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A novel parameter-dependent polynomial approach for robust automated lane keeping

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Abstract This paper is concerned with a novel control technique for automated lane keeping of a vehicle, which takes advantage of an exact fuzzy modelling of bounded parametric uncertainties –both constant and varying– for a convex treatment of local characteristic polynomials, put together via parameter-dependent Lyapunov analysis. It is shown that the specificity of the proposed technique enlarges the feasibility chances of synthesizing a robust steering control law in contrast with only-Lyapunov-based designs. The proposal is put at test in simulation for the perturbed bicycle model.

Keywords Takagi-Sugeno Fuzzy Model · Robust Lane Keeping Control · Parameter-Dependent Lyapunov Functions · Characteristic Polynomial

1 Introduction

Steering control for automated lane keeping has become an important research topic due to the positive impact on driver safety resulting in a reduced number of car accidents. Several works have been developed considering the vehicle lateral dynamics model; some of them assume that the longitudinal velocity v_x remains constant during a given maneuver [1,2]; some others consider it varying, but bounded, which allows using the *sector nonlinearity* approach [3] to write it as a convex sum [4,5]. The use of convex modeling is common within the linear parameter varying (LPV) and Takagi-Sugeno (TS) fuzzy control communities; it is usually combined with the direct Lyapunov method to produce design conditions in the form of linear matrix inequalities (LMIs)

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[6,7]; the resulting control law is known as parallel distributed compensation (PDC), which enables the designer to use the system nonlinearities which are assumed available [8]. In [9] a linear state feedback controller is synthesized through robust yaw rate control by driving the sideslip angle and the yaw rate to zero; despite the fact that time-varying parameters (the vehicle mass m and the moment of inertia I_z) are convexly modeled using knowledge about its bounds, the control law remains linear as the parameters are unknown and therefore cannot be used for control purposes. In addition to mass and inertia, some papers consider uncertainty in the cornering stiffness coefficients; under these conditions, a controller implementing \mathcal{H}_∞ control via LMIs can be found in [10].

In [11] a linear output feedback controller is designed, which includes validation of the lane-keeping system. In [12] a linear time invariant (LTI) model parameterized by the vehicle longitudinal velocity is used; while the experimental results assume the longitudinal velocity remains approximately constant, the controller design of the single-input two-output (SITO) system is transformed into the design of a controller for a SISO one via μ -synthesis, which guarantees robustness with respect to model uncertainties such as vehicle mass, moment of inertia, the rear and front cornering stiffness, and the longitudinal velocity. In [13] the same linear model parameterized with respect to the lateral velocity is employed; it also assumes the velocity is a slow-varying parameter and includes experimental validation.

In [14], a Takagi-Sugeno (TS) observer with unknown inputs is used to estimate the sideslip and steering angles for control purposes; otherwise the approach is no different from the LTI assumptions above. Similarly, [15] uses an observer to estimate the sideslip angle, but the control law employs a nonlinear model of the vehicle tyres as well as nonlinear terms. In [16], a PDC controller is designed with the steering angle and the external yaw moment as inputs; it is assumed the time-varying parameters are $1/v_x$, $1/v_x^2$, and an expression depending on the yaw rate, which in turn depends on the longitudinal speed: no uncertainties are considered in the problem formulation. In [17] a TS model, norm-bounded uncertainty, and \mathcal{H}_∞ techniques are employed; the number of polytopes obtained from convexly modeling v_x and $1/v_x$ is reduced from 4 to 3 using a triangulation technique. In [18] a TS modeling is applied only to model the vehicle mass as a time-varying parameter; its control law has a PDC structure, but assumes the mass is available at every instant of time. Some other works on autonomous vehicles can be found in the literature [19, 20], but the techniques referred therein –sliding modes, attention assistance– are out of the scope of this work.

Motivation: The prevalent use of LTI models which oversee the nonlinear nature of the vehicle lateral dynamics and the use of Lyapunov-based methodologies for any unknown constant parameter, has limited the feasibility chances for controller design: in the first case by limiting the control law to be linear (thus being operational only around a given point); in the second case by using a tool that applies unchanged to time-varying parameters (thus lacking specificity). This work intends to alleviate this burden by using a specific tool

for unknown constant parameters, and a more generic one for the time-varying velocity v_x .

Contribution: This work presents a novel methodology for designing a robust convex steering control law for automated lane keeping; it employs LMI polynomial approaches recently appeared which are specifically addressed to parameters which are unknown, but constant [21], and Lyapunov-based methodologies to include time-varying velocity v_x as a nonlinear variable in a PDC-like control law [7]. It is shown that the scheme allows using parameter-dependent Lyapunov functions to ease the feasibility of the whole scheme without further adjustments.

Organization: The vehicle lateral dynamics and its transformation into an exact TS fuzzy model via the sector nonlinearity approach are presented in section 2; section 3 presents a first approach based on an adaptation of a PDC control law of the sort employed in [4, 16, 17], which makes use of available signals for control purposes while taking into account the possible range of unknown, but fixed parameters: it belongs to the classical TS framework; section 4 constitutes the core of this report as a novel polynomial approach is developed and combined with parameter-dependent Lyapunov functions to handle fixed and varying parameters: it is contrasted with former PDC approaches; finally, conclusions are discussed in section 5.

Notation: Throughout this manuscript, (\cdot) and $(\bar{\cdot})$ denote the lower and upper bounds of (\cdot) , respectively. θ denotes the vector of constant uncertain parameters. ζ_i and z denote the terms made of constant uncertain parameters and time-varying measurable parameters appearing in the system dynamics, respectively. $(\cdot)^0$ and $(\cdot)^1$ denote the respective lower and upper bounds of ζ_i and z . $P > 0$ ($P \geq 0$) means that P is a positive definite (positive semidefinite) matrix. Similarly, $P < 0$ ($P \leq 0$) means that P is negative definite (negative semidefinite). \mathbb{B}^n stands for the set of n -bit binary numbers.

2 Error dynamics for automated lane keeping

The model considered in this work is that of [22]; it is based on the linearized dynamics of two error signals: the distance from the center of the road lane to the vehicle center of gravity (e_1), and the difference between the vehicle orientation and the center of the road lane (e_2). The system parameters corresponding to the total cornering stiffness at the front and rear wheels ($C_{\alpha f}$ and $C_{\alpha r}$, respectively), the vehicle total mass (m) and total moment of inertia around the z axis (I_z) will be considered constant, but unknown; they will be grouped in a vector $\theta = [m \ I_z \ C_{\alpha r} \ C_{\alpha f}]^T$. The distance from the center of the vehicle front wheel axis and from the vehicle rear wheel axis to the center of gravity (l_f and l_r , respectively), will be considered fixed and given. The vehicle longitudinal speed ($v_x(t)$) will be considered measurable and time-dependent. Thus, the error dynamics are:

$$\dot{x}(t) = A(v_x(t), \theta) x(t) + B(\theta) u(t) + B_2(v_x(t), \theta) \dot{\psi}_{\text{des}}(t), \quad (1)$$

where the state vector is $x = [e_1 \dot{e}_1 e_2 \dot{e}_2]^T$, $u(t)$ is the front wheel steering angle which acts as the control input of the system, $\dot{\psi}_{\text{des}}$ is the desired yaw rate (given by the road curvature), and the system matrices are

$$A(v_x(t), \theta) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f} + 2C_{\alpha r}}{mv_x} & \frac{2C_{\alpha f} + 2C_{\alpha r}}{m} & -\frac{2C_{\alpha f} l_f + 2C_{\alpha r} l_r}{mv_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_{\alpha f} l_f - 2C_{\alpha r} l_r}{I_z v_x} & \frac{2C_{\alpha f} l_f - 2C_{\alpha r} l_r}{I_z} & -\frac{2C_{\alpha f} l_f^2 + 2C_{\alpha r} l_r^2}{I_z v_x} \end{bmatrix}, \quad (2)$$

$$B(\theta) = \begin{bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2C_{\alpha f} l_f}{I_z} \end{bmatrix}, \quad B_2(v_x(t), \theta) = \begin{bmatrix} 0 \\ -\frac{2C_{\alpha f} l_f - 2C_{\alpha r} l_r}{m v_x} - v_x \\ 0 \\ -\frac{2C_{\alpha f} l_f^2 + 2C_{\alpha r} l_r^2}{I_z v_x} \end{bmatrix}. \quad (3)$$

Lane-keeping control intends to drive the error signals to zero, i.e., to stabilize (1) by means of $u(t)$. This task will be done under the following assumptions:

Assumption 1 $C_{\alpha f}$, $C_{\alpha r}$, m , and I_z are unknown parameters with known bounds whose values remain fixed during the lane-keeping system operation [2, 9, 23].

Assumption 2 v_x is a measurable time-varying parameter bounded by known values during the lane-keeping system operation. Its lower and upper bound will be denoted as \underline{v}_x and \bar{v}_x , respectively [17, 5].

Note that the unknown parameters considered in Assumption 1 actually vary during the vehicle's journey, for example, the road and the state of the tires influence the cornering stiffness values whereas vehicle load and the way it is distributed influence the mass and moment of inertia. However, it is reasonable to consider that these parameters remain constant during a given maneuver [2, 9, 23]. Controller design is usually based on the assumption that yaw rate $\dot{\psi}_{\text{des}} = 0$, since it is an exogenous variable and as such it is not controlled: it can be seen as a reference driving the system towards the center of the lane. This assumption simplifies (1) to

$$\dot{x}(t) = A(v_x(t), \theta) x(t) + B(v_x(t), \theta) u(t). \quad (4)$$

According to assumption 1, the parametric uncertainties belong to a compact, i.e., $\theta \in \mathcal{C} \equiv [\underline{m}, \bar{m}] \times [\underline{I}_z, \bar{I}_z] \times [\underline{C}_{\alpha r}, \bar{C}_{\alpha r}] \times [\underline{C}_{\alpha f}, \bar{C}_{\alpha f}] \subset \mathbb{R}^4$. Bounded expressions can be written as convex sums of their bounds [3], which means (4) can be convexly rewritten by means of the nonlinear sector methodology prevalent in TS fuzzy systems literature [7]. In order to do so, let us first identify all the terms in $A(v_x(t), \theta)$ and $B(v_x(t), \theta)$ that are unknown, but constant (i.e., depending on θ), namely:

$$\zeta_1(\theta_1) = 1/m, \quad \zeta_2(\theta_2) = 1/I_z, \quad \zeta_3(\theta_3) = C_{\alpha r}, \quad \zeta_4(\theta_4) = C_{\alpha f}, \quad (5)$$

and those which are known, but time-varying (i.e., depending on $v_x(t)$):

$$z(v_x) = 1/v_x. \tag{6}$$

This split of signals intends to consider the bounds of those which are unknown for stability analysis as well as using the available signals for control purposes; based on it, the following pairs of convex functions will be defined for each $\zeta_i(\theta) \in [\zeta_i^0, \zeta_i^1]$

$$\omega_0^i(\zeta_i(\theta)) = \frac{\zeta_i^1 - \zeta_i(\theta)}{\zeta_i^1 - \zeta_i^0}, \quad \omega_1^i(\zeta_i(\theta)) = \omega_0^i - 1, \tag{7}$$

with $i = \{1, 2, 3, 4\}$, $\omega_0^i(\zeta) + \omega_1^i(\zeta) = 1$, $0 \leq \omega_k^i(\zeta) \leq 1$, $k = \{0, 1\}$, for all $\theta \in \mathcal{C}$; similarly, for $z(v_x) \in [z^0, z^1]$ we have

$$w_0(z(v_x)) = \frac{z^1 - z(v_x)}{z^1 - z^0}, \quad w_1(z(v_x)) = w_0 - 1, \tag{8}$$

with $w_0(z) + w_1(z) = 1$, $0 \leq w_k(z) \leq 1$, $k = \{0, 1\}$, for all $v_x \in [v_x, \bar{v}_x]$.

Thus, with (7) and (8) it is possible to rewrite (5) and (6) in a convex manner:

$$\zeta_i(\theta) = \underbrace{\left(\frac{\zeta_i^1 - \zeta_i(\theta)}{\zeta_i^1 - \zeta_i^0}\right)}_{\omega_0^i(\zeta_i(\theta))} \zeta_i^0 + \underbrace{\left(\frac{\zeta_i(\theta) - \zeta_i^0}{\zeta_i^1 - \zeta_i^0}\right)}_{\omega_1^i(\zeta_i(\theta))} \zeta_i^1, \tag{9}$$

$$z(v_x) = \underbrace{\left(\frac{z^1 - z(v_x)}{z^1 - z^0}\right)}_{w_0(z(v_x))} z^0 + \underbrace{\left(\frac{z(v_x) - z^0}{z^1 - z^0}\right)}_{w_1(z(v_x))} z^1. \tag{10}$$

In order to ease the notation, let us define $\mathbb{B} = \{0, 1\}$, $\mathbf{i} = (i_1, i_2, i_3, i_4) \in \mathbb{B}^4$ and $\omega_{\mathbf{i}}(\zeta) = \omega_{i_1}^1 \omega_{i_2}^2 \omega_{i_3}^3 \omega_{i_4}^4$. Thus, (4) can be rewritten as the following convex sum:

$$\dot{x}(t) = \sum_{\mathbf{i} \in \mathbb{B}^4} \sum_{j \in \mathbb{B}} \omega_{\mathbf{i}}(\zeta) w_j(z) (A_{\mathbf{i}j} x(t) + B_{\mathbf{i}} u(t)), \tag{11}$$

with $A_{\mathbf{i}j} = A(v_x, \theta)|_{\omega_{\mathbf{i}}=1, w_j=1}$, $B_{\mathbf{i}} = B(v_x, \theta)|_{\omega_{\mathbf{i}}=1}$. Convex rewriting (11) is advantageous because it allows designing controllers that simultaneously stabilize the infinite number of dynamics contained in (4).

3 A partial parallel distributed compensation

For comparison purposes, let us consider a PDC-like control law of the sort employed in [4, 16, 17], that makes use of the fact that $v_x(t)$ is a measurable time-varying parameter, i.e., it is based only on partial information of the convex functions:

$$u(t) = \sum_{j \in \mathbb{B}} w_j(z) F_j x(t), \tag{12}$$

with $w_j(z)$ as defined in (8) and where F_j , $j \in \mathbb{B}$, are controller gains to be designed. Stabilizing gains F_j for (12) can be computed by solving the LMI feasibility problem using the following result, which is an adaptation of the Lyapunov-based one in [7]:

Theorem 1 *The origin $x = 0$ of the uncertain linear time-varying system (4) under the control law (12) is asymptotically stable with decay rate β for all $\theta \in \mathcal{C}$ if there exist matrices $X = X^T > 0$ and $M_j \in \mathbb{R}^{1 \times 4}$ such that the following LMIs are satisfied:*

$$A_{\mathbf{i}j}X + B_{\mathbf{i}}M_j + (A_{\mathbf{i}j}X + B_{\mathbf{i}}M_j)^T + 2\beta X < 0 \quad (13)$$

for all $\mathbf{i} \in \mathbb{B}^4$, $j \in \mathbb{B}$. The control gains are computed as $F_j = M_j \cdot X^{-1}$.

Proof Considering (12) and the convex rewriting of (4), (11), we have the closed-loop system

$$\dot{x} = \sum_{\mathbf{i} \in \mathbb{B}^4} \sum_{j \in \mathbb{B}} \omega_{\mathbf{i}}(\zeta) w_j(z) (A_{\mathbf{i}j} + B_{\mathbf{i}}F_j) x.$$

Using a quadratic Lyapunov function candidate $V(x) = x^T P x$, $P = P^T > 0$ whose time derivative is

$$\begin{aligned} \dot{V} &= 2\dot{x}^T P x = x^T P \dot{x} + \dot{x}^T P x, \\ &= \sum_{\mathbf{i} \in \mathbb{B}^4} \sum_{j \in \mathbb{B}} \omega_{\mathbf{i}}(\zeta) w_j(z) x^T [P (A_{\mathbf{i}j} + B_{\mathbf{i}}F_j) + (A_{\mathbf{i}j}^T + F_j^T B_{\mathbf{i}}^T) P] x, \end{aligned}$$

we can guarantee $\dot{V}(x) < -2\beta V(x)$ if

$$A_{\mathbf{i}j}X + B_{\mathbf{i}}M_j + (A_{\mathbf{i}j}X + B_{\mathbf{i}}M_j)^T + 2\beta X < 0, \quad (14)$$

where the convexity of $\omega_{\mathbf{i}}$ and w_j has been taken into account, thus concluding the proof.

Remark 1 Computational complexity: Generalizing Theorem 1 to r_{ζ} constant unknown parameters and r_z measurable parameters, its computational complexity amounts to solving a number of LMIs bounded by $\mathcal{O}(2^{r_{\zeta}+r_z})$ with a number of variables bounded by $\mathcal{O}(n^2 + m \times n \times 2^{r_z})$.

Remark 2 Conservativeness: Notice that the theorem above is entirely based on the direct Lyapunov method, which guarantees the controller to stabilize the system regardless of the nature of the bounded terms. Indeed, the fact that most of the parameters are fixed is immaterial to the approach: if found feasible, Theorem 1 provides a valid controller for the family of systems with the same structure and bounds, even if the parameters are state-, time-, or exogenous-dependent. Thus, the lack of specificity of this approach points to its conservativeness.

Example 1 Consider the convex system (11) under the control law (12), where the uncertain terms (5) and the time-dependent term (6) are bounded according to table 1.

LMIs (13) are programmed in the LMI Toolbox of Matlab altogether with the LMIs corresponding to an input restriction $|u(t)| < \mu$:

$$\begin{bmatrix} X & M_j^T \\ M_j & \mu^2 I \end{bmatrix} \geq 0, \quad \begin{bmatrix} 1 & x^T(0) \\ x(0) & X \end{bmatrix} \geq 0,$$

with $\mu = 0.1047$ (6 degrees), which corresponds to the maximum input for a J-turn maneuver [4][9]. Thus, the following gains are obtained:

$$F_1 = [35.461 \ 4.092 \ 128.468 \ 0.333],$$

$$F_2 = [34.04 \ 3.823 \ 123.724 \ 0.447],$$

with $\beta_{\max} = 1.286$. Simulating system (1)-(3) with initial conditions $x(0) = [0 \ 0 \ 0 \ 0]^T$ and uncertain terms chosen as $m = 1730.3$, $I_z = 2442.1$, $C_{\alpha_f} = 60000$, and $C_{\alpha_r} = 100000$, the time-dependent term as $v_x(t) = 15 \sin(\pi t/10) + 25$, the disturbance given by $\dot{\psi}_{des} = 0$ when $t < 1$ and $\frac{v_x}{R}$ (rad/s) when $t \geq 1$ (which corresponds to a road that is assumed straight until $t = 1$ and circular with a radius $R = 1000$ meters afterwards), the results in Fig. 1 are obtained; they show the time evolution of the error signals and the control input, thus effectively keeping the road lane.

As said before, Theorem 1 [possesses](#) a certain degree of conservativeness, which may affect (i) the amount of uncertainty for which a controller can be synthesized, i.e., the size of \mathcal{C} , and (ii) the closed-loop system performance, i.e., it may affect the maximum value of β that can be obtained. In the next sections we pursue less conservative solutions.

Symbol	Nominal value	Uncertainty	Units
m	1573	$\pm 20\%$	kg
I_z	2873	$\pm 20\%$	kg·m ²
l_f	1.1	-	m
l_r	1.58	-	m
C_{α_f}	$8 \cdot 10^4$	$\pm 50\%$	N/rad
C_{α_r}	$8 \cdot 10^4$	$\pm 50\%$	N/rad
v_x	[10, 40]	-	m/s

Table 1: Parameter values including uncertainty.

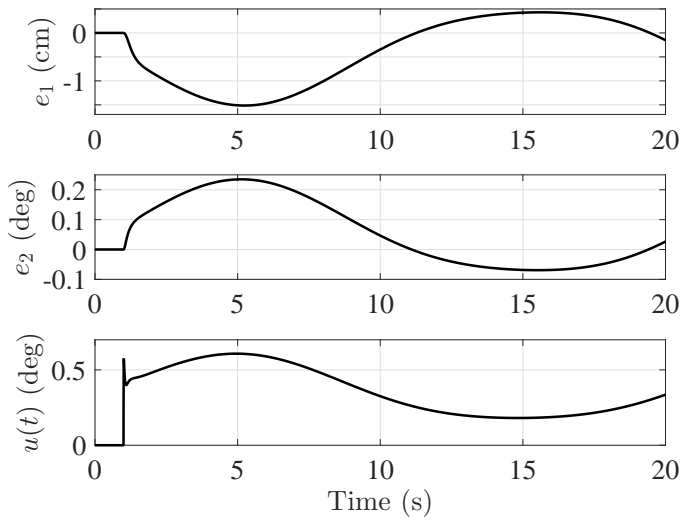


Fig. 1: Time evolution of states e_1 , e_2 and the control signal $u(t)$ for example 1.

4 A novel parameter-dependent polynomial approach

Recall the uncertain time-varying system (4), reproduced below for convenience

$$\dot{x}(t) = A(v_x(t), \theta) x(t) + B(v_x(t), \theta) u(t),$$

along with the control law (12)

$$u(t) = F(v_x)x(t) = \sum_{k \in \mathbb{B}} w_k(z(v_x)) F_k x(t).$$

Thus, the closed-loop generalized characteristic polynomial is

$$p(s, v_x, \theta) = \det(sI - A(v_x, \theta) - B(\theta)F(v_x)),$$

$$p(s, v_x, \theta) = s^4 + a_3(v_x, \theta) s^3 + a_2(v_x, \theta) s^2 + a_1(v_x, \theta) s + a_0(v_x, \theta), \quad (15)$$

with

$$\begin{aligned} a_3(v_x, \theta) &= 2z(v_x)\zeta_1(\theta)\zeta_4(\theta) - 2l_f f_4(v_x)\zeta_2(\theta)\zeta_4(\theta) - 2f_2(v_x)\zeta_1(\theta)\zeta_4(\theta) \\ &\quad + 2z(v_x)\zeta_1(\theta)\zeta_3(\theta) + 2l_f^2 z(v_x)\zeta_2(\theta)\zeta_4(\theta) + 2l_r^2 z(v_x)\zeta_2(\theta)\zeta_3(\theta), \\ a_2(v_x, \theta) &= 2\zeta_2(\theta)\zeta_3(\theta) - 2(l_f + f_3(v_x)l_f)\zeta_2(\theta)\zeta_4(\theta) - 2f_1(v_x)\zeta_1(\theta)\zeta_4(\theta) \\ &\quad + 4(l_f^2 + l_r^2 + 2l_f l_r)\zeta_1(\theta)\zeta_2(\theta)\zeta_3(\theta)\zeta_4(\theta)z^2(v_x) \\ &\quad - 4(f_4(v_x)(l_f + l_r) + f_2(v_x)(l_r^2 + l_f))\zeta_1(\theta)\zeta_2(\theta)\zeta_3(\theta)\zeta_4(\theta)z(v_x) \\ a_1(v_x, \theta) &= -4f_2(v_x)\zeta_1(\theta)\zeta_2(\theta)\zeta_3(\theta)\zeta_4(\theta)(l_f + l_r) \\ &\quad - 4\zeta_1(\theta)\zeta_2(\theta)\zeta_3(\theta)\zeta_4(\theta)z(v_x)(f_3(v_x)(l_f + l_r) + f_1(v_x)(l_r^2 + l_f l_r)) \\ a_0(v_x, \theta) &= -4(l_f + l_r)f_1(v_x)\zeta_1(\theta)\zeta_2(\theta)\zeta_3(\theta)\zeta_4(\theta). \end{aligned}$$

Note that the term $z^2(v_x)$ appears in $a_2(v_x, \theta)$.

Consider the convex rewriting of $z(v_x) = \sum_{j \in \mathbb{B}} w_j z^j$, with $w_0 + w_1 = 1$, $w_j \in [0, 1]$, presented in Section 2. It allows writing

$$z^2(v_x) = z(v_x) z(v_x) = \left(\sum_{j_1 \in \mathbb{B}} w_{j_1} z^{j_1} \right) \left(\sum_{j_2 \in \mathbb{B}} w_{j_2} z^{j_2} \right) = \sum_{j_1=0} \sum_{j_2=0} w_{j_1} w_{j_2} z^{j_1} z^{j_2}. \quad (16)$$

Likewise, the terms $z(v_x) f_i(v_x)$, $i \in \{1, 2, 3, 4\}$, can be convexly rewritten as follows:

$$z(v_x) f_i(v_x) = \left(\sum_{j_1 \in \mathbb{B}} w_{j_1} z^{j_1} \right) \left(\sum_{j_3 \in \mathbb{B}} w_{j_3} f_i^{j_3} \right) = \sum_{j_1 \in \mathbb{B}} \sum_{j_3 \in \mathbb{B}} w_{j_1} w_{j_3} z^{j_1} f_i^{j_3}. \quad (17)$$

The notation can be eased by defining $\mathbf{j} = (j_1, j_2, j_3) \in \mathbb{B}^3$ and $\mathbf{w}_{\mathbf{j}}(z) = w_{j_1} w_{j_2} w_{j_3}$. Considering the convex models described above and following the methodology of Section 2, the generalized characteristic polynomial can be rewritten as a convex sum of polynomials:

$$p(s, v_x, \theta) = \sum_{\mathbf{i} \in \mathbb{B}^4} \sum_{\mathbf{j} \in \mathbb{B}^3} \omega_{\mathbf{i}}(\zeta) \mathbf{w}_{\mathbf{j}}(z) p_{\mathbf{ij}}(s), \quad (18)$$

with $p_{\mathbf{ij}}(s) = p(s, v_x, \theta)|_{\omega_{\mathbf{i}}=\mathbf{w}_{\mathbf{j}}=1}$. Since $\mathbf{w}_{001}(z) = \mathbf{w}_{010}(z) = \mathbf{w}_{100}(z)$, the following association is possible:

$$\begin{aligned} \sum_{\mathbf{i} \in \mathbb{B}^4} \omega_{\mathbf{i}}(\zeta) (\mathbf{w}_{001}(z) p_{\mathbf{i},001}(s) + \mathbf{w}_{010}(z) p_{\mathbf{i},010}(s) + \mathbf{w}_{100}(z) p_{\mathbf{i},100}(s)) = \\ \sum_{\mathbf{i} \in \mathbb{B}^4} \omega_{\mathbf{i}}(\zeta) \mathbf{w}_{001}(z) (p_{\mathbf{i},001} + p_{\mathbf{i},010} + p_{\mathbf{i},100}); \end{aligned}$$

thus, the three groups of polynomials can be replaced by a single one denoted as $\mathbf{p}_{\mathbf{i},001} = p_{\mathbf{i},001} + p_{\mathbf{i},010} + p_{\mathbf{i},100}$. The same can be done for the other terms in $\mathbf{w}_{\mathbf{j}}(z)$ which are algebraically identical, namely, $\mathbf{w}_{000}(z)$, $\mathbf{w}_{011}(z) = \mathbf{w}_{101}(z) = \mathbf{w}_{110}(z)$, and $\mathbf{w}_{111}(z)$.

Let us define $J = \{000, 001, 011, 111\}$, then (18) can be rewritten as follows:

$$p(s, v_x, \theta) = \sum_{\mathbf{i} \in \mathbb{B}^4} \sum_{\mathbf{j} \in J} \omega_{\mathbf{i}}(\zeta) \mathbf{w}_{\mathbf{j}}(z) \mathbf{p}_{\mathbf{ij}}(s), \quad (19)$$

with $\mathbf{p}_{\mathbf{i},000}(s) = p_{\mathbf{i},000}(s)$, $\mathbf{p}_{\mathbf{i},001}(s) = p_{\mathbf{i},001}(s) + p_{\mathbf{i},010}(s) + p_{\mathbf{i},100}(s)$, $\mathbf{p}_{\mathbf{i},011}(s) = p_{\mathbf{i},011}(s) + p_{\mathbf{i},101}(s) + p_{\mathbf{i},110}(s)$, and $\mathbf{p}_{\mathbf{i},111}(s) = p_{\mathbf{i},111}(s)$.

Going from convex modeling (18) to convex modeling (19) not only reduces the number of vertex polynomials, from 128 to 64, it can also reduce the conservativeness of the approach. Via polynomial methods [21], the equation (19) can be used to synthesize the gains F_j in (12), which ensure the asymptotic stability of the uncertain system as long as v_x is time-invariant. Nevertheless,

considering the convex rewriting (11), the control law (12) with gains F_k obtained from (19), the quadratic Lyapunov function candidate $V(x) = x^T P x$, $P = P^T > 0$ and its derivative, we have:

$$\dot{V} = \sum_{\mathbf{i} \in \mathbb{B}^4} \sum_{j \in \mathbb{B}} \omega_{\mathbf{i}}(\zeta) w_j(z) x^T [P (A_{\mathbf{i}j} + B_{\mathbf{i}} F_j) + (A_{\mathbf{i}}^T + F_j^T B_{\mathbf{i}}^T) P] x,$$

thus, if a matrix $P = P^T > 0$ is found such that the following LMIs hold

$$P (A_{\mathbf{i}j} + B_{\mathbf{i}} F_j) + (A_{\mathbf{i}}^T + F_j^T B_{\mathbf{i}}^T) P + 2\beta P,$$

the asymptotic stability of the uncertain system with v_x being time-dependent is guaranteed.

Now consider splitting the compact \mathcal{C} into s partitions and applying a different quadratic Lyapunov function to each of them: this is possible without being concerned by continuity or piecewise considerations because the parameters are fixed, i.e., the different Lyapunov functions relax the analysis, but there are no transitions between them. Thus, the gains F_j obtained by the polynomial approach guarantee the stability of the uncertain system with v_x varying over time if the following bilinear matrix inequalities hold:

$$\begin{aligned} P_1 = P_1^T > 0, P_1 A_{\mathbf{i}j}^1 + P_1 B_{\mathbf{i}}^1 F_j + (P_1 A_{\mathbf{i}j}^1 + P_1 B_{\mathbf{i}}^1 F_j)^T + 2\beta P_1 < 0, \\ P_2 = P_2^T > 0, P_2 A_{\mathbf{i}j}^2 + P_2 B_{\mathbf{i}}^2 F_j + (P_2 A_{\mathbf{i}j}^2 + P_2 B_{\mathbf{i}}^2 F_j)^T + 2\beta P_2 < 0, \\ & \vdots \\ P_s = P_s^T > 0, P_s A_{\mathbf{i}j}^s + P_s B_{\mathbf{i}}^s F_j + (P_s A_{\mathbf{i}j}^s + P_s B_{\mathbf{i}}^s F_j)^T + 2\beta P_s < 0, \end{aligned} \quad (20)$$

with $A_{\mathbf{i}j}^i$ and $B_{\mathbf{i}}^i$ being the matrices that correspond to the convex rewriting of (4) in the partition \mathcal{C}_i . Considering assumption 1 and 2, (19) and (20) can be used to design (12) via the following result:

Theorem 2 [24][21] *The origin of the uncertain time-varying linear system (4), having a closed-loop characteristic polynomial (19) in the compact set \mathcal{C} under the control law (12), is stable with decay rate β if there exist matrices $P_r = P_r^T$, $Q_{\mathbf{ij}} = Q_{\mathbf{ij}}^T \in \mathbb{R}^{n \times n}$, $\mathbf{i} \in \mathbb{B}^4$, and gain entries $F_j \in \mathbb{R}^{1 \times n}$, $j \in \mathbb{B}$, such that the following LMIs are feasible:*

$$P_r > 0, P_r A_{\mathbf{i}j}^r + P_r B_{\mathbf{i}}^r F_j + (P_r A_{\mathbf{i}j}^r + P_r B_{\mathbf{i}}^r F_j)^T + 2\beta P_r < 0 \quad (21)$$

with $r \in \{1, 2, \dots, s\}$ and F_j obtained from the following LMIs

$$\begin{aligned} \sum_{g=1}^n \sum_{h=1}^n \left(\Pi_g^T \begin{bmatrix} 2\beta & 1 \\ 1 & 0 \end{bmatrix} \Pi_h + \Pi_h^T \begin{bmatrix} 2\beta & 1 \\ 1 & 0 \end{bmatrix} \Pi_g \right) q_{\mathbf{ij}}^{(g,h)} \\ + a_{\mathbf{ij}}^T d + d^T a_{\mathbf{ij}} - 2\gamma d^T d \geq 0, \end{aligned} \quad (22)$$

for an arbitrarily small $\gamma > 0$, with $\mathbf{j} \in J = \{000, 001, 011, 111\}$, $\Pi_i \in \mathbb{R}^{2 \times (n+1)}$ being matrices with ones at entries $(1, i)$ and $(2, i + 1)$, and zeros

elsewhere, $q_{ij}^{(g,h)} \in \mathbb{R}$ the (g, h) -th entry of Q_{ij} , $d = [d_0 \ d_1 \ \dots \ d_{n-1} \ d_n] \in \mathbb{R}^{1 \times n}$ the coefficients of a given β -stable polynomial, and a_{ij} the coefficients of the \mathbf{p}_{ij} vertex polynomial in ascending order.

Proof Using convex sum properties on (22), i.e., taking into account that $\sum_i \omega_i = 1$, $0 \leq \omega_i \leq 1$, $\sum_j \mathbf{w}_j = 1$, $0 \leq \mathbf{w}_j \leq 1$, and omitting arguments when convenient, yields

$$\begin{aligned} & \sum_{i \in \mathbb{B}^4} \sum_{j \in J} \omega_i(\zeta) \mathbf{w}_j(z) \left\{ a_{ij}^T d + d^T a_{ij} - 2\gamma d^T d \right. \\ & \left. + \sum_{g=1}^n \sum_{h=1}^n \left(\Pi_g^T \begin{bmatrix} 2\beta & 1 \\ 1 & 0 \end{bmatrix} \Pi_h + \Pi_h^T \begin{bmatrix} 2\beta & 1 \\ 1 & 0 \end{bmatrix} \Pi_g \right) q_{ij}^{gh} \right\} \\ & = \left(\sum_{i,j} \omega_i \mathbf{w}_j a_{ij} \right)^T d + d^T \underbrace{\left(\sum_{i,j} \omega_i \mathbf{w}_j a_{ij} \right)}_c - 2\gamma d^T d \\ & \quad + \sum_{g=1}^n \sum_{h=1}^n \left(\Pi_g^T \begin{bmatrix} 2\beta & 1 \\ 1 & 0 \end{bmatrix} \Pi_h + \Pi_h^T \begin{bmatrix} 2\beta & 1 \\ 1 & 0 \end{bmatrix} \Pi_g \right) \underbrace{\sum_{i,j} \omega_i \mathbf{w}_j q_{ij}^{gh}}_Q \geq 0, \end{aligned}$$

which, by Theorem 1 in [25], allows establishing the desired result with c and Q as defined above with underbraces.

Note that up to this point, the gains F_j obtained guarantee the stability of the system for a fixed v_x , but splitting the compact \mathcal{C} into s partitions and applying a different Lyapunov function to each partition, we have the following conditions:

$$\begin{aligned} P_1 &= P_1^T > 0, \quad P_1 A_{ij}^1 + P_1 B_i^1 F_j + (P_1 A_{ij}^1 + P_1 B_i^1 F_j)^T + 2\beta P_1 < 0, \\ P_2 &= P_2^T > 0, \quad P_2 A_{ij}^2 + P_2 B_i^2 F_j + (P_2 A_{ij}^2 + P_2 B_i^2 F_j)^T + 2\beta P_2 < 0, \\ & \vdots \\ P_s &= P_s^T > 0, \quad P_s A_{ij}^s + P_s B_i^s F_j + (P_s A_{ij}^s + P_s B_i^s F_j)^T + 2\beta P_s < 0, \end{aligned}$$

with A_{ij}^i and B_i^i being the matrices that correspond to the convex rewriting of (4) in the partition \mathcal{C}_i . If these conditions are satisfied with the gains F_j obtained by polynomial methods, the stability of the system is guaranteed with decay rate β and the time-dependent parameter $v_x(t)$.

Remark 3 Computational complexity: Generalizing Theorem 2 to r_ζ constant unknown parameters and r_z measurable parameters, the number of LMIs is bounded by $\mathcal{O}(s \times n^2 + m \times n \times 2^{r_z} + n^2 \times 2^{n(r_\zeta+r_z)})$ with a number of variables bounded by $\mathcal{O}(s \times 2^{r_\zeta+r_z} + 2^{n \times (r_\zeta+r_z)})$.

Remark 4 Conservativeness: The main handicap of the proposed approach is that the solution is computed in two steps and that there is no *optimal* methodology to compute the parameters during the first step. First, controller gains are computed via LMIs (22) with parameters γ and d chosen by the designer. If a solution exists, i.e., matrices Q_{ij} and F_j that satisfy the LMI, the gains are used in step 2 to compute P_r via LMIs (21). However, if the LMIs of the first step are not feasible, there might still exist a solution for a different set of parameters γ and d .

Algorithm 1 is now presented; it allows finding the maximum feasible β in Theorem 2, i.e., the maximum decay rate, from a given known feasible β_0 and a parameter $\epsilon > 0$ denoting the tolerance gap between the maximum feasible and the minimum unfeasible β values.

Algorithm 1: Algorithm for computing the maximum feasible β parameter of Theorem 2.

Input: β_0, ϵ ;
Output: β_{\max} ;
 $\beta := \beta_0$;
 $\beta_{\text{feas,max}} := \beta_0$;
 $\beta_{\text{infeas}} := \infty$;
while $\beta_{\text{infeas}} - \beta_{\text{feas,max}} > \epsilon$ **do**
 primal = solveLMIs(β);
 if primal > 0 **then**
 $\beta_{\text{feas,max}} := \beta$;
 if $\beta_{\text{infeas}} = \infty$ **then**
 $\beta := 2 \cdot \beta_{\text{feas,max}}$;
 else
 $\beta := (\beta_{\text{feas,max}} + \beta_{\text{infeas}}) / 2$;
 end
 else
 $\beta_{\text{infeas}} := \beta$;
 $\beta := (\beta_{\text{feas,max}} + \beta_{\text{infeas}}) / 2$;
 end
end

As in the approach presented in section 3, the control effort can be bounded by minimizing the norm of the gains $\|F_j\| \leq \mu$ via the LMIs:

$$\min \mu : \begin{bmatrix} \mu & F_j \\ F_j^T & \mu I \end{bmatrix} \geq 0, \quad j \in \mathbb{B}. \quad (23)$$

Example 2 In order to benchmark Theorem 2, (22) will be solved with $\gamma = 0.5$, $d = [1784.069 \ 1470.388 \ 353.527 \ 32.621 \ 1]$, and the bounds and parameters of Table 1 while maximizing β using Algorithm 1 and minimizing the norm of F_j

by adding (23). Using Yalmip/Mosek [26], the following results are obtained:

$$F_1 = \begin{bmatrix} -0.976 & -0.335 & -7.400 & -0.703 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -0.818 & -0.019 & -3.000 & -0.203 \end{bmatrix}.$$

with $\beta_{\max} = 1.801$ and $\mu = 7.505$. Using the above gains, $\beta = 1.402$, and 9 partitions of \mathcal{C} , conditions (21) are feasible, which means the results obtained from Theorem 2 guarantee the asymptotic stability of the origin of the uncertain linear time-varying system (4). For illustrative purposes, some of the computed P_r matrices are given:

$$P_1 = \begin{bmatrix} 0.247 & 0.033 & 0.372 & -0.025 \\ 0.033 & 0.007 & 0.064 & -0.005 \\ 0.3724 & 0.0637 & 1.3825 & 0.0012 \\ -0.025 & -0.005 & 0.001 & 0.015 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0.370 & 0.0284 & 0.4901 & -0.022 \\ 0.028 & 0.009 & 0.017 & -0.006 \\ 0.490 & 0.017 & 3.088 & 0.032 \\ -0.022 & -0.006 & 0.032 & 0.021 \end{bmatrix},$$

$$P_6 = \begin{bmatrix} 0.891 & 0.044 & 1.125 & -0.034 \\ 0.044 & 0.017 & -0.042 & -0.012 \\ 1.125 & -0.042 & 8.656 & 0.103 \\ -0.034 & -0.012 & 0.103 & 0.036 \end{bmatrix}, \quad P_9 = \begin{bmatrix} 0.484 & 0.017 & 0.585 & -0.013 \\ 0.018 & 0.006 & -0.020 & -0.005 \\ 0.585 & -0.020 & 4.491 & 0.040 \\ -0.013 & -0.005 & 0.040 & 0.013 \end{bmatrix}.$$

Simulating system (1)-(3) with initial conditions $x(0) = [0 \ 0 \ 0]^T$ and uncertain terms chosen as $m = 1730.3$, $Iz = 2442.1$, $C_{\alpha_f} = 60000$, and $C_{\alpha_r} = 100000$, the time-dependent term as $v_x = 15 \sin(\pi t/10) + 25$, the disturbance given by $\dot{\psi}_{des} = 0$ when $t < 1$ and $\frac{v_x}{R}$ (rad/s) when $t \geq 1$, the results in Fig. 2 are obtained: they show the time evolution of the error signals and the control input. Clearly, the road lane is kept as desired.

Comparing β_{\max} : Unlike example 1 for which $\beta_{\max} = 1.286$ was the maximum decay rate, now $\beta_{\max} = 1.402$ has been achieved, which represents a significant increase in the speed of convergence. Nevertheless, β_{\max} in Theorem 1 is a valid decay rate even if all the constant uncertain parameters were replaced by time-varying parameters with identical bounds, which is not the case for β_{\max} obtained via Theorem 2. Thus, another way of comparing the controllers synthesized in each approach is to compute β_{\max} under (20) by replacing the gains computed in Example 1 via Theorem 1, obtaining LMI conditions, with the same number of partitions used in Example 2 for Theorem 2. Following this procedure, $\beta_{\max} = 1.718$ is obtained, which might explain the better performance for $e_1(t)$ in Fig. 1 when compared with the same variable in Fig. 2. Naturally, this procedure is not a purely PDC-like approach as those in [4, 16, 17], but suggests another path for future work.

Comparing the size of \mathcal{C} : In order to compare the maximum compact set \mathcal{C} compact for which each theorem can synthesize a controller, a level of uncertainty denoted as $\lambda \in \mathbb{N}$ will be added to the uncertainty values in Table 1. Setting $\beta = 0$, the size of the compact set \mathcal{C} will be compared for each approach by comparing the maximum λ value obtained in each theorem. For

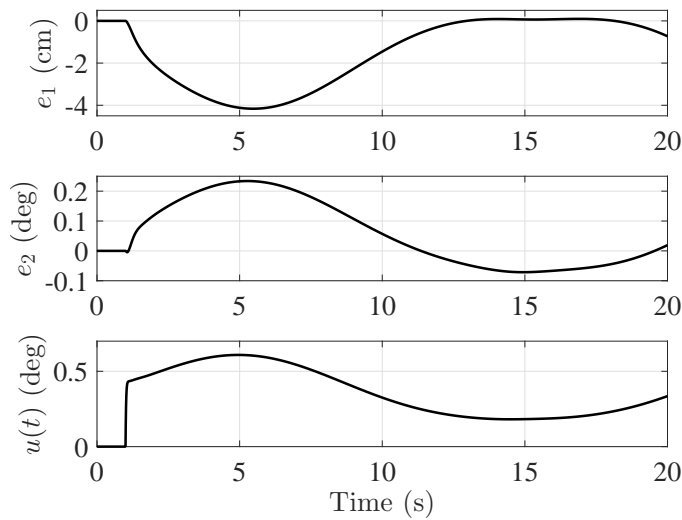


Fig. 2: Time evolution of states e_1 , e_2 and the control signal $u(t)$ for example 2.

instance, $\lambda = 1$ means $\pm 21\%$ uncertainty for m and I_z , and $\pm 51\%$ uncertainty for C_{α_f} and C_{α_r} . Considering $\mu = 0.1047$, $\lambda = 21$ is obtained via Theorem 1: it is independent of d . Considering $d = [1784.069\ 1470.388\ 353.527\ 32.621\ 1]$ and $\gamma = 0.5$, $\lambda = 4$ is obtained via Theorem 2: it depends on d . Therefore, the advantages of β_{\max} in the polynomial approach when compared with pure PDC ones are not as easily extended to greater compact sets.

Feasibility: Importantly, if no partitions of the parameter space are considered for the Lyapunov function (i.e., a common P is used for the compact set \mathcal{C}), the LMIs (21) and (22) are not feasible for a variety of design parameters such as the coefficients of the desired characteristic polynomial d and β . This stresses the importance of combining the polynomial approach with a parameter-dependent Lyapunov function which takes advantage of the unknown, but constant nature of some parametric uncertainties.

5 Conclusions

A novel control technique for automated lane keeping of a vehicle has been presented. It has been shown that conservativeness of Lyapunov-based traditional approaches can be significantly reduced by providing specific analysis for unknown constant parameters, via novel techniques based on the generalized characteristic polynomial, which in turn allow parameter-dependent Lyapunov analysis to be used. The proposal has been successfully tested in simulation for the uncertain bicycle model and contrasted with PDC-like approaches already available in the literature. As future work, real-time implementation will be considered.

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Conflict of interest

The authors declare that they have no conflict of interest.

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