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► **To cite this version:**

van Anh Nguyen Thi, Laurent Vermeiren, Antoine Dequidt, Tran Anh-Tu Nguyen, Michel Dambrine, et al.. Takagi-Sugeno fuzzy descriptor approach for trajectory control of a 2-DOF serial manipulator. 13th IEEE Conference on Industrial Electronics and Applications (ICIEA), May 2018, Wuhan, China. pp.1284-1289, 10.1109/ICIEA.2018.8397907 . hal-03411200

HAL Id: hal-03411200

<https://uphf.hal.science/hal-03411200>

Submitted on 21 Jul 2023

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Takagi-Sugeno fuzzy descriptor approach for trajectory control of a 2-DOF serial manipulator

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Abstract—This paper presents a Takagi-Sugeno fuzzy descriptor approach for nonlinear control of a 2-DoF serial manipulator. The design goal is to achieve high tracking performance in case of circular trajectory while significantly reducing the numerical complexity of the designed controller through an original robot control scheme. Based on Lyapunov stability theory, the control design is reformulated as an LMI (linear matrix inequality) optimization which can be easily solved with available solvers. Simulation results carried out with SimMechanics environment clearly demonstrate the effectiveness of the proposed control approach.

I. INTRODUCTION

The pick and place manipulators have many applications in all areas of industry during the last decades. They have several advantages such as speed, compactness, precision and reliability. In pick and place operation, one of important tasks is to obtain high-performance control for trajectory tracking of 2-DoF robot. The main purpose of this paper is to present a based Takagi-Sugeno (T-S) fuzzy model approach for the control of robotic manipulators. For several years, T-S fuzzy model [1] are widely applied in the control of dynamic systems [2]–[6]. T-S fuzzy models are non-linear systems in a compact set of the state variables which is blending of linear models via nonlinear functions, which are called membership functions verifying the convex sum property [3]. It can be represented by a set of IF-THEN rules which the consequent parts are local linear representation.

The main objective of controlling 2-DoF robot is to achieve trajectory tracking and stabilization this robot. In literature, many research investigations have been devoted to the control of 2-DoF (degree of freedom) robots using conventional fuzzy logic controller [7], proportional-integral-derivative (PID) controller [8], fuzzy PD control and sliding mode control [9], online adaptive MIMO switching control [10], etc.

For trajectory tracking control, the work in [11] is concerned with the tracking control of a 2-DoF inverted pendulum. An adaptive dynamic controller for autonomous mobile robot trajectory tracking has been proposed in [12]. Another adaptive switching learning PD control for trajectory tracking of robot manipulators [13]. A the trajectory tracking solution using model-based predictive control of a nonholonomic wheeled mobile robot has been presented in [14]. The authors in [15] have proposed a sliding mode control of Biglide Planar Parallel manipulator. Trajectory tracking of a 2-DoF helicopter system

has been also discussed in [16] using neuro-fuzzy system and in [17] using PID controllers.

In this work, a T-S fuzzy descriptor control approach is proposed for circular trajectory tracking of a 2-DoF serial manipulator [8], [18]. The dynamics of this type of robot is highly nonlinear which represents a challenging control problem. We directly exploit the original descriptor form of the 2-DoF robot for the control design to keep the control structure as simple as possible, see [3], [19]. In addition, compared to classical state-space representation, the descriptor modeling form can describe a wider class of dynamical systems including physical models and non-dynamic constraint [20]. For control design, the direct Lyapunov method will be used to study the stability of the closed-loop system. The tracking controller is designed based on the concept of parallel distributed compensation (PDC) which consists of a set of local linear state-feedback controllers interconnected by the membership functions of the T-S fuzzy models [3]. In particular, we show that the proposed descriptor T-S fuzzy approach can guarantee a high tracking performance while being able to reduce significantly the numerical complexity of the control structure through a robust compensation scheme. The control design is reformulated as an LMI (linear matrix inequality) optimization which can be efficiently solved with numerical solvers [21].

Notation: Ω_r denotes the number set $\{1, 2, \dots, r\}$. For a square matrix X , X^\top denotes its transpose, $X > 0$ means that X is positive definite, $X_{(i)}$ denotes its i th row and $\text{He } X = X + X^\top$. $\text{diag}(X_1, X_2)$ denotes a block-diagonal matrix composed of X_1, X_2 . I is the identity matrix of appropriate dimension. The scalar functions η_i , $i \in \Omega_r$, satisfy the convex sum property if $\eta_i \geq 0$ and $\sum_{i=1}^r \eta_i = 1$. For brevity, we denote $X_h = \sum_{i=1}^r h_i X_i$ and $Y_{hv} = \sum_{i=1}^r \sum_{k=1}^{r_e} h_i v_k Y_{ik}$ where X_i and Y_{ik} are matrices of appropriate dimension and the scalar functions h_i and v_k , $i \in \Omega_r$, $k \in \Omega_{r_e}$, of any argument satisfy the convex sum property.

II. SYSTEM MODELING

This section provides a brief description on the studied 2-DoF serial manipulator, see Fig. 1. The parameters of the robot are given in Table I.

The first arm of length L_1 and mass m_1 rotates about the z -axis. The second arm of length L_2 and mass m_2 is

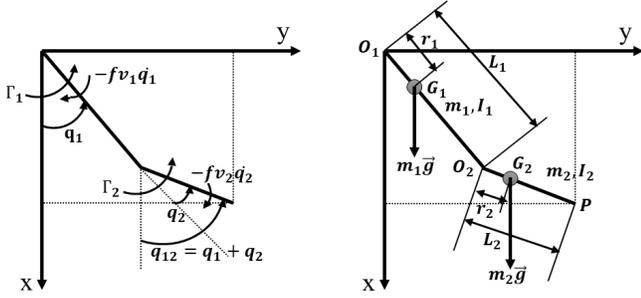


Fig. 1. Description of a 2-DoF robot.

attached to the first arm by a pivot link at point O_2 . Let us denote q_1 the rotational angle of the first arm about the z -axis measured counter-clockwise, q_2 the rotational angle of the second arm about the z -axis measured clockwise from the first arm position, and $q_{12} = q_1 + q_2$. The torques at Joint 1 and Joint 2 are respectively denoted by Γ_1 and Γ_2 .

Based on the Euler-Lagrange principle, the nonlinear dynamics of a serial 2-DoF robot is expressed as follows [18]:

$$\begin{aligned}\Gamma_1 &= (c_1 + 2c_2 \cos q_2)\ddot{q}_1 + (c_3 + c_2 \cos q_2)\ddot{q}_2 - c_2\dot{q}_2^2 \sin q_2 \\ &\quad + (fv_1 - 2c_2\dot{q}_2 \sin q_2)\dot{q}_1 + c_4 \sin q_1 + c_5 \sin q_{12} \\ \Gamma_2 &= (c_3 + c_2 \cos q_2)\ddot{q}_1 + c_3\ddot{q}_2 + c_2 \sin q_2 \dot{q}_1^2 + fv_2\dot{q}_2 \\ &\quad + c_5 \sin q_{12}\end{aligned}\quad (1)$$

where $c_1 = m_1 r_1^2 + I_1 + m_2 L_1^2 + m_2 r_2^2 + I_2$, $c_2 = m_2 L_1 r_2$, $c_3 = m_2 r_2^2 + I_2$, $c_4 = m_1 g r_1 + m_2 g L_1$, $c_5 = m_2 g r_2$.

Let us define the state vector as $x = [q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2]^\top$, the input $u = [\Gamma_1 \ \Gamma_2]^\top$, the output $y = [q_1 \ q_2]^\top$, the nonlinear system (1) can be represented in the form

$$\begin{aligned}E(x)\dot{x}(t) &= A(x)x(t) + Bu(t) \\ y(t) &= Cx(t)\end{aligned}\quad (2)$$

where

$$\begin{aligned}E(x) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_1 + 2c_2 z_1(x) & c_3 + c_2 z_1(x) \\ 0 & 0 & c_3 + c_2 z_1(x) & c_3 \end{bmatrix} \\ A(x) &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ z_2(x) & z_3(x) & 2z_4(x) - fv_1 & z_4(x) \\ z_3(x) & z_3(x) & z_5(x) & -fv_2 \end{bmatrix} \\ B &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^\top, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\end{aligned}$$

and $z_1(x) = \cos q_2$, $z_2(x) = -c_4 \frac{\sin q_1}{q_1} - c_5 \frac{\sin q_{12}}{q_{12}}$, $z_3(x) = -c_5 \frac{\sin q_{12}}{q_{12}}$, $z_4(x) = c_2 \dot{q}_2 \sin q_2$, $z_5(x) = -c_2 \sin q_2 \dot{q}_1$. We note that the matrix $E(x)$ is non-singular.

In this paper, the control goal is to achieve circular trajectory tracking and stabilization of the 2-DoF robot (1). We can observe that the dynamics of the considered serial manipulator is highly nonlinear and its model is represented in a descriptor

form. Due to these model features, the related control design is very challenging. We propose here a systematic control approach for this type of nonlinear systems. Especially, based on a robust control scheme, the numerical complexity of the designed controllers can be drastically reduced for real-time implementation.

TABLE I
SYSTEM NOMENCLATURE

Symbol	Description	Value
L_1, L_2	Lengths of robot arms (m)	0.5, 0.5
m_1, m_2	Masses of robot arms (kg)	15, 9
I_1, I_2	Inertia of robot arms (kgm ²)	0.313, 0.313
r_1, r_2	Distances between joints and mass center (m)	0.25, 0.25
f_{v1}, f_{v2}	Coefficients of friction (Nms/rd)	1/7, 1/7
g	Gravitational acceleration (m/s ²)	9.81

III. ROBUST CONTROL DESIGN FOR FUZZY TAKAGI-SUGENO DESCRIPTOR SYSTEMS

Hereafter, the control law based on T-S descriptor approach is applied of the 2-DoF robot.

A. T-S Fuzzy Descriptor Model

The so-called Takagi-Sugeno (T-S) fuzzy systems belong to a class of quasi-LPV systems [3]. The T-S model can exactly represent a nonlinear system in a compact set of the state space. It consists in a collection of linear descriptor model interconnected with nonlinear membership functions. The fuzzy T-S descriptor system subject to modeling uncertainties can be represented in the following form [19], [22]:

$$\begin{aligned}\sum_{k=1}^{r_e} v_k(z) \hat{E}_k \dot{x}(t) &= \sum_{i=1}^r h_i(z) \left(\hat{A}_i x(t) + \hat{B}_i u(t) \right) \\ y(t) &= \sum_{i=1}^r h_i(z) C_i x(t)\end{aligned}\quad (3)$$

where $\hat{E}_k = E_k + \Delta E_k$, $\hat{A}_i = A_i + \Delta A_i$, $\hat{B}_i = B_i + \Delta B_i$, $k \in \Omega_{r_e}$, $i \in \Omega_r$. In (3), $z_j(t)$, for $j \in \Omega_p$, are the premise variables, r_e and r are respectively the number of nonlinear functions for the left and right parts of the state equation. The membership functions $v_k(z) \geq 0$, $k \in \Omega_{r_e}$, $h_i(z) \geq 0$, $i \in \Omega_r$ satisfy the convex sum property. We suppose that the descriptor matrix \hat{E}_v is non-singular.

It is assumed that the system uncertainties can be represented in the form

$$\Delta E_k = H_{ek}^\top \Delta_e W_{ek}, \Delta A_i = H_{ai}^\top \Delta_a W_{ai}, \Delta B_i = H_{bi}^\top \Delta_b W_{bi}$$

for $k \in \Omega_{r_e}$, $i \in \Omega_r$, where $\Delta_l^\top \Delta_l(t) \leq I$ with $l \in \{e, a, b\}$. The T-S fuzzy system (3) can be equivalently rewritten in the following compact form:

$$\begin{aligned}E^* \dot{x}^*(t) &= A_{hv}^* x^*(t) + B_h^* u(t) \\ y(t) &= C_h^* x^*(t)\end{aligned}\quad (4)$$

where $E^* = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $A_{ik}^* = \begin{bmatrix} 0 & I \\ \hat{A}_i & -\hat{E}_k \end{bmatrix}$, $B_i^* = \begin{bmatrix} 0 \\ \hat{B}_i \end{bmatrix}$ and $C_i^* = [C_i \ 0]$. For control design of system (4), we consider the following extended parallel distributed compensation (PDC) control law [19]:

$$u(t) = - \sum_{i=1}^r \sum_{k=1}^{re} h_i(z(t)) v_k(z(t)) F_{ik}^* x^*(t) \quad (5)$$

where the local control gains $F_{ik}^* = [F_{ik} \ 0]$ are to be designed. From (4) and (5), the closed-loop descriptor T-S fuzzy system in extended form can be rewritten in the form

$$E^* \dot{x}^*(t) = \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^{re} h_i(z) h_j(z) v_k(z) \mathbb{A}_{ijk} x^*(t) \quad (6)$$

with $\mathbb{A}_{ijk} = A_{ik}^* - B_i^* F_{jk}^*$.

B. LMI-Based Robust Control Design

The following theorem provides an LMI-based solution for the robust control design of 2-DoF robot.

Theorem 1. Given an uncertain T-S fuzzy descriptor model (3) and a positive scalar α . The close-loop system (6) is asymptotically stable with a decay rate α under the effects of the PDC controller (5) if there exist matrices of appropriate dimensions $P_1 > 0$, P_{3ij} , P_{4ij} , M_{jk} , and positive scalars τ_{ijk}^a , τ_{ijk}^b , τ_{ijk}^e such that

$$\Xi_{iik} < 0, \quad \Xi_{ijk} + \Xi_{jik} < 0, \quad i, j \in \Omega_r, \quad k \in \Omega_{re}, \quad i < j \quad (7)$$

The quantity Ξ_{ijk} is defined as

$$\Xi_{ijk} = \begin{bmatrix} \Psi_{ij} & (*) & (*) & (*) & (*) \\ \Lambda_{ijk} & \Omega_{ijk} & 0 & 0 & (*) \\ W_{ai} P_1 & 0 & -\tau_{ijk}^a I & 0 & 0 \\ -W_{bi} M_{jk} & 0 & 0 & -\tau_{ijk}^b I & 0 \\ -W_{ek} P_{3ij} & -W_{ek} P_{4ij} & 0 & 0 & -\tau_{ijk}^e I \end{bmatrix}$$

where $\Psi_{ij} = P_{3ij} + P_{3ij}^\top + 2\alpha P_1$, $\Lambda_{ijk} = A_i P_1 - B_i F_{jk} - E_k P_{3ij} + P_{4ij}^\top$, $\Omega_{ijk} = -E_k P_{4ij} - (E_k P_{4ij})^\top + \tau_{ijk}^a H_{ai}^\top H_{ai} + \tau_{ijk}^b H_{bi}^\top H_{bi} + \tau_{ijk}^e H_{ek}^\top H_{ek}$. Furthermore, the control gains of the PDC controller (5) can be computed as follows:

$$F_{jk} = M_{jk} P_1^{-1}. \quad (8)$$

Proof. Let

$$P_{hh} = \begin{bmatrix} P_1 & 0 \\ P_{3hh} & P_{4hh} \end{bmatrix} = \sum_{i=1}^r \sum_{j=1}^r h_i(z) h_j(z) \begin{bmatrix} P_1 & 0 \\ P_{3ij} & P_{4ij} \end{bmatrix}.$$

It will be shown above that P_{4hh} is non-singular. Hence, the matrix P_{hh} is also non-singular. Consider the following Lyapunov function candidate:

$$V(x^*) = x^{*\top} E^* P_{hh}^{-1} x^* \quad (9)$$

The time-derivative of the Lyapunov function (9) along the trajectory of system (6) is given as follows:

$$\begin{aligned} \dot{V}(x^*) &= \dot{x}^{*\top} E^{*\top} P_{hh}^{-1} x^* + x^{*\top} (P_{hh}^{-1})^\top E^* \dot{x}^* \\ &\quad + x^{*\top} E^{*\top} \dot{P}_{hh}^{-1} x^* \end{aligned} \quad (10)$$

Due to the definition of E^* and P_{hh} , it follows that $E^{*\top} \dot{P}_{hh}^{-1} = 0$. This allows to avoid the *unknown* time-derivatives of the membership functions. It should be noticed that dealing with *unknown* time-derivatives of the membership functions in the T-S fuzzy control framework still remains an open research issue, see [23]. The closed-loop system (6) is asymptotically stable with a decay rate α if

$$\dot{V}(x^*) < -2\alpha V(x^*) \quad (11)$$

From (9) and (10), condition (11) is guaranteed if

$$\hat{\Upsilon} = \text{He} \left((A_{hv}^* - B_h^* F_{hv}^*)^\top P_{hh}^{-1} + \alpha E^* P_{hh}^{-1} \right) < 0 \quad (12)$$

Pre- and post-multiplying (12) with P_{hh}^\top and its transpose, it follows that

$$\hat{\Upsilon} = \text{He} (A_{hv}^* P_{hh} - B_h^* F_{hv}^* P_{hh} + \alpha E^* P_{hh}) < 0 \quad (13)$$

After some simple computations, we can represent inequality (13) in the form

$$\hat{\Upsilon} = \Upsilon + \Delta \Upsilon < 0$$

where

$$\begin{aligned} \Upsilon &= \text{He} \begin{bmatrix} P_{3hh} + \alpha P_1 & P_{4hh} \\ A_{hv} P_1 - B_h M_{hv} - E_h P_{3hh} & -E_v P_{4hh} \end{bmatrix} \\ \Delta \Upsilon &= \bar{H}^\top \Delta(t) \bar{W} + \bar{W}^\top \Delta(t) \bar{H}. \\ M_{hv} &= F_{hv} P_1 \end{aligned}$$

The matrices characterizing the model uncertainties are given as follows:

$$\begin{aligned} \bar{H} &= \begin{bmatrix} 0 & H_{a_h} \\ 0 & H_{b_h} \\ 0 & H_{e_v} \end{bmatrix}, \quad \bar{W} = \begin{bmatrix} W_{a_h} P_1 & 0 \\ -W_{b_h} M_{hv} & 0 \\ -W_{e_v} P_{3hh} & -W_{e_v} P_{4hh} \end{bmatrix} \\ \Delta(t) &= \text{diag} (\Delta_a(t), \Delta_b(t), \Delta_e(t)). \end{aligned}$$

Let

$$S_{hhv} = \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^{re} h_i(z) h_j(z) v_k(z) \text{diag} \left(\tau_{ijk}^a I, \tau_{ijk}^b I, \tau_{ijk}^e I \right).$$

Using the following well-known matrix property (completion of squares)

$$\mathbf{S} = \mathbf{S}^\top > 0, \quad \mathbf{X}^\top \mathbf{Y} + \mathbf{Y}^\top \mathbf{X} \leq \mathbf{X}^\top \mathbf{S} \mathbf{X} + \mathbf{Y}^\top \mathbf{S}^{-1} \mathbf{Y}$$

with $\mathbf{S} = S_{hhv}$, $\mathbf{X} = \bar{H}$, $\mathbf{Y} = \Delta(t) \bar{W}$, while considering the fact that $\Delta(t)^\top \Delta(t) \leq I$ (from the assumption on modeling uncertainty), it can be deduced that

$$\begin{aligned} \Upsilon + \Delta \Upsilon &\leq \Upsilon + \bar{H}^\top S_{hhv} \bar{H} + \bar{W}^\top \Delta^\top(t) S_{hhv}^{-1} \Delta(t) \bar{W} \\ &\leq \Upsilon + \bar{H}^\top S_{hhv} \bar{H} + \bar{W}^\top S_{hhv}^{-1} \bar{W} \end{aligned} \quad (14)$$

Since $S_{hhv} > 0$, by Schur complement lemma, inequality (14) is equivalent to

$$\Xi_{hhv} = \begin{bmatrix} \Upsilon + \bar{H}^\top S_{hhv} \bar{H} & \bar{W}^\top \\ \bar{W} & -S_{hhv} \end{bmatrix} < 0 \quad (15)$$

From the relaxation lemma in [3], the matrix inequalities (7) imply (15). It just now remains to prove that the matrix P_{4hh} is non-singular. The second diagonal block of matrix

$\Xi_{h_h v}$ is equal to the sum of $-\text{He}(E_v P_{4hh})$ and a semi-definite positive matrix and is definite-negative. This implies that $\text{He}(E_v P_{4hh}) > 0$, and consequently that P_{4hh} is non-singular. \square

For the nominal case, i.e. $\Delta E_v = 0$, $\Delta A_h = 0$, $\Delta B_h = 0$, the design conditions of the T-S fuzzy descriptor model can be easily deduced as in the following corollary.

Corollary 1. Given a nominal T-S fuzzy descriptor model (3) and a positive scalar α . The close-loop system (6) is asymptotically stable with a decay rate α under the effects of the PDC controller (5) if there exist matrices of appropriate dimensions $P_1 > 0$, P_{3ij} , P_{4ij} , M_{jk} , and positive scalars τ_{ijk}^a , τ_{ijk}^b , τ_{ijk}^e such that

$$\Pi_{iik} < 0, \quad \Pi_{ijk} + \Pi_{jik} < 0, \quad i < j \in \Omega_r, \quad k \in \Omega_{r_e}$$

The quantity Π_{ijk} is defined as

$$\Pi_{ijk} = \text{He} \begin{bmatrix} P_{3ij} & P_{4ij} \\ A_i P_1 - B_i F_{jk} - E_k P_{3ij} + \alpha P_1 & -E_k P_{4ij} \end{bmatrix}.$$

The feedback gains are given in (8).

Remark 1. A decay rate performance is incorporated in the Lyapunov stability condition to improve the closed-loop convergence of the robot system. A large value of the decay rate leads to a quick convergence, however it may cause aggressive closed-loop behaviors.

IV. APPLICATION TO ROBOT FUZZY TRACKING CONTROL

In the sequel, we apply the T-S fuzzy descriptor control approach presented in the previous section to the trajectory tracking of the 2-DoF robot (1). For tracking control purposes, we make use of an integral control structure, see Fig. 2. This amounts adding a new system state $x_l(t)$ whose dynamics is defined as follows:

$$\dot{x}_l(t) = y_d(t) - y(t)$$

where $y_d(t) = [q_{1d}(t) \ q_{2d}(t)]^\top$. Here, $q_{1d}(t)$ and $q_{2d}(t)$ represent the desired angular positions to be tracked by the studied robot. Let us denote the extended state vector $\bar{x}(t)^\top = [x^*(t)^\top \ x_l(t)^\top]$, the T-S fuzzy descriptor used for tracking control purposes can be represented as

$$\begin{aligned} \bar{E} \dot{\bar{x}}(t) &= \bar{A}_{hv} \bar{x}(t) + \bar{B}_h u(t) + B_0 y_d(t) \\ y(t) &= \bar{C}_h \bar{x}(t) \end{aligned}$$

with $\bar{E} = \begin{bmatrix} E^* & 0 \\ 0 & I \end{bmatrix}$, $\bar{A}_{hv} = \begin{bmatrix} A_{hv}^* & 0 \\ -C_h^* & 0 \end{bmatrix}$, $\bar{B}_h = \begin{bmatrix} B_h^* \\ 0 \end{bmatrix}$, $B_0 = \begin{bmatrix} 0 & I \end{bmatrix}^\top$, $\bar{C}_h = \begin{bmatrix} C_h^* & 0 \end{bmatrix}$. Then, the corresponding extended PDC controller is of the form

$$u(t) = - \sum_{i=1}^r \sum_{k=1}^{r_e} h_i(z) v_k(z) \bar{F}_{ik} \bar{x}(t)$$

where the local control gains $\bar{F}_{ik} = [F_{ik}^* \ L_{ik}]$, $i \in \Omega_r$, $k \in \Omega_{r_e}$ can be straightforwardly computed from Theorem 1 or Corollary 1.

Remark 2. The design conditions in both Theorem 1 and Corollary 1 are recast as an LMI optimization problem which can be easily solved with available numerical solvers. In this work, Matlab LMI Toolbox was used to compute the feedback gains of the tracking controllers.

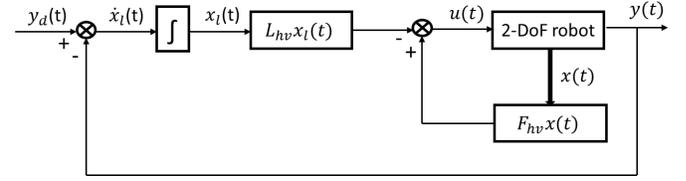


Fig. 2. Integral PDC control structure for robot tracking purposes.

A. Exact T-S Fuzzy Descriptor Representation

The five premise variables (i.e. system nonlinearities) of the nonlinear descriptor model (2) can be naturally defined as $z_j(t)$ where $z_{j \min} \leq z_j(t) \leq z_{j \max}$, for $j \in \Omega_5$. In this paper, the workspace of the studied manipulator is defined as

$$\begin{aligned} |q_1| &\leq \pi \text{ (rad)}, & |q_2| &\leq \pi \text{ (rad)} \\ |\dot{q}_1| &\leq 15 \text{ (rad/s)}, & |\dot{q}_2| &\leq 15 \text{ (rad/s)} \end{aligned}$$

From the physical parameters given in Table I, we can obtain the following bounds on the premise variables: $z_{1 \max} = -z_{1 \min} = 1$, $z_{2 \max} = z_{3 \max} = 4.79$, $z_{2 \min} = -103.01$, $z_{3 \min} = -22.07$, $z_{4 \max} = z_{5 \max} = -z_{4 \min} = -z_{5 \min} = 16.87$. Then, using the sector nonlinearity approach in [3], an *exact* T-S fuzzy descriptor representation for the 2-DoF robot can be easily derived with $2^5 = 32$ local linear subsystems. This 32-rules exact T-S fuzzy representation is not given here due to the space limitation.

B. T-S Fuzzy Descriptor Model with Reduced Complexity

Note that the control design method proposed in Section III can be theoretically used to deal with T-S fuzzy descriptor systems with any number of rules. However, a large rule number may cause costly computational burden for real-time applications. Therefore, it is worth reducing the numerical complexity of the T-S fuzzy models to cope with the hardware limitations. To achieve this goal, some system nonlinearities will be considered as modeling uncertainties to reduce the number of premise variables, thus the rule number. Then, these uncertainties can be dealt with using the proposed robust control scheme. We present here an example when deriving a 8-rules T-S fuzzy system from the descriptor system (2). To this end, we consider

$$z_4 = z_{4 \max} f_1(t), \quad z_5 = z_{5 \max} f_2(t)$$

with $|f_i(t)| \leq 1$, $i = 1, 2$. Then, the system (2) can be represented in the form

$$E(x) \dot{x}(t) = (\bar{A}(x) + \Delta \bar{A}(t)) x(t) + B u(t) \quad (16)$$

where $\Delta\bar{A}(t) = H_a\Delta_a(t)W_a$ and

$$\bar{A}(x) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ z_2(x) & z_3(x) & -fv_1 & 0 \\ z_3(x) & z_3(x) & 0 & -fv_2 \end{bmatrix}, \quad H_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Delta_a(t) = \text{diag}(f_1(t), f_2(t)), \quad W_a = \begin{bmatrix} 0 & 0 & 2z_{4 \max} & z_{4 \max} \\ 0 & 0 & z_{5 \max} & 0 \end{bmatrix}$$

Observe that the nonlinear uncertain descriptor system (16) has now only 3 premise variables $z_1(t)$, $z_2(t)$ and $z_3(t)$. Hence, using the sector nonlinearity approach, the corresponding T-S fuzzy representation (3) of the nonlinear uncertain system (16) has $2^3 = 8$ rules where the state-space matrices are given as

$$E_1 = E(z_{1 \min}), \quad E_2 = E(z_{1 \max})$$

$$A_1 = \bar{A}(z_{2 \min}, z_{3 \min}), \quad A_2 = \bar{A}(z_{2 \max}, z_{3 \min})$$

$$A_3 = \bar{A}(z_{2 \min}, z_{3 \max}), \quad A_4 = \bar{A}(z_{2 \max}, z_{3 \max})$$

Let us define

$$\omega_{i1} = \frac{z_i - z_{i \min}}{z_{i \max} - z_{i \min}}, \quad \omega_{i2} = 1 - \omega_{i1}, \quad i \in \Omega_3$$

The membership functions of the above T-S fuzzy descriptor system are expressed as follows:

$$v_1(z) = \omega_{11}, \quad h_1(z) = \omega_{21}\omega_{31}, \quad h_2(z) = \omega_{22}\omega_{31}$$

$$v_2(z) = \omega_{12}, \quad h_3(z) = \omega_{21}\omega_{32}, \quad h_4(z) = \omega_{22}\omega_{32}$$

Similarly, we can obtain a 4-rules (respectively 2-rules) T-S fuzzy model by considering z_2, z_4, z_5 (respectively z_1, z_2, z_4, z_5) as system uncertainties. It is clear that a *linear* uncertain descriptor system can be easily obtained following the same line while considering all premise variables as model uncertainties. The details of these uncertain T-S fuzzy descriptor systems are not given here for brevity reasons.

V. ILLUSTRATIVE RESULTS AND DISCUSSIONS

This section provides numerical results carried out with SimMechanics environment in Matlab/Simulink software to demonstrate the effectiveness of the proposed control method. Especially, a comparative study between the nominal case and different robust controllers derived from reduced-complexity T-S fuzzy representation is given to emphasize the interest of the proposed robust control scheme for real-time applications. It should be stressed that when solving LMI conditions in Theorem 1, feasible control solutions can only be obtained with T-S fuzzy descriptor models in Section IV-B having more than 4 rules. This means that the reduction of the numerical model complexity is directly related to the design conservatism. The simplest PDC tracking controller that can be obtained from Theorem 1 in our case has 4 fuzzy rules.

Some control feedback gains obtained from LMI conditions in Theorem 1 with $\alpha = 0.85$ are given as follows:

$$M_{11} = 10^3 \times \begin{bmatrix} 2.57 & 0.57 & 0.31 & 0.08 \\ 0.67 & 1.33 & 0.07 & 0.13 \end{bmatrix} \begin{bmatrix} -5.39 & -1.11 \\ -1.45 & -3.07 \end{bmatrix}$$

$$M_{12} = 10^3 \times \begin{bmatrix} 2.70 & 0.53 & 0.32 & 0.07 \\ 0.73 & 1.40 & 0.08 & 0.14 \end{bmatrix} \begin{bmatrix} -5.44 & -0.95 \\ -1.52 & -3.08 \end{bmatrix}$$

$$M_{22} = 10^3 \times \begin{bmatrix} 2.79 & 0.17 & 0.34 & 0.04 \\ 0.38 & 2.16 & 0.03 & 0.21 \end{bmatrix} \begin{bmatrix} -5.77 & -0.20 \\ -0.82 & -4.86 \end{bmatrix}$$

The above feedback gains are significantly different which also justifies a posteriori the interest of a T-S control approach compared to a linear one in terms of performance improvement and/or design relaxation.

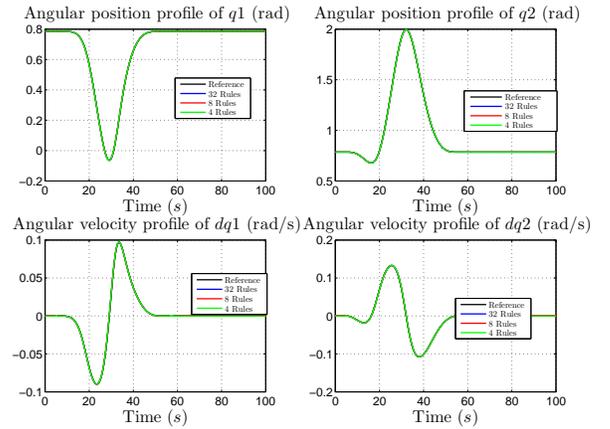


Fig. 3. Angular position and velocity profile.

We note that the tracking positions in the xy -plane can be expressed in function of the desired angular positions as

$$x_d = L_1 \cos(q_{1d}) + L_2 \cos(q_{1d} + q_{2d})$$

$$y_d = L_1 \sin(q_{1d}) + L_2 \sin(q_{1d} + q_{2d})$$

The desired angular positions q_{1d} and q_{2d} that the 2-DoF manipulator should follow are depicted in Fig. 3. This corresponds to a *circular* trajectory with a radius of $R = 0.2$ (m), see Fig. 4. We can observe that the 4-rules PDC controller can provide a similar tracking performance compared to 32-rules and 8-rules PDC controllers. This is confirmed by small tracking errors shown in Fig. 6.

From the comparative results between three cases (32-rules, 8-rules and 4-rules), we can see that the proposed robust control approach can reduce drastically the numerical complexity from 32-rules to 4-rules of T-S fuzzy descriptor representation while guaranteeing a high tracking performance.

VI. CONCLUSIONS

A robust T-S fuzzy descriptor approach was applied to control a nonlinear 2-DoF serial manipulator. This approach can not only guarantee a high tracking performance but also helps to reduce drastically the numerical complexity of the controllers via a robust compensation scheme. The illustrative

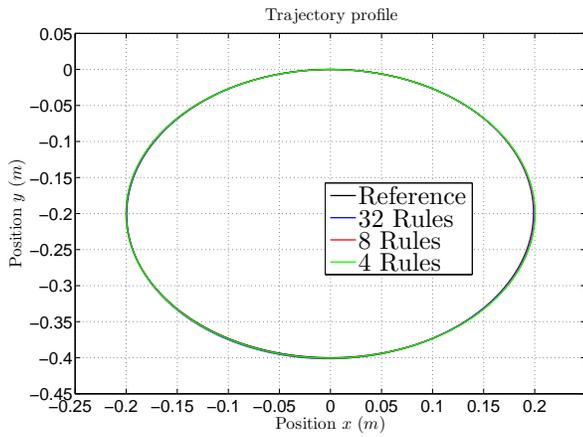


Fig. 4. Circular trajectory tracking.

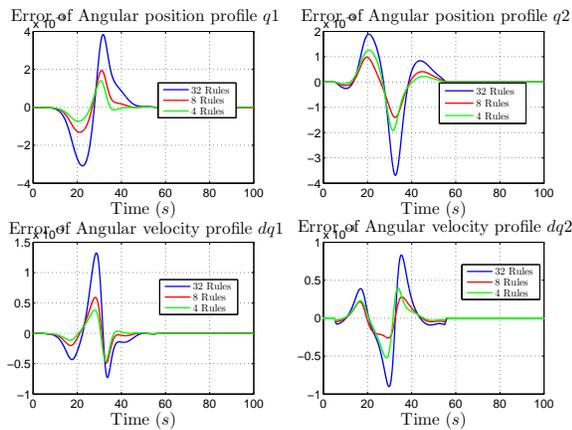


Fig. 5. Tracking errors of angular position and velocity profile.

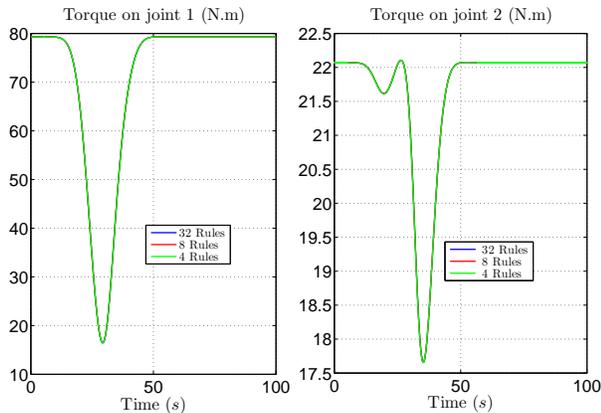


Fig. 6. Torques at Joint 1 and Joint 2.

results have proved the effectiveness of the proposed robust T-S fuzzy descriptor approach in the case of circular trajectory tracking. Since the angular velocities of the robot cannot be measured in practice, future works focus on a robust output

feedback scheme requiring only available measurements for real-time implementation.

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