 \\ 0 & 0 & 0 & 1 \\ -156.77 & 0.70 & 147.63 & 0 \end{bmatrix},$$

$$A_5=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.55 & 0.35 \\ 0 & 0 & 0 & 1 \\ -157.94 & -0.71 & 147.27 & 0 \end{bmatrix},$$

$$A_6=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.55 & -0.35 \\ 0 & 0 & 0 & 1 \\ -157.94 & 0.71 & 147.27 & 0 \end{bmatrix},$$

$$A_7=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.54 & 0.35 \\ 0 & 0 & 0 & 1 \\ -156.77 & -0.71 & 147.27 & 0 \end{bmatrix},$$

$$A_8=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -157.54 & -0.35 \\ 0 & 0 & 0 & 1 \\ -156.77 & 0.71 & 147.27 & 0 \end{bmatrix},\quad B=\begin{bmatrix}0\\1\\0\\0\end{bmatrix};$$ the MFs are $v_1=z_4/0.0111$, $v_2=1-v_1$, $h_1=w_0^1w_0^2w_0^3$, $h_2=w_0^1w_0^2w_1^3$, $h_3=w_0^1w_1^2w_0^3$, $h_4=w_0^1w_1^2w_1^3$, $h_5=w_1^1w_0^2w_0^3$, $h_6=w_1^1w_0^2w_1^3$, $h_7=w_1^1w_1^2w_0^3$, $h_8=w_1^1w_1^2w_1^3$,

$w_0^1=(z_1-0.9975)/0.0025$, $w_1^1=1-w_0^1$,

$w_0^2=(z_2-0.9926)/0.0074$, $w_1^2=1-w_0^2$,

$w_0^3=(z_3+0.0220)/0.044$, $w_1^3=1-w_0^3$.

In order to use the approach in (Estrada-Manzo et al. 2015) we consider a 2nd order integrator to capture the dynamic of the unknown input, i.e. $\ddot{d}\approx 0$. Therefore we add the following dynamics:

$$I_2\times\dot{\bar{d}}=S\times\bar{d}\quad\Leftrightarrow\quad\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\dot{d}\\\ddot{d}\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}d\\\dot{d}\end{bmatrix}\tag{7}$$

Thus (6) yields the following system

$$E_v^e\dot{X}^e=A_h^eX^e,\qquad y=C^eX^e,\tag{8}$$

with $X^e=\begin{bmatrix}X\\\bar{d}\end{bmatrix}$, $E_v^e=\begin{bmatrix}E_v&0\\0&I_2\end{bmatrix}$, $M=\begin{bmatrix}B&0_{2\times1}\end{bmatrix}$ $A_h^e=\begin{bmatrix}A_h&M\\0_{2\times4}&S\end{bmatrix}$, and $C^e=\begin{bmatrix}C&0_{1\times2}\end{bmatrix}$. From (8), it can be seen that $\left(E_v^e\right)^{-1}$ exists for all $X\in\Omega_X$.

3. MAIN RESULTS

By means of the descriptor redundancy approach, (8) can be expressed as the following augmented TS descriptor:

$$\bar{E}\dot{\bar{X}}=\bar{A}_{hv}\bar{X},\qquad y=\bar{C}\bar{X},\tag{9}$$

where $\bar{X}=\begin{bmatrix}X^e\\\dot{X}^e\end{bmatrix}$, $\bar{E}=\begin{bmatrix}I&0\\0&0\end{bmatrix}$, $\bar{A}_{hv}=\begin{bmatrix}0&I\\A_h^e&-E_v^e\end{bmatrix}$, and $\bar{C}=\begin{bmatrix}C^e&0\end{bmatrix}$. In what follows, the augmented TS descriptor system (9) will be used in order to derive LMI conditions. Hence, the following augmented observer is proposed (Estrada-Manzo et al. 2015):

$$\bar{E}\dot{\hat{\bar{X}}}=\bar{A}_{hv}\hat{\bar{X}}+P^{-T}\bar{L}_{hv}\left(y-\hat{y}\right),\qquad\hat{y}=\bar{C}\hat{\bar{X}},\tag{10}$$

where $\hat{\bar{X}}=\begin{bmatrix}\hat{X}^e\\\beta^e\end{bmatrix}\in\mathbb{R}^{(2n_x+2)}$ is the estimated vector and $\bar{L}_{hv}=\begin{bmatrix}L_{1hv}\\L_{2hv}\end{bmatrix}\in\mathbb{R}^{(2n_x+2)\times n_y}$ is the observer gain to be designed, the matrix $P=\begin{bmatrix}P_1&0\\P_3&P_4\end{bmatrix}\in\mathbb{R}^{(2n_x+2)\times(2n_x+2)}$ comes from the Lyapunov function candidate. Thus, the augmented estimation error is

$$\bar{e}=\bar{x}-\hat{\bar{x}}=\begin{bmatrix}x^e-\hat{x}^e\\\dot{x}^e-\beta^e\end{bmatrix}=\left[\begin{array}{c}x-\hat{x}\\\bar{d}-\hat{\bar{d}}\\\hline\dot{x}-\beta_1^e\\\dot{\bar{d}}-\beta_2^e\end{array}\right].\tag{11}$$

Therefore, its dynamics are

$$\bar{E}\dot{\bar{e}}=\left(\bar{A}_{hv}-P^{-T}\bar{L}_{hv}\bar{C}_h\right)\bar{e}.\tag{12}$$

Consider the following Lyapunov function:

$$V\left(\bar{e}\right)=\bar{e}^T\bar{E}^TP\bar{e},\qquad\bar{E}^TP=P^T\bar{E}\geq0,\tag{13}$$

where P as in (10), $P_1=P_1^T>0$, P_4 being a regular matrix.

Theorem 1. Consider the system (9) together with the observer (10). If there exist matrices $P_1=P_1^T>0$, P_3, P_4, L_{1jk}, and L_{2jk}, $i,j\in\{1,2,\ldots,r_a\}$, $k\in\{1,2,\ldots,r_e\}$ such that

$$\Upsilon_{ii}^k<0,\quad\forall i,k;\qquad\frac{2}{r_a-1}\Upsilon_{ii}^k+\Upsilon_{ij}^k+\Upsilon_{ji}^k<0,\quad\forall k,i\neq j,\tag{14}$$

holds with

$$\Upsilon_{ij}^k=\begin{bmatrix}P_3^TA_i^e-L_{1jk}C_i^e+(*)&(*)\\P_4^TA_i^e-L_{2jk}C_i^e+P_1-\left(E_k^e\right)^TP_3&-P_4^TE_k^e+(*)\end{bmatrix},\tag{15}$$

then the estimation error e is asymptotically stable.

Proof. The time-derivative of the Lyapunov function (13) is

$$\dot{V}\left(\bar{e}\right)=\bar{e}^TP^T\left(\bar{A}_{hv}-P^{-T}\bar{L}_{hv}\bar{C}_h\right)\bar{e}+(*).\tag{16}$$

Hence

$$\Upsilon_{hhv} := \begin{bmatrix} P_3^T A_h^e - L_{1hv} C_h^e + (*) & (*) \\ P_4^T A_h^e - L_{2hv} C_h^e + P_1 - \left(E_v^e\right)^T P_3 & -P_4^T E_v^e + (*) \end{bmatrix} < 0, \quad (17)$$

which directly leads to (15) via a relaxation lemma scheme such as (Tuan et al. 2001). The final form of the TS descriptor observer is obtained as follows: using (10), define

$$\begin{bmatrix} N_{1hv} \\ N_{2hv} \end{bmatrix} = P^{-T} \bar{L}_{hv} = \begin{bmatrix} P_1^{-1}\left(L_{1hv} - P_3^T P_4^{-T} L_{2hv}\right) \\ P_4^{-T} L_{2hv} \end{bmatrix}. \quad (18)$$

Then,

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\hat{x}}^e \\ \dot{\beta}^e \end{bmatrix} = \begin{bmatrix} 0 & I \\ A_h^e & -E_v^e \end{bmatrix} \begin{bmatrix} \hat{x}^e \\ \beta^e \end{bmatrix} + \begin{bmatrix} N_{1hv} C_h^e \\ N_{2hv} C_h^e \end{bmatrix} \left(x^e - \hat{x}^e\right), \quad (19)$$

or equivalently

$$\begin{aligned} \dot{\hat{x}}^e &= \beta^e + N_{1hv} C_h^e \left(x^e - \hat{x}^e\right) \\ E_v^e \beta^e &= A_h^e \hat{x}^e + N_{2hv} C_h^e \left(x^e - \hat{x}^e\right). \end{aligned} \quad (20)$$

From (20), we can define $\beta^e = \dot{\hat{x}}^e - N_{1h} C_h^e \left(x^e - \hat{x}^e\right)$. Thus, (20) gives

$$E_v^e \left(\dot{\hat{x}}^e - K_{1hv}\left(y - \hat{y}\right)\right) = A_h^e \hat{x}^e + K_{2hv}\left(y - \hat{y}\right), \quad (21)$$

At last, by grouping terms, it produces the final observer form

$$E_v^e \dot{\hat{x}}^e = A_h^e \hat{x}^e + \begin{bmatrix} E_v^e & I \end{bmatrix} P^{-T} \begin{bmatrix} L_{1hv} \\ L_{2hv} \end{bmatrix} \left(y - \hat{y}\right). \quad (22)$$

The proof is concluded. ■

Remark 2. In order to control the convergence speed of the observer, a decay rate performance can be implemented using, i.e., $\dot{V}\left(\bar{e}\right) \le -2\alpha V\left(\bar{e}\right)$, with $\alpha > 0$.

4. SIMULATIONS

In this section, simulation results are presented, that is, the TS descriptor model (6) under the control law (3) with $K = \begin{bmatrix} 8126 & 1629 & -6954 & -1346 \end{bmatrix}$ and the observer (10) in order to estimate both the state vector and the unknown input $d(t)$, recall that in this case $d(t) = u(t)$. When implementing conditions in Theorem 1 together with a decay rate performance, the choice of $\alpha = 20$ (see Remark 2) exhibits a good trade off and gives a feasible solution. Some of the matrices obtained from the result are given:

$$L_{11}^{(1)} = 10^5 * \begin{bmatrix} 503.99 & 24.98 \\ -19.35 & -2.16 \\ 859.11 & 21.27 \\ -15.67 & -4.01 \\ 1.22 & -3.73 \\ 3.73 & 0 \end{bmatrix}, \; L_{11}^{(2)} = 10^5 * \begin{bmatrix} 0 & 21.94 \\ -16.79 & -3.04 \\ 608.37 & 30.49 \\ -25.21 & 2.44 \\ -0.08 & 1.53 \\ -1.53 & 0 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 2.52\text{e+}6 & * & * & * & * & * \\ -4.80\text{e+}4 & 1.94\text{e+}3 & * & * & * & * \\ 2.15\text{e+}6 & -4.09\text{e+}4 & 3.04\text{e+}6 & * & * & * \\ -3.85\text{e+}4 & 1.89\text{e+}3 & -6.32\text{e+}4 & 2.67\text{e+}3 & * & * \\ 2.94\text{e+}2 & -2.36\text{e+}1 & 4.16\text{e+}2 & -3.01\text{e+}1 & 4.70\text{e-}1 & * \\ -4.86\text{e+}0 & 6.46\text{e-}1 & -8.06\text{e+}0 & 8.74\text{e-}1 & -1.72\text{e-}2 & 7.45\text{e-}4 \end{bmatrix}$$

Fig. 3 shows the time evolution of the states for initial conditions $X(0) = \begin{bmatrix} 0 & 0 & 0.005 & 0 \end{bmatrix}^T$, $\hat{X}(0) = 0_{4\times 1}$ Furthermore, Fig. 4 plots the estimation of the unknown input $d(t) = F(t-\tau)$. As expected, the UIO estimates the control input (3).

To exhibit the interest of the proposed results, a sinusoidal disturbance is added on the state variable $X_3 = \theta(t)$ from $t_{initial} = 1s$ to $t_{final} = 13s$. Fig. 5 and Fig. 6 show the results. The pikes on X_2, X_4 and $d(t)$ correspond to $t_{initial}$ and t_{final}, and the figures show the good capability of the observer to reconstruct the unknown input.

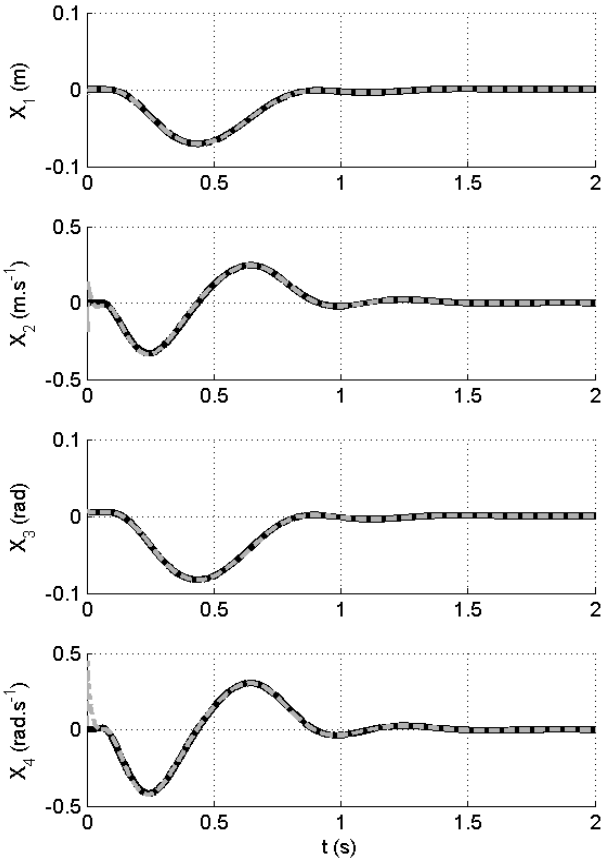

Fig. 3. States (black) and their estimates (dashed-grey)

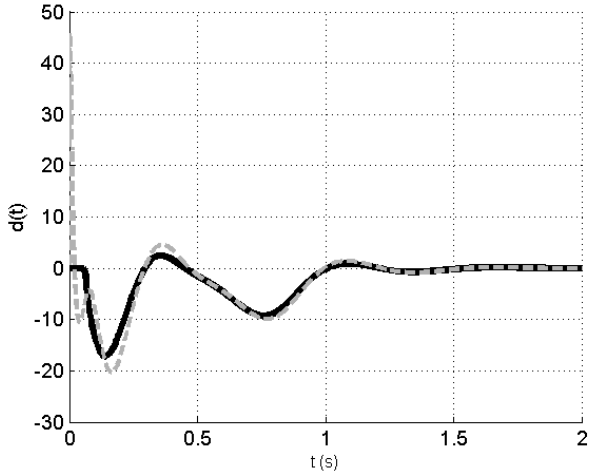

Fig. 4. The unknown input d(t) (black) and its estimate (dashed-grey)

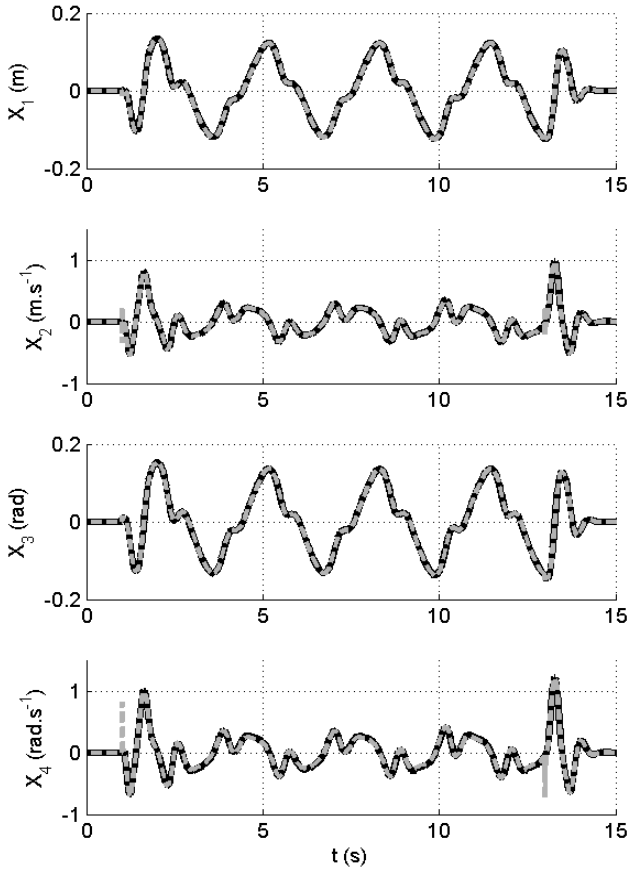

Fig. 5. States (black) and their estimates (dashed-grey) with perturbation

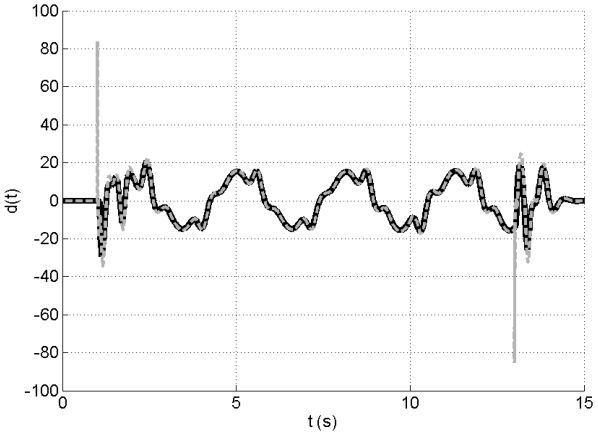

Fig. 6. The unknown input d(t) (black) and its estimate (dashed-grey) with perturbation

5. DISCUSSIONS

The main objective of the work was to study the sitting of complete thoracic SCI people. To this end, a variation of double inverted pendulum has been formulated, then a fuzzy T–S unknown-inputs-observer in descriptor form was designed to estimate the generated force to stabilize the system. This method applied on descriptor has been put under LMI conditions in the recent works of (Guerra et al. 2015) and (Estrada-Manzo et al. 2015). Although the H2AT model is a simplification of the upper body joints, its use combined with experimental data will allow us to compare different strategies of stabilization for persons with SCI when submitted to perturbations.

Several assumptions have been made in order to cope with this first approach; we consider a perfect lumbar joint without passive resistance to flexion. Naturally, future research is directed to increase the complexity of the model in order to describe the stability of the upper body as the combination of three subsystems: active (e.g. joint voluntary activation), passive (e.g. viscoelasticity of the joint) and neural (e.g. activation delay…) (Panjabi 1992). Taking into account these parameters would allow us to derive a control law that stabilizes the original nonlinear problem with a better coherence with human behaviour.

The Unknown Input Observer technique is very seldom used in biomechanics but its advantages are to be emphasized. We can estimate the force in our model without having to compute the velocities and accelerations of the segments, which is one of the drawbacks of inverse dynamics: the classical method in biomechanics, which reports successively the measurement errors at each new joint.

6. CONCLUSION

An approach coming from the control theory community has been applied to a biomechanics problem. This is, to our knowledge, the first attempt to understand how people with complete thoracic SCI maintain their sitting via an unknown input observer in descriptor form.

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