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Simultaneous Design of Parallel Distributed Output Feedback and Anti-windup Compensators for Constrained Takagi-Sugeno Fuzzy Systems

Anh-Tu Nguyen, Michel Dambrine and Jimmy Lauber

Abstract

This paper is devoted to develop a novel approach dealing with constrained continuous-time nonlinear systems in the form of Takagi-Sugeno fuzzy models. Here, the disturbed systems are subject to both input and state constraints. The one-step design method is used to simultaneously synthesize the dynamic output feedback controller and its anti-windup strategy. A parameter-dependent version of the generalized sector condition is used together with Lyapunov stability theory to derive LMI design conditions. Based on this result and for different design specifications, the synthesis of an anti-windup based dynamic output feedback controller is expressed on the form of convex optimization problems. A physically motivated example is given to illustrate the effectiveness of the proposed method.

Index Terms

Takagi-Sugeno fuzzy systems, dynamic output feedback controller, anti-windup compensator, control input saturation, state constraints, domain of attraction.

I. INTRODUCTION

Computationally intelligent techniques have been demonstrated as powerful tools to cope with real-world complex systems [1]–[3]. Especially, control methods based on neural networks and fuzzy logic have received more and more attention [1]. Due to their outstanding features such as adaptation, learning and universal approximation, neural network control has been successfully applied to many practical systems [1], [2], [4]. However, there still remains a number of drawbacks for classical neural network approach, for instance: the existence of multiple local minima solutions, the heavy computational cost [3], [5]. Support vector machines (SVMs) [6] has appeared as an alternative to neural networks. The main advantages of SVM compared to neural network consists in its capacity to provide global optimal solutions [3], [6]. Additionally, SVM algorithms offer a higher generalization ability and they more suitable for machine learning in small sample condition [5]. Therefore, SVMs have become now an efficient methodology in many areas, e.g. classification, function regression, time series prediction [6]. It should be stressed that despite some important advances in the field of control systems [3], [5], systematic framework for *rigorous* stability analysis of both neural network and SVM control systems is not yet available in the literature [1].

Over the past two decades, model-based fuzzy control has been received great interest in the research community [2], [7]–[11]. In particular, control technique based on Takagi-Sugeno (T-S) fuzzy models [12] becomes nowadays one of the most popular and promising approach for complex nonlinear dynamics [2]. Indeed, T-S fuzzy models have been widely used to represent complex nonlinear systems as they are a universal approximator. Furthermore, using the sector nonlinear approach [13], an exact T-S fuzzy model can be deduced from a sufficiently smooth nonlinear system. With a T-S representation, a model-based control may be designed to guarantee the stability and achieve some performance requirements for nonlinear systems. Thanks to its polytopic structure, the main interest of T-S control approach is to extend some linear concepts to the case of nonlinear systems [13]. Based on Lyapunov stability tools, this control technique provides a *general and systematic* framework to cope with complex nonlinear systems [2], [9]–[11], [14].

Among all nonlinear phenomena, actuator saturation is unavoidable in almost all real applications. It can severely degrade closed-loop system performance and in some cases may lead to the system instability [15]. Motivated by this practical control aspect, a great deal of effort has been recently focused on saturated systems, see [4], [15]–[17] and references therein. In [16], the saturation effect is explicitly considered into the design procedure by using a polytopic representation. Norm-bounded approach is proposed in [17] to deal with T-S systems subject to control input constraints. These approaches often lead to conservative design conditions [15]. A novel adaptive neural controller based on auxiliary error compensation scheme has been recently proposed to deal with flexible air-breathing hypersonic vehicle with magnitude constraints on actuators [4]. It is worth mentioning that among all available control approaches, anti-windup (AW) based control scheme [18] is known as the most popular one to handle the input nonlinearity. There are two categories in this case: *one-step* or *two-step* design methods. For the two-step method, first a controller is designed without regarding control input nonlinearity and then an additional anti-windup compensator is introduced in order to minimize the undesirable degradation of closed-loop performance caused by input saturation [15], [18], [19]. Two-step design methods generally prove to be satisfactory and have an abundant literature,

see e.g. [15] for a quick overview. However, the main disadvantages of such a method are that the effect of the nominal controller on the closed-loop performance under saturation is completely ignored [20] and the inherent sub-optimality since the nominal controller and the AW compensator are separately designed [21]. The one-step design methods [20] can be used to overcome these drawbacks. For such methods, the dynamic output feedback controller (DOFC) and its associated AW compensator are simultaneously designed. Up to now, this method still remains an open research topic [21]. Among some few recent results existing in linear control framework, the following works can be notably cited. In [20], a convex synthesis method guaranteeing global stability and performance was developed for only open-loop stable systems. The authors in [22] proposed a local synthesis result which is based on parameter elimination approach. Only sub-optimal solution can be achieved with this method since two separated LMI problems must be solved successively to obtain the DOFC and its associated AW compensator, see Procedure 1 in [22]. A change-of-variable approach for local context was also proposed in [21] for undisturbed linear systems. A great advantage of this method is that it can be naturally extended to multi-objective control problem which is not the case in [22]. However, the Lyapunov matrix has to be imposed with a special structure. Furthermore, it should be stressed that most of works on AW design are available for linear systems; there is a serious lack of literature on the study for nonlinear cases, especially when exogenous disturbance signals are involved in the system dynamics. Recently, a one-step method based on piecewise fuzzy AW dynamic output feedback controller (DOFC) for discrete-time T-S fuzzy systems has been proposed in [23]. Note that those results seem to be valid only for systems that are stable in open-loop since the estimate of the closed-loop domain of attraction is not explicitly defined even with the presence of system disturbances. Another interesting one-step method has been proposed in [24] which can be applied to open-loop unstable T-S systems subject to persistent disturbances. However, this method can be only considered as a "mixed" between one-step and two-step methods since the local \mathcal{L}_2 -gain performance *cannot* be guaranteed whenever the control input is saturated. Besides control input saturation, the models are usually valid on a given subset of the state space. This is not only true for any system in practice due to physical/safety reasons, but also fundamental when using the nonlinear sector decomposition approach to obtain T-S fuzzy models [13]. This validity domain of T-S models can be represented by some constraints on the state variables. It is particularly important to consider explicitly these state constraints in the control design to guarantee a desirable closed-loop behavior in response of disturbances [8]. However, this control issue is usually neglected in T-S control framework [8], [25].

In T-S control framework, state-feedback control based on the concept of parallel distributed compensation (PDC) is usually applied to derive the design conditions [13]. However, all system state variables are not always available in many practical cases. Therefore, output feedback control has been intensively investigated in the literature, see for instance [26]–[29]. Most of works concern observer-based controller design [27], [28]. However, observer-based control approaches become much more complicated when dealing with T-S fuzzy systems subject to control input and system state constraints. This control issue has not been well addressed in the literature [25]. Motivated by the above control issues, we propose in this paper a novel LMI-based method to simultaneously design a DOFC and an AW compensator for a given disturbed T-S fuzzy system subject to control input saturation and also system state constraints. It will be shown that the obtained dynamic controller can satisfy several regional closed-loop requirements, often conflicting. Anti-windup based control design in the presence of energy-bounded disturbances and state constraints is quite novel in T-S fuzzy control framework. The proposed method provides a systematic tool to deal with a very large class of constrained nonlinear systems, which is our main contribution.

The paper is organized as follows. Section II describes the design problem and recalls some preliminaries results needed for the control synthesis. The main result is stated in Section III. In Section IV, different convex optimization problems are formulated to deal with the trade-off between several conflicting control objectives. The effectiveness of the proposed method is clearly demonstrated via an illustrative example in Section V. Finally, Section VI provides some concluding remarks.

Notation. For an integer number r , Ω_r denotes the set $\{1, 2, \dots, r\}$. I denotes the identity matrix of appropriate dimensions. For any square matrix X , $\text{He}(X) = X + X^T$. $X > 0$ means that the matrix X is symmetric and positive definite. The i -th element of a vector u is denoted $u_{(i)}$ and $X_{(i)}$ denotes the i -th row of matrix X . $(*)$ stands for matrix blocks that can be deduced by symmetry. For a matrix $P > 0$ and a positive scalar ρ , we denote $\mathcal{E}(P, \rho) \triangleq \{x : x^T P x \leq \rho\}$, and for brevity $\mathcal{E}(P) \equiv \mathcal{E}(P, 1)$. The scalar real functions η_1, \dots, η_r are said to verify the convex sum property on a set \mathcal{D} , if $\forall \theta \in \mathcal{D}$,

$\forall i \in \Omega_r$, $\eta_i(\theta) \geq 0$ and $\sum_{i=1}^r \eta_i(\theta) = 1$. For such functions, we will denote: $Y_\theta \triangleq \sum_{i=1}^r \eta_i(\theta) Y_i$, $Y_\theta^{-1} \triangleq \left(\sum_{i=1}^r \eta_i(\theta) Y_i \right)^{-1}$,

$Z_{\theta\theta} \triangleq \sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) Z_{ij}$, where the matrices Y_i and Z_{ij} are of appropriate dimensions.

II. PROBLEM DEFINITION AND PRELIMINARIES RESULTS

A. Problem Formulation

Consider the following continuous-time T-S fuzzy system described by [13]

$$\begin{cases} \dot{x} = \sum_{i=1}^r \eta_i(\theta) (A_i x + B_i^u u + B_i^w w) \\ y = \sum_{i=1}^r \eta_i(\theta) (C_i^y x + D_i^{yw} w) \end{cases} \quad (1)$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $w \in \mathbb{R}^{n_w}$ and $y \in \mathbb{R}^{n_y}$ are respectively the state, the control input, the disturbance and the measured output vectors of the system. In this paper, it is assumed that all components of the premise variable vector $\theta \in \mathbb{R}^k$ are measured. For $i \in \Omega_r$, the matrices $A_i \in \mathbb{R}^{n_x \times n_x}$, $B_i^u \in \mathbb{R}^{n_x \times n_u}$, $B_i^w \in \mathbb{R}^{n_x \times n_w}$, $C_i^y \in \mathbb{R}^{n_y \times n_x}$, $D_i^{yw} \in \mathbb{R}^{n_y \times n_w}$ represent the set of r local linear subsystems and the nonlinear scalar functions $\eta_i(\theta)$ satisfy the convex sum property. For system (1), we consider the following assumptions.

Assumption 1. *The input vector u is subject to symmetric magnitude limitations*

$$-u_{\max(l)} \leq u(l) \leq u_{\max(l)}, \quad u_{\max(l)} > 0, \quad i \in \Omega_{n_u}. \quad (2)$$

Assumption 2. *The disturbance w is a bounded energy signal, i.e. it belongs to the following set of functions*

$$\mathcal{W}_\delta = \left\{ w : \mathbb{R}^+ \rightarrow \mathbb{R}^{n_w}, \quad \int_0^\infty w^T w dt \leq \delta \right\} \quad (3)$$

where the bound $\delta > 0$ is given.

Consider now the unconstrained dynamic output feedback controller (DOFC) of the form

$$\begin{cases} \dot{x}_c = \sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) A_{ij}^c x_c + \sum_{i=1}^r \eta_i(\theta) B_i^c y + v \\ u_c = \sum_{i=1}^r \eta_i(\theta) C_i^c x_c + D^c y \end{cases} \quad (4)$$

where $x_c \in \mathbb{R}^{n_x}$, $u_c \in \mathbb{R}^{n_u}$ are respectively the state and output vectors of the controller. The additional input signal v is used to compensate the *windup* effect. Controller (4) has to be designed to guarantee the stability and some performance requirements for the closed-loop system, see Figure 1.

Because of control input limitations, the actual control signal injected into the system is subject to the saturation effect, i.e. $u = \text{sat}(u_c)$, where each component of the saturation function $\text{sat}(\cdot)$ is given by

$$\text{sat}(u_{c(l)}) \triangleq \text{sign}(u_{c(l)}) \min(|u_{c(l)}|, u_{\max(l)}), \quad l \in \Omega_{n_u}. \quad (5)$$

Remark 1. Without loss of generality, we consider symmetric saturation functions in this paper. Asymmetric saturations can be easily handled with the proposed method with some level of conservativeness by taking $u_{\max(l)}$ as the minimum absolute value of the negative and positive saturations levels. We note that the symmetrization strategy presented in [30] can be also used to deal with asymmetric saturation functions. Then, the proposed design procedure is straightforwardly applied.

The interactions between system (1) and the constrained dynamic controller (4) are given as

$$u = \text{sat}(u_c), \quad v = \sum_{i=1}^r \eta_i(\theta) E_{ij}^c (\text{sat}(u_c) - u_c). \quad (6)$$

From (4) and (6), the DOFC combined with the anti-windup strategy can be expressed as

$$\begin{cases} \dot{x}_c = \sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) A_{ij}^c x_c + \sum_{i=1}^r \eta_i(\theta) (B_i^c y - E_{ij}^c \psi(u_c)) \\ u_c = \sum_{i=1}^r \eta_i(\theta) C_i^c x_c + D^c y \end{cases} \quad (7)$$

where E_{ij}^c are the anti-windup gains to be designed and $\psi(u_c) \triangleq u_c - \text{sat}(u_c)$. The l -th component of the decentralized dead-zone nonlinearity $\psi(u_c)$ is defined as

$$\psi(u_{c(l)}) = \begin{cases} 0, & \text{if } |u_{c(l)}| \leq u_{\max(l)} \\ u_{c(l)} - \text{sign}(u_{c(l)}) u_{\max(l)}, & \text{if } |u_{c(l)}| > u_{\max(l)} \end{cases} \quad (8)$$

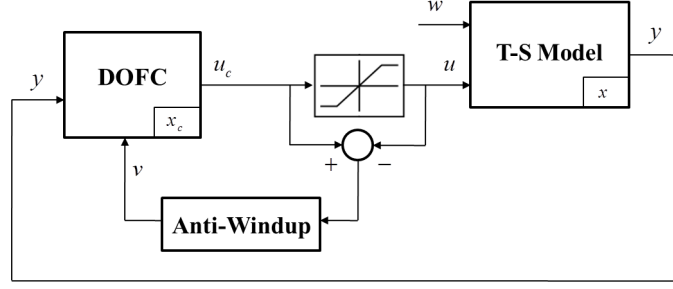


Fig. 1. Closed-loop system with anti-windup strategy.

Define $x_{cl}^T = [x^T \ x_c^T]$, the constrained closed-loop system can be obtained from (1) and (7)

$$\begin{cases} \dot{x}_{cl} = \sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) \left(\mathbb{A}_{ij} x_{cl} + \mathbb{B}_{ij}^w w - \left(\mathbb{B}_i^\psi + R_\psi E_{ij}^c \right) \psi(u_c) \right) \\ y = \sum_{i=1}^r \eta_i(\theta) (\mathbb{C}_i x_{cl} + \mathbb{D}_i w) \end{cases} \quad (9)$$

where

$$\mathbb{A}_{ij} = \begin{bmatrix} A_i + B_i^u D^c C_j^y & B_i^u C_j^c \\ B_i^c C_j^y & A_{ij}^c \end{bmatrix}, \quad \mathbb{B}_{ij}^w = \begin{bmatrix} B_i^w + B_i^u D^c D_j^{yw} \\ B_i^c D_j^{yw} \end{bmatrix}, \quad \mathbb{B}_i^\psi = \begin{bmatrix} B_i^u \\ 0 \end{bmatrix}, \quad \mathbb{C}_i = [C_i^y \ 0], \quad \mathbb{D}_i = [D_i^{yw}], \quad R_\psi = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

and the controller output is given by

$$u_c = \sum_{i=1}^r \eta_i(\theta) (\mathbb{K}_i x_{cl} + K_i^w w), \quad (10)$$

where $\mathbb{K}_i = [D^c C_i^y \ C_i^c]$ and $K_i^w = D^c D_i^{yw}$. Using notations in Section I, the closed-loop system (9) is rewritten as

$$\begin{cases} \dot{x}_{cl} = \mathbb{A}_{\theta\theta} x_{cl} + \mathbb{B}_{\theta\theta}^w w - \left(\mathbb{B}_\theta^\psi + R_\psi E_{\theta\theta}^c \right) \psi(u_c) \\ y = \mathbb{C}_\theta x_{cl} + \mathbb{D}_\theta w \end{cases} \quad (11)$$

and the DOFC output (10) is rewritten as

$$u_c = \mathbb{K}_\theta x_{cl} + K_\theta^w w. \quad (12)$$

In this paper, we aim at proposing a systematic method to design a dynamic output feedback controller together with its anti-windup strategy of the form (7) such that the closed-loop system satisfies the following properties.

• **Property 1 [State constraints]:** For any admissible initial state, the states of the closed-loop system (9) are required to remain inside the polyhedral region described by linear inequalities

$$\mathcal{P}_x = \{x_{cl} \in \mathbb{R}^{2n_x} : h_k^T x_{cl} \leq 1, \quad \forall k \in \Omega_q\}, \quad (13)$$

where $h_k^T = [h_{1k}^T \ 0_{1 \times n_x}]$ and the vectors h_{1k} are given. In this paper, the set \mathcal{P}_x represents the validity of (1).

• **Property 2 [Regional quadratic α -stability]:** When $w = 0$, there exist a positive-definite function $\mathbb{V}(x_{cl}) = x_{cl}^T P x_{cl}$, $P > 0$, and a scalar $\alpha > 0$, such that $\dot{\mathbb{V}}(x_{cl}) < -2\alpha \mathbb{V}(x_{cl})$ along the closed-loop trajectories for $\forall x_{cl}(0) \in \mathcal{E}(P)$. This fact implies that these trajectories will converge exponentially to the origin with a decay rate α .

• **Property 3 [Regional \mathcal{L}_2 -performance]:** For a given positive scalar δ , there exist positive real numbers γ, ρ such that, for any energy-bounded signal $w \in \mathcal{W}_\delta$ and for all initial conditions in $\mathcal{E}(P, \rho)$, the corresponding trajectories of the closed-loop system will never escape the ellipsoid $\mathcal{E}(P) \supset \mathcal{E}(P, \rho)$. Moreover, the \mathcal{L}_2 -norm of the output signal y is bounded by

$$\int_0^\infty y^T y dt < \gamma \int_0^\infty w^T w dt + \rho. \quad (14)$$

B. Preliminary Results

In what follows, some useful lemmas needed for design problem in Section III are presented.

Lemma 1. Given matrices $\mathbb{K}_i \in \mathbb{R}^{n_u \times 2n_x}$ and $\mathbb{G}_i = [G_1^i \ G_2^i] \in \mathbb{R}^{n_u \times 2n_x}$, with $G_1^i \in \mathbb{R}^{n_u \times n_x}$, $G_2^i \in \mathbb{R}^{n_u \times n_x}$, for $i \in \Omega_r$, we define the polyhedral set \mathcal{P}_u as follows

$$\mathcal{P}_u = \bigcap_{i=1}^r \Xi(\mathbb{K}_i - \mathbb{G}_i), \quad (15)$$

where

$$\Xi(\mathbb{K}_i - \mathbb{G}_i) \triangleq \{x_{cl} \in \mathbb{R}^{2n_x} : |(\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)}) x_{cl}| \leq u_{\max(l)}, \forall l \in \Omega_{n_u}\}. \quad (16)$$

Consider the function $\psi(u_c)$ defined in (8) with u_c defined in (12). If $x_{cl} \in \mathcal{P}_u$, then the following condition is verified

$$\psi^T(u_c) \left(\sum_{i=1}^r \eta_i S_i \right)^{-1} \left[\psi(u_c) - \begin{bmatrix} \sum_{i=1}^r \eta_i \mathbb{G}_i & \sum_{i=1}^r \eta_i K_i^w \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix} \right] \leq 0, \quad (17)$$

for any positive diagonal matrices $S_i \in \mathbb{R}^{n_u \times n_u}$, and for any scalar functions η_i , $i \in \Omega_r$, satisfying the convex sum property.

Proof. Assume that $x_{cl} \in \mathcal{P}_u$, it implies $x_{cl} \in \Xi \left(\sum_{i=1}^r \eta_i (\mathbb{K}_i - \mathbb{G}_i) \right)$ since scalar functions η_i , $\forall i \in \Omega_r$, satisfy the convex sum propriety. Hence, it follows that

$$-u_{\max(l)} \leq \sum_{i=1}^r \eta_i (\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)}) x_{cl} \leq u_{\max(l)}, \quad \forall (i, l) \in \Omega_r \times \Omega_{n_u}. \quad (18)$$

Let $S_{i(l,l)} > 0$, $(i, l) \in \Omega_r \times \Omega_{n_u}$, is the element of the l -th row and l -th column of diagonal matrix S_i , we will show that the inequality

$$\psi(u_{c(l)}) \left(\sum_{i=1}^r \eta_i S_{i(l,l)} \right)^{-1} \left[\psi(u_{c(l)}) - \begin{bmatrix} \sum_{i=1}^r \eta_i \mathbb{G}_{i(l)} & \sum_{i=1}^r \eta_i K_{i(l)}^w \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix} \right] \leq 0 \quad (19)$$

holds, which implies obviously (17). For that, there exist only three possible cases according to the value of $u_{c(l)}$.

- **Case 1:** $|u_{c(l)}| \leq u_{\max(l)}$. It follows that $\psi(u_{c(l)}) = 0$ and so, inequality (19) holds trivially.
- **Case 2:** $u_{c(l)} > u_{\max(l)}$. Then,

$$\psi(u_{c(l)}) = u_{c(l)} - u_{\max(l)} = \sum_{i=1}^r \eta_i (\mathbb{K}_{i(l)} x_{cl} + K_{i(l)}^w w) - u_{\max(l)} > 0. \quad (20)$$

From (18), it follows that $\sum_{i=1}^r \eta_i (\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)}) x_{cl} \leq u_{\max(l)}$. Hence,

$$\psi(u_{c(l)}) - \begin{bmatrix} \sum_{i=1}^r \eta_i \mathbb{G}_{i(l)} & \sum_{i=1}^r \eta_i K_{i(l)}^w \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix} = \sum_{i=1}^r \eta_i (\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)}) x_{cl} - u_{\max(l)} \leq 0. \quad (21)$$

Since in this case $\psi(u_{c(l)}) > 0$, inequality (19) holds.

- **Case 3:** $u_{c(l)} < -u_{\max(l)}$. It follows that

$$\psi(u_{c(l)}) = u_{c(l)} + u_{\max(l)} = \sum_{i=1}^r \eta_i (\mathbb{K}_{i(l)} x_{cl} + K_{i(l)}^w w) + u_{\max(l)} < 0. \quad (22)$$

Then, from (18) it follows that $\sum_{i=1}^r \eta_i (\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)}) x_{cl} \geq -u_{\max(l)}$, and then

$$\psi(u_{c(l)}) - \begin{bmatrix} \sum_{i=1}^r \eta_i \mathbb{G}_{i(l)} & \sum_{i=1}^r \eta_i K_{i(l)}^w \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix} = \sum_{i=1}^r \eta_i (\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)}) x_{cl} + u_{\max(l)} \geq 0. \quad (23)$$

Since in this case $\psi(u_{c(l)}) < 0$, inequality (19) holds again.

From the results of these three cases, the proof of Lemma 1 can be concluded. \square

Notice that Lemma 1 presents an extended version of the generalized sector bound condition for input saturation proposed in [31]. In this version, all involved matrices are allowed to be parameter-dependent which contributes to reduce the conservatism of the results. Moreover, it accounts also the effect of system disturbance signal w into the sector condition.

Lemma 2. The ellipsoid $\mathcal{E}(P)$ is included in the polyhedral set \mathcal{P}_u defined in (15) if and only if [32]

$$(\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)}) P^{-1} (\mathbb{K}_{i(l)} - \mathbb{G}_{i(l)})^T \leq u_{\max(l)}^2, \quad \forall (i, l) \in \Omega_r \times \Omega_{n_u}. \quad (24)$$

Lemma 3. The ellipsoid $\mathcal{E}(P)$ is included in the polyhedral set \mathcal{P}_x defined in (13) if and only if [32]

$$\begin{bmatrix} P & h_k \\ h_k^T & 1 \end{bmatrix} \geq 0, \quad \forall k \in \Omega_p. \quad (25)$$

Lemma 4. [27] Let Υ_{ij} , $i, j \in \Omega_r$ be symmetric matrices of appropriate dimensions and η_1, \dots, η_r be a family of functions satisfying the convex sum property. If there exist matrices Q_{ij} with $Q_{ij} = Q_{ji}^T$, for $i, j \in \Omega_r$ such that

$$\begin{cases} \Upsilon_{ii} + Q_{ii} < 0, & \forall i \in \Omega_r, \\ \Upsilon_{ij} + \Upsilon_{ji} + Q_{ij} + Q_{ji} < 0, & \forall i, j \in \Omega_r, \quad i < j, \\ \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1r} \\ Q_{21} & Q_{22} & & \vdots \\ \vdots & & \ddots & Q_{(r-1)r} \\ Q_{r1} & \cdots & Q_{r(r-1)} & Q_{rr} \end{bmatrix} > 0. \end{cases} \quad (26)$$

Then, it follows that $\sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) \Upsilon_{ij} < 0$.

The relaxation result in Lemma 4 has been further improved in [26], [33] at the expense of higher computational cost.

III. MAIN RESULTS

The theorem below provides LMI conditions to design the DOFC together with its anti-windup strategy of the form (7) which solves the control problem defined in Section II.

Theorem 1. Given T-S fuzzy system (1) and positive scalars α, δ . Assume there exist positive definite matrices $P_{11} \in \mathbb{R}^{n_x \times n_x}$, $X_{11} \in \mathbb{R}^{n_x \times n_x}$, positive diagonal matrices $S_i \in \mathbb{R}^{n_u \times n_u}$, matrices $Q_{ij} \in \mathbb{R}^{(2n_x + n_u + n_y + n_w) \times (2n_x + n_u + n_y + n_w)}$, $W_{ij} \in \mathbb{R}^{n_x \times n_u}$, $U_i \in \mathbb{R}^{n_u \times n_x}$, $V_i \in \mathbb{R}^{n_u \times n_x}$, $\hat{A}_{ij} \in \mathbb{R}^{n_x \times n_x}$, $\hat{B}_i \in \mathbb{R}^{n_x \times n_y}$, $\hat{C}_i \in \mathbb{R}^{n_u \times n_x}$ and $\hat{D} \in \mathbb{R}^{n_u \times n_y}$, for $i, j \in \Omega_r$, and positive scalars γ, ρ such that

$$\begin{bmatrix} X_{11} & I \\ I & P_{11} \end{bmatrix} > 0, \quad (27)$$

$$\begin{bmatrix} X_{11} & I & X_{11} h_{1k} \\ * & P_{11} & h_{1k} \\ * & * & 1 \end{bmatrix} \geq 0, \quad \forall k \in \Omega_q, \quad (28)$$

$$\begin{bmatrix} X_{11} & * & * \\ I & P_{11} & * \\ \hat{C}_{i(l)} - U_{i(l)} & (\hat{D} C_y^i)_{(l)} - V_{i(l)} & u_{\max(l)}^2 \end{bmatrix} \geq 0, \quad \forall (i, l) \in \Omega_r \times \Omega_{n_u}, \quad (29)$$

$$1 - \rho - \gamma\delta \geq 0, \quad (30)$$

and conditions (26) hold with

$$\Upsilon_{ij} \triangleq \begin{bmatrix} \text{He}(\mathbb{H}_{ij}) & \mathbb{J}_{ij} & U_i^T - B_i^u S_j & \mathbb{L}_{ij} & X_{11} (C_i^y)^T \\ * & \text{He}(\mathbb{M}_{ij}) & V_i^T - W_{ij} & \mathbb{N}_{ij} & (C_i^y)^T \\ * & * & -2S_j & \hat{D} D_j^{yw} & 0 \\ * & * & * & (D_i^{yw})^T (D_i^{yw}) - \gamma I & 0 \\ * & * & * & * & -I \end{bmatrix}, \quad (31)$$

and

$$\begin{aligned} \mathbb{H}_{ij} &\triangleq A_i X_{11} + B_i^u \hat{C}_j + \alpha X_{11}, & \mathbb{J}_{ij} &\triangleq \hat{A}_{ij}^T + A_i + B_i^u \hat{D} C_j^y + 2\alpha I, & \mathbb{M}_{ij} &\triangleq P_{11} A_i + \hat{B}_i C_j^y + \alpha P_{11}, \\ \mathbb{L}_{ij} &\triangleq B_i^u \hat{D} D_j^{yw} + B_i^w + X_{11} (C_i^y)^T D_i^{yw}, & \mathbb{N}_{ij} &\triangleq P_{11} B_i^u + \hat{B}_i D_j^{yw} + (C_i^y)^T D_i^{yw}. \end{aligned}$$

Let P_{12} and X_{12} be two matrices satisfying the condition

$$P_{11} X_{11} + P_{12} X_{12}^T = I. \quad (32)$$

Then, the DOFC together with its anti-windup compensation (7) given by

$$\begin{aligned} E_{ij}^c &= P_{12}^{-1} W_{ij} S_j^{-1} - P_{12}^{-1} P_{11} B_i^u, \\ D^c &= \hat{D}, \\ C_i^c &= (\hat{C}_i - D^c C_i^y X_{11}) X_{12}^{-T}, \\ B_i^c &= P_{12}^{-1} (\hat{B}_i - P_{11} B_i^u D^c), \\ A_{ij}^c &= P_{12}^{-1} (\hat{A}_{ij} - P_{11} (A_i + B_i^u D^c C_j^y) X_{11} - P_{12} B_i^c C_j^y X_{11} - P_{11} B_i^u C_j^c X_{12}^T) X_{12}^{-T}, \quad \forall i, j \in \Omega_r. \end{aligned} \quad (33)$$

solves the control problem stated in Section II.

Proof. We use the linearizing approach of [34]: properties (27) and (32) imply the existence of two matrices P_{22} and X_{22} such that the block matrices P and X given by

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}, \quad X = P^{-1} = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad (34)$$

are positive definite. Note also that the same properties imply that the matrices P_{12} and X_{12} are regular. Let us now introduce the matrices

$$\Pi_1 \triangleq \begin{bmatrix} X_{11} & I \\ X_{12}^T & 0 \end{bmatrix}, \quad \Pi_2 \triangleq P\Pi_1 = \begin{bmatrix} I & P_{11} \\ 0 & P_{12}^T \end{bmatrix}. \quad (35)$$

By congruence transformation with $\text{diag}(\Pi_1, I)$, inequality (25) is shown to be equivalent to (28). This implies that the ellipsoid $\mathcal{E}(P)$ is included in the polyhedral set \mathcal{P}_x defined in (13). Similarly, by Schur complement lemma and congruence transformation with $\text{diag}(\Pi_1, I)$, inequality (29) is shown to be equivalent to (24), so that the ellipsoid $\mathcal{E}(P)$ is included in the polyhedral set \mathcal{P}_u defined in (15) with $G_1^i \triangleq V_i$ and $G_2^i \triangleq U_i X_{12}^{-T} - G_1^i X_{11} X_{12}^{-T}$.

By Lemma 4, inequalities (26) imply clearly that $\Upsilon_{\theta\theta} \triangleq \sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) \Upsilon_{ij} < 0$. After a congruence transformation with $\text{diag}(\Pi_1, S_\theta^{-1}, I, I)$, this inequality is proved to be equivalent to

$$\begin{bmatrix} \mathbb{A}_{\theta\theta}^T P + P \mathbb{A}_{\theta\theta} + 2\alpha P & \mathbb{G}_\theta^T S_\theta^{-1} - P(\mathbb{B}_\theta^\psi + R_\psi E_{\theta\theta}^c) & P \mathbb{B}_{\theta\theta}^w + \mathbb{C}_\theta^T \mathbb{D}_\theta & \mathbb{C}_\theta^T \\ * & -2S_\theta^{-1} & S_\theta^{-1} K_\theta^w & 0 \\ * & * & \mathbb{D}_\theta^T \mathbb{D}_\theta - \gamma I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (36)$$

which, by Schur complement lemma, is equivalent to

$$\begin{bmatrix} \text{He}(P \mathbb{A}_{\theta\theta} + \alpha P) + \mathbb{C}_\theta^T \mathbb{C}_\theta & \mathbb{G}_\theta^T S_\theta^{-1} - P(\mathbb{B}_\theta^\psi + R_\psi E_{\theta\theta}^c) & P \mathbb{B}_{\theta\theta}^w + \mathbb{C}_\theta^T \mathbb{D}_\theta \\ * & -2S_\theta^{-1} & S_\theta^{-1} K_\theta^w \\ * & * & \mathbb{D}_\theta^T \mathbb{D}_\theta - \gamma I \end{bmatrix} < 0. \quad (37)$$

Pre- and post-multiplying (37) by $\begin{bmatrix} x_{cl}^T & w^T & \psi^T(u_c) \end{bmatrix}$ and its transpose, the following condition can be obtained after some algebraic manipulations

$$\dot{\mathbb{V}}(x_{cl}) + 2\alpha V(x_{cl}) + y^T y - \gamma w^T w - 2\psi^T(u_c) S_\theta^{-1} \left[\psi(u_c) - \begin{bmatrix} \mathbb{G}_\theta & K_\theta^w \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix} \right] < 0. \quad (38)$$

Since $\mathcal{E}(P) \subseteq \mathcal{P}_u$, by Lemma 1, the satisfaction of condition (38) implies that

$$\dot{\mathbb{V}}(x_{cl}) + 2\alpha \mathbb{V}(x_{cl}) + y^T y - \gamma w^T w < 0. \quad (39)$$

We distinguish the two following cases.

- **Case 1.** If $w = 0$, $\forall t \geq 0$, it follows from (39) that: $\dot{\mathbb{V}}(x_{cl}) < -2\alpha \mathbb{V}(x_{cl})$. This inequality implies that the set $\mathcal{E}(P)$ is positively invariant with respect to the closed-loop system (9), and that, for any initial state in this set, the corresponding trajectory converges exponentially to the origin with a decay rate α . This fact proves Property 2.
- **Case 2.** If $w \neq 0$, $\forall t \geq 0$ and $w \in \mathcal{W}_\delta$, integrating both sides of (39) from 0 to T_f , with $T_f > 0$, it follows that

$$\mathbb{V}(x_{cl}(T_f)) - \mathbb{V}(x_{cl}(0)) + 2\alpha \int_0^{T_f} \mathbb{V}(x_{cl}) dt - \gamma \int_0^{T_f} w^T w dt + \int_0^{T_f} y^T y dt < 0. \quad (40)$$

From (30) and (40), it can be deduced that, for any initial state in $\mathcal{E}(P, \rho)$, we have $\mathbb{V}(x_{cl}(T_f)) < \rho + \gamma\delta \leq 1$. This means that the corresponding closed-loop trajectories are confined in the set $\mathcal{E}(P) \subseteq \mathcal{P}_x$. Furthermore, considering the limit case $T_f \rightarrow \infty$ in (40), then we obtain $\int_0^\infty y^T y dt < \gamma \int_0^\infty w^T w dt + \mathbb{V}(x(0))$, this means that the \mathcal{L}_2 -norm of the output signal y is upper bounded by $\|y\|_2^2 < \gamma \|w\|_2^2 + \rho$. These facts proves Properties 1 and 3 of the closed-loop system.

From the results of the two above cases, the proof of the theorem can be concluded. \square

Remark 2. When $w \in \mathcal{W}_\delta \setminus \{0\}$, $\forall t \geq 0$, and $x_{cl}(0) \neq 0$, there is a trade-off between the size of the set of initial conditions and the maximal level energy of disturbances characterized by δ . Indeed, the lower is the admissible δ (i.e. the lower is the admissible energy of the disturbance signal), the larger is the estimate of the set of admissible initial states $\mathcal{E}(P, \rho)$, and by extension, the ellipsoid $\mathcal{E}(P)$ [8].

Remark 3. Using quadratic Lyapunov function may make the results conservative. To overcome this drawback, multiple Lyapunov function approach [35] could be applied. Although this latter has recently received increasing attention, it still remains an open research topic for continuous-time T-S systems. This fact is due to the presence of the time-derivatives of the

membership functions involved in the theory development (which is not the case of discrete-time T-S systems [36]). It should be stressed that these non-quadratic Lyapunov based approaches generally lead to extremely complex design conditions with heavy computation cost even with a static state-feedback controller [37]. Moreover, in terms of control law implementation, the control structure is also very complex requiring online parameter-dependent matrix inversion. Therefore, for practical uses this work keeps using quadratic Lyapunov function regardless of its inducing conservativeness.

IV. ANTI-WINDUP BASED DYNAMIC OUTPUT FEEDBACK CONTROL DESIGN

Theorem 1 provides conditions to check easily the feasibility of the control problem defined in Section II. Since the results are developed for regional stability and performance, there is a trade-off between different requirements. According to the design purposes, two following optimization problems can be formulated.

Optimization Problem 1. Find a DOFC and its AW strategy (7) such that the closed-loop system satisfies the properties defined in Section II-A and the disturbance rejection effect is maximized for a given bound δ on the \mathcal{L}_2 -norm of admissible disturbances. This problem can be straightforwardly solved with the following LMI optimization

$$\min_{P_{11}, X_{11}, S_i, U_i, V_i, W_{ij}, \hat{A}_{ij}, \hat{B}_i, \hat{C}_i, \hat{D}, \gamma, \rho} \gamma, \quad (41)$$

subject to: $\gamma > 0$ and LMI conditions in Theorem 1.

Optimization Problem 2. Find a DOFC and its AW strategy (7) such that the closed-loop system satisfies the properties defined in Section II-A and both ellipsoids $\mathcal{E}(P, \rho)$ and $\mathcal{E}(P)$ can be maximized. In order to avoid obtaining a flat estimate of the domain of attraction, we can minimize the following optimization function [15], [32], [38]

$$\alpha_R = \lambda \text{trace}(R) + (\lambda - 1) \rho, \quad (42)$$

where $0 \leq \lambda \leq 1$ is a tuning parameter and the matrix $R \triangleq \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix}$ satisfies $R > P$. Using successively Schur complement lemma and congruence transformation with $\text{diag}(I, \Pi_2^T)$, it can be proved that

$$R > P \Leftrightarrow \begin{bmatrix} R_{11} & R_{12} & I & P_{11} \\ * & R_{22} & 0 & P_{12}^T \\ * & * & X_{11} & I \\ * & * & * & P_{11} \end{bmatrix} > 0. \quad (43)$$

From (43), it follows that $\text{trace}(R) > \text{trace}(P)$. Then, the minimization of $\text{trace}(R)$ implies the minimization of $\text{trace}(P)$. Then, Problem 2 can be solved through the convex optimization problem

$$\min_{P_{11}, X_{11}, P_{12}, S_i, U_i, V_i, W_{ij}, \hat{A}_{ij}, \hat{B}_i, \hat{C}_i, \hat{D}, \gamma, \rho, R} \alpha_R, \quad (44)$$

subject to: $R > 0$, LMI conditions in Theorem 1 and LMI (43).

Several remarks on the obtained results can be reported as follows.

Remark 4. The equation (32) has an infinite number of solutions P_{12} and X_{12} parameterizing an infinite number of controllers with the same closed-loop performance. Indeed, the choice of matrices P_{12} , X_{12} is irrelevant for control design since it corresponds to a change of coordinates of the controller states [39]. Up to now this degree of freedom has not been yet exploited for the selection of the most desirable controller [22]. However, with the proposed method it should be stressed from (43) that P_{12} can be considered as a slack variable of the optimization problem (44) which aims at selecting, among all possible solutions of (32), the most desirable one maximizing the domain of attraction with respect to a predefined optimization criterion. The corresponding "optimal" DOFC with its AW compensation (7) can be now computed by (44) and (33).

Remark 5. It follows from (27) that $P_{11} > X_{11}^{-1}$. Consequently, the set $\mathcal{E}(P_{11}, \rho)$ is always contained in $\mathcal{E}(X_{11}^{-1}, \rho)$. Assume that $x_c(0) = 0$, then $x_{cl}(0) \in \mathcal{E}(P, \rho)$ if $x(0) \in \mathcal{E}(P_{11}, \rho)$. In the absence of disturbances, the set $\mathcal{E}(P)$ is positively invariant, and so $x_{cl}(t) \in \mathcal{E}(P)$ for $\forall t \geq 0$. Therefore, $x(t)$ will never escape the projection of $\mathcal{E}(P)$ onto the plane defined by $x_c = 0$. This projection is nothing but the ellipsoid $\mathcal{E}(X_{11}^{-1})$.

V. ILLUSTRATIVE EXAMPLE

In this section, the effectiveness of the proposed method is demonstrated through a physically motivated nonlinear system. All LMI problems are solved with the YALMIP toolbox in Matlab R2013b. Let us consider the following nonlinear mass-spring-damper mechanical system

$$\begin{cases} \dot{x}_1 = -x_1 - (1 + 0.05x_2^2)x_2 + (0.5 + 0.075x_2^3)u + 0.1w \\ \dot{x}_2 = x_1 + 0.1w \\ y = x_2 \end{cases} \quad (45)$$

where x_1 , x_2 and w are respectively the velocity, the position and the system disturbance. The following assumptions are considered for this example. First, the state vector $x^T \triangleq [x_1 \ x_2]$ always remains in the validity domain described by $\mathcal{P}_x = \{x \in \mathbb{R}^2 : |x_1| \leq 1.5, |x_2| \leq 1.5\}$. Second, the control saturation limit is: $u_{\max} = 0.5$. Third, only the position is measured as system output. Finally, the system disturbance w is energy-bounded and of the following form

$$w(t) = \begin{cases} 0.1 \cos(t), & \text{if } 0 \leq t \leq 15 \\ 0, & \text{otherwise} \end{cases} \quad (46)$$

where the corresponding energy bound is given $\delta \approx 0.07$. Since $x \in \mathcal{P}_x$, two nonlinearities $f(x_2) = 1 + 0.05x_2^2$ and $g(x_2) = 0.5 + 0.075x_2^3$ of system (45) are then bounded $f_{\min} \leq f(x_2) \leq f_{\max}$ and $g_{\min} \leq g(x_2) \leq g_{\max}$. Using sector nonlinearity approach [13], these nonlinearities can be represented as follows

$$f(x_2) = \omega_1 f_{\min} + \omega_2 f_{\max}, \quad g(x_2) = \omega_3 g_{\min} + \omega_4 g_{\max}, \quad (47)$$

where

$$\omega_1(x_2) = \frac{f_{\max} - f(x_2)}{f_{\max} - f_{\min}}, \quad \omega_2(x_2) = 1 - \omega_1(x_2), \quad \omega_3(x_2) = \frac{g_{\max} - g(x_2)}{g_{\max} - g_{\min}}, \quad \omega_4(x_2) = 1 - \omega_3(x_2).$$

The normalized nonlinear membership functions of T-S model can be now obtained by

$$\begin{cases} \eta_1(x_2) = \omega_1(x_2)\omega_3(x_2), & \eta_2(x_2) = \omega_1(x_2)\omega_4(x_2), \\ \eta_3(x_2) = \omega_2(x_2)\omega_3(x_2), & \eta_4(x_2) = \omega_2(x_2)\omega_4(x_2). \end{cases} \quad (48)$$

Then, the nonlinear system (45) can be exactly represented by T-S model (1) in the polyhedral set \mathcal{P}_x , where the subsystem matrices are given by

$$\begin{aligned} A_1 &= \begin{bmatrix} -1 & -f_{\min} \\ 1 & 0 \end{bmatrix}, & B_1^u &= \begin{bmatrix} g_{\min} \\ 0 \end{bmatrix}, & A_2 &= \begin{bmatrix} -1 & -f_{\min} \\ 1 & 0 \end{bmatrix}, & B_2^u &= \begin{bmatrix} g_{\max} \\ 0 \end{bmatrix}, & B_i^w &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -1 & -f_{\max} \\ 1 & 0 \end{bmatrix}, & B_3^u &= \begin{bmatrix} g_{\min} \\ 0 \end{bmatrix}, & A_4 &= \begin{bmatrix} -1 & -f_{\max} \\ 1 & 0 \end{bmatrix}, & B_4^u &= \begin{bmatrix} g_{\max} \\ 0 \end{bmatrix}, & \forall i \in \Omega_4. \end{aligned}$$

For numerical experiments, it is noticed that the decay rate α in Property 2 is related to the time performance of the closed-loop system [13]. A large value of this tuning parameter leads to a fast convergence time; however the corresponding controller could induce some aggressive closed-loop behaviors. Especially, this situation can get worst if the disturbance is directly involved in the system dynamics. In this work, we take $\alpha = 0.01$ to illustrate the results. Moreover, without loss of generality the effectiveness of the proposed method will be demonstrated through the solution of the optimization problem (44). To this end, let us take the weighting factor $\lambda = 0.6$, solving this convex optimization yields

$$\gamma = 1.2134, \quad \rho = 0.9879, \quad P_{11} = \begin{bmatrix} 0.6579 & 0.4821 \\ 0.4821 & 1.0888 \end{bmatrix}, \quad X_{11} = \begin{bmatrix} 2.2500 & -0.9962 \\ -0.9962 & 1.3596 \end{bmatrix}. \quad (49)$$

In what follows, we will show that the corresponding DOFC satisfies the three predefined properties stated in Section II.

Regional quadratic α -stability and set inclusion properties. In the absence of disturbances, i.e. $w(t) = 0$, it can be seen in Figure 2 that the projection of $\mathcal{E}(P)$ onto the plane defined by $x_c = 0$, that is $\mathcal{S}_1 \triangleq \mathcal{E}(X_{11}^{-1})$, is an invariant set (corresponding to system states) of the closed-loop system, i.e. all system trajectories initialized in this set will never escape it. From (49), one has $X_{11}^{-1} \approx P_{11}$. Then, we note that both ellipsoids $\mathcal{E}(X_{11}^{-1})$ and $\mathcal{E}(P_{11}, \rho)$ are (almost) superimposed in Figure 2 and the set $\mathcal{E}(X_{11}^{-1})$ includes, of course, $\mathcal{E}(P_{11}, \rho)$ since $\rho < 1$. Furthermore, as can be also observed, the ellipsoid \mathcal{S}_1 is maximized along the polyhedral set $\mathcal{S}_2 \triangleq \mathcal{P}_u \cap \mathcal{P}_x$ which is, in turn, contained in the $\mathcal{S}_3 \triangleq \Xi \left(\sum_{i=1}^r \eta_i (\mathbb{K}_i - \mathbb{G}_i) \right)$. Finally, all closed-loop trajectories converge to the origin.

Closed-loop finite \mathcal{L}_2 -gain performance. It is assumed now that the disturbance w defined in (46) is directly involved in the dynamics of system (45), Figure 3 (up) shows the state evolution of the closed-loop system (45) obtained with initial condition $x_0^T = [1 \ -0.5]$. As can be observed, the energy-bounded disturbance w is well attenuated and the ratio $\frac{\int_0^{T_f} y^T y dt}{\gamma \int_0^{T_f} w^T w dt + \rho}$, with $T_f > 0$, is always bounded by 1, see Figure 3 (bottom). This latter guarantees the closed-loop \mathcal{L}_2 -gain performance defined in (14). The corresponding control signal u , the controller output u_c and the controller state x_c are depicted in Figure 4. We can see that despite an important level of control input saturation at the beginning, the proposed AW-based dynamic controller induces stable closed-loop behaviors. It is noticed that in this case, the classical DOFC (without anti-windup structure) proposed in [13] is no longer effective as shown in Figure 5.

Now, let us examine a limit case where the initial condition $x_0^T = [1.5 \ 1.5]$ is a vertex of the validity domain \mathcal{P}_x . It can be clearly observed in Figure 6 that the proposed controller can provide stable behaviors despite a huge level of saturation of input signal u and also system state x_2 . Notice that the classical DOFC in [13] is not able to guarantee the closed-loop stability in such a situation.

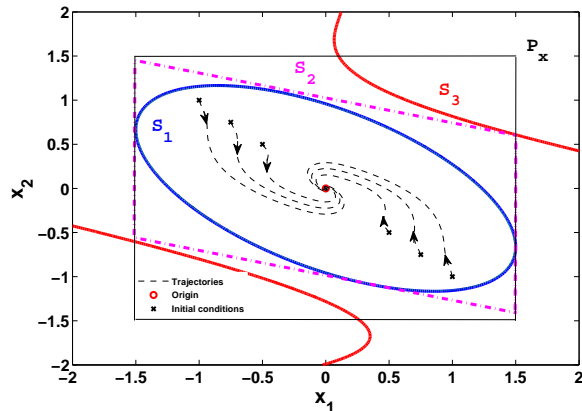


Fig. 2. Projection of $\mathcal{E}(P)$ onto the plane defined by $x_c = 0$ and system trajectories.

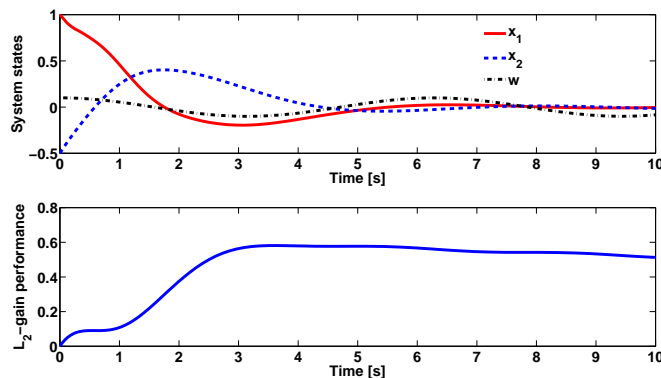


Fig. 3. Closed-loop system states (up) and finite \mathcal{L}_2 -gain performance (bottom) with $x_0^T = [1 \ -0.5]$.

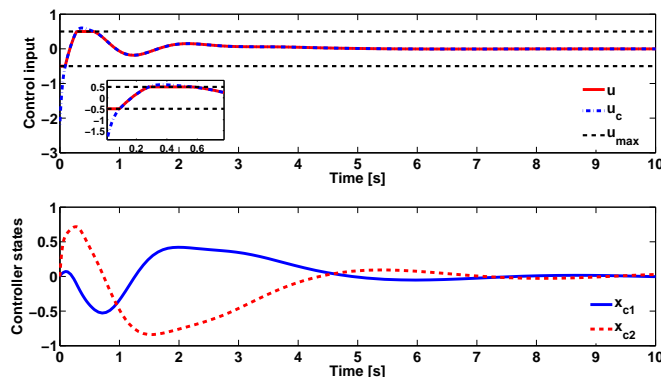


Fig. 4. Control signal response (up) and evolution of the controller state of the proposed controller (bottom).

VI. CONCLUDING REMARKS

A novel approach to design a dynamic output feedback controller together with anti-windup strategy for continuous-time nonlinear systems in T-S form has been proposed. In this approach, the disturbed systems are subject to control input and system state constraints. Based on Lyapunov stability theory, a constructive procedure is given to design simultaneously the dynamic output feedback controller and its anti-windup compensator. The control design is reformulated as convex optimization problems such that several regional closed-loop specifications can be achieved. The interests of the proposed method are clearly demonstrated by means of an physically motivated example. Moreover, the proposed design conditions are relatively simple which can be applied to a wide class of nonlinear disturbed systems. Notice that if properly extended, the proposed control method also provides possible application prospects for T-S fuzzy systems subject to both input magnitude and rate saturations,

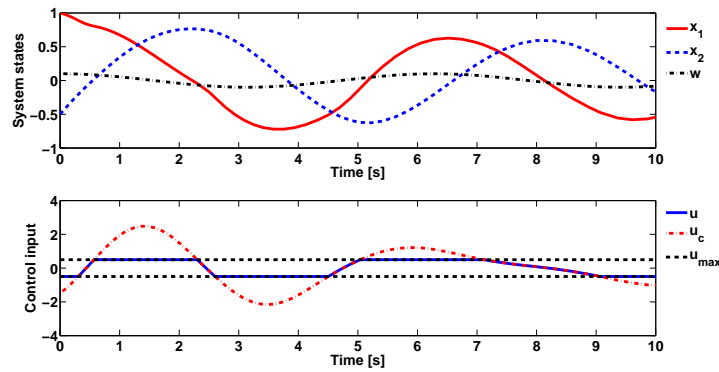


Fig. 5. Closed-loop state responses (top); evolution of the control input and controller output (bottom) obtained with classical DOFC [13].

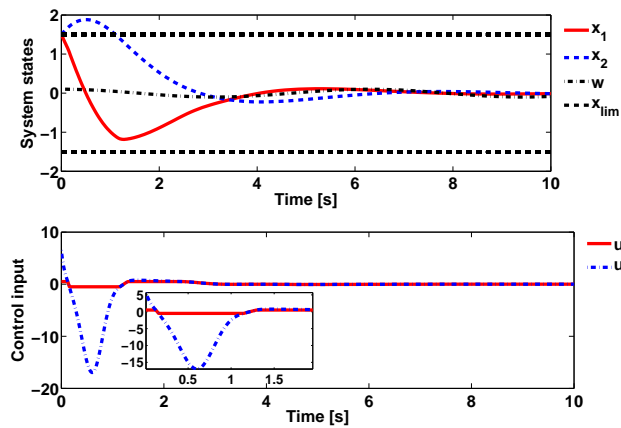


Fig. 6. Closed-loop behaviors obtained with the proposed AW-based dynamic controller for $x_0^T = [1.5 \ 1.5]$.

or more generally nested saturations [15], which will be pursued in future works.

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