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Simplified soft output demapper for non-binary LDPC codes

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A B S T R A C T

Combining non-binary LDPC codes and high order constellation, namely Bit-Interleaved Coded Modulation (BICM), is a simple but effective way to improve the bandwidth efficiency. Since the non-binary LDPC codes require soft decisions as the input information, the constellation has to provide soft information (log-likelihood ratio) to it. In this paper, a simplified algorithm to calculate LLR for binary code is applied for non-binary code, which will remarkably reduce the computational complexity, and has nearly no performance loss.

Keywords:

Low-density parity-check (LDPC) codes

Non-binary

Log-likelihood ratio (LLR)

QAM

Gray mapping

1. Introduction

Soft decision channel decoders have been widely used for modern error correcting codes such as: turbo decoder and LDPC (low-density parity-check) decoder. The soft decisions are typically represented by LLR (log-likelihood ratio) values, which represent the reliability of a correct decision. The superiority LLR computation comes at the expense of higher complexity. The LLR expression generating from the soft output demapper to the decoder can be simplified to reduce the complexity.

Binary LDPC codes proposed by Gallager in 1962 [4] and rediscovered by MacKay [5], approach Shannon capacity [6]. Unfortunately, binary LDPC code shows performance weaknesses when the code size is small or moderate, and higher order modulation is used for transmission. Non-binary LDPC code, designed in high order Galois Fields $GF(q)$ where q is the cardinality of the Galois field, is investigated by Davey and MacKay [7] to avoid this weaknesses.

Also in [7] the authors proposed the first practical iterative decoding algorithm for non-binary LDPC codes. This algorithm, called Sum-Product Algorithm (SPA), is an optimal iterative decoding with computational complexity. Several algorithms have been proposed to reduce the complexity of the non-binary SPA: FFT-SPA [1] in the frequency domain based on the Fast Fourier Transform (FFT); log-domain SPA [8] such as Min-Sum Algorithm (MSA); a mixed-domain SPA [9] can take advantage of both domains.

Furthermore, all these three algorithms need to keep all q messages. Consequently, to reduce complexity and memory requirements two simplified algorithms, based MSA, have been proposed: the Extended Min-Sum Algorithm (EMSA) [10,11] and the Min-Max Algorithm (MMA) [12]. From the EMSA, based on the combinatorial optimization, the Simplified Min-Sum Algorithm (SMSA) is derived. Compared to the SPA, the SMSA shows small SNR loss, which is similar to that of the EMSA and MMA [2].

The initial message used by the decoder is the soft output demapping of the received signal. We use in this paper a decoder that uses the LLR calculation, such as EMSA, to compare between exact and simplified LLR for non-binary LDPC code. We show that, with square Gray-QAM constellation and under Gaussian channel, the simplified LLR-EMSA has no performance degradation compared to the exact LLR-EMSA.

The rest of the paper is organized as follows. Section 2 introduces the non-binary LDPC coding and decoding and the EMSA. In Section 3, the exact and the simplification of the LLR calculation for square QAM constellation under Gaussian channel is investigated. Finally, the simulation results and concluding remarks are given in Sections 4 and 5, respectively.

2. Non-binary LDPC coding and decoding

In this paper, we will focus on the Galois Field of q elements $GF(q)$ with characteristic 2, i.e. $q = 2^p$.

Non-binary LDPC codes are defined by their non-binary parity check matrix i.e. the non-zero element in this matrix are numbers in the Galois field $GF(q)$ ($q > 2$). The entries in the parity-check matrix

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of a non-binary LDPC code belong to $GF(q)$. A codeword can be expressed as a sequence of symbols in $GF(q)$ [13].

The $(M \times N) - H$ parity-check matrix can be represented graphically by a Tanner graph, which is a bipartite graph with two disjoint variable node and check node classes. These nodes are connected by the edges. The connections correspond to the non-zero element in the parity check matrix. The number of variable nodes N_m ($m \in \{1, \dots, M\}$) and check nodes M_n ($n \in \{1, \dots, N\}$) corresponds, respectively, to the number of matrix columns N and rows M . Each node generates and propagates messages to its neighbours based on its current incoming messages except the input message on the edge where the output message will be sent.

Let, for all symbol a , where $a \in GF(q)$, $\alpha_{m,n}$ be the soft messages from variable nodes v_n to check nodes c_m and $\beta_{m,n}$ be the soft messages from check nodes c_m to variable nodes v_n . The initial message γ_n sent from variable nodes v_n to check nodes c_m is a soft demapping of the received signal y_n given knowledge of the channel properties. It can be: a priori information: $\gamma_n(a) = \Pr(v_n = a/y_n)$, where $\Pr(v_n = a/y_n)$ is the probability that $v_n = a$ given the received signal y_n , or a log-likelihood ratio (LLR): $\gamma_n(a) = \log(\Pr(v_n = 0/y_n)/\Pr(v_n = a/y_n))$.

2.1. Extended Min-Sum Algorithm (EMSA)

The Extended Min-Sum decoding Algorithm (EMSA) performs the following operations [12]:

- Initialization

$$\gamma_n(a) = \log \left(\frac{\Pr(v_n = 0/y_n)}{\Pr(v_n = a/y_n)} \right) \quad (1)$$

- Variable node messages

$$\alpha_{m,n}(a) = \gamma_n(a) \quad (2)$$

- Iterations

- Check node processing:

$$\beta_{m,n}(a) = \min_{(a_{n'})_{n' \in N_m/n}} / \left(\sum_{n' \in N_m/n} \alpha_{m,n'}(a_{n'}) \right) \quad (3)$$

- Variable node processing

$$\alpha'_{m,n}(a) = \gamma_n(a) + \sum_{m' \in M_n/m} \beta_{m',n}(a) \quad (4)$$

$$\alpha_{m,n}(a) = \alpha'_{m,n}(a) - \alpha'_{m,n}(0) \quad (5)$$

- A posteriori information

$$\tilde{\gamma}(a) = \gamma_n(a) + \sum_{m \in M_n} \beta_{m,n}(a) \quad (6)$$

- Decision

$$z_n = \arg \max_{a \in GF(q)} (\tilde{\gamma}_n(a)) \quad (7)$$

Finally, the algorithm stops if $(z_{v_j} H^T = 0)$ i.e. $z_v = [z_{v_1}, \dots, z_{v_N}]$ is a valid codeword or a maximum number of iterations is reached.

3. Log-likelihood ratio (LLR)

At time k , each point of the 2^m -QAM that has square constellation is represented by $C_k = A_k + jB_k$, where A_k and $B_k \in \{\pm 1, \pm 3, \pm 5, \dots, \pm(m-1)\}$, coded by a set $\{u_{k,i}\}$, $i \in \{1, \dots, m\}$, of m bits according to Gray mapping.

The priori probability $\hat{u}_{k,i}$ of $u_{k,i}$, where the received symbol $C'_k = A'_k + jB'_k$ corresponds to $C_k = A_k + jB_k$, is:

$$\hat{u}_{k,i} = \Pr \left(\frac{u_{k,i} = x}{A'_k, B'_k} \right), \quad x \in GF(2) \quad (8)$$

where $\Pr(u_{k,i} = x/A'_k, B'_k)$ is the probability that $u_{k,i} = x$ given symbols A'_k and B'_k .

The LLR $\Lambda(u_{k,i})$ of $u_{k,i}$, are obtained by using the relation [3]:

$$\Lambda(u_{k,i}) = \frac{\sigma^2}{2} \log \frac{\Pr(u_{k,i} = 1/A'_k, B'_k)}{\Pr(u_{k,i} = 0/A'_k, B'_k)} \quad (9)$$

where σ^2 is the noise variance.

Applying Bayes' rule twice from the previous expression and assuming that $\Pr(u_{k,i} = 1) = \Pr(u_{k,i} = 0)$, we can also write:

$$\Lambda(u_{k,i}) = \frac{\sigma^2}{2} \log \frac{\Pr(A'_k, B'_k/u_{k,i} = 1)}{\Pr(A'_k, B'_k/u_{k,i} = 0)} \quad (10)$$

where $\Pr(A'_k, B'_k/u_{k,i} = x)$ is the probability of the couple $\{A'_k, B'_k\}$ assuming that bit $u_{k,i}$ is equal to x .

3.1. Exact LLR for Gray-QAM with square constellation under Gaussian channel

Consider that the modulation scheme is an M -QAM with square constellation. Since Gray mapping is used, such a modulation consists of two identical $M^{1/2}$ -ASK in quadrature. Symbol A_k is associated with the subset $\{u_{k,i}\}$, $i \in \{1, \dots, (m/2)\}$ while symbol B_k is associated with the subset $\{u_{k,i}\}$, $i \in \{(m/2)+1, \dots, m\}$. Therefore Eq. (10) yields to two separated Eqs. (11) and (12) [3]:

$$\Lambda(u_{k,i}) = \frac{\sigma^2}{2} \log \frac{\Pr(A'_k/u_{k,i} = 1)}{\Pr(A'_k/u_{k,i} = 0)} \quad (11)$$

$$\Lambda(u_{k,i}) = \frac{\sigma^2}{2} \log \frac{\Pr(B'_k/u_{k,i} = 1)}{\Pr(B'_k/u_{k,i} = 0)} \quad (12)$$

Assuming that a Gaussian channel is used, we use:

$$\Pr(A'_k/u_{k,i} = x) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{n=1}^{2^{m/2}} \exp - \frac{1}{2\sigma^2} (A'_k - A_{n,x})^2 \quad (13)$$

$$\Pr(B'_k/u_{k,i} = x) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{n=2^{(m/2)+1}}^{2^m} \exp - \frac{1}{2\sigma^2} (B'_k - B_{n,x})^2 \quad (14)$$

Eqs. (4) and (5) yield respectively to:

$$\Lambda(u_{k,i}) = \frac{\sigma^2}{2} \log \frac{\sum_{n=1}^{2^{m/2}} \exp - (1/2\sigma^2)(A'_k - A_{n,1})^2}{\sum_{n=1}^{2^{m/2}} \exp - (1/2\sigma^2)(A'_k - A_{n,0})^2}, \quad i \in \left\{ 1, \dots, \frac{m}{2} \right\} \quad (15)$$

$$\Lambda(u_{k,i}) = \frac{\sigma^2}{2} \log \frac{\sum_{n=2^{(m/2)+1}}^{2^m} \exp - (1/2\sigma^2)(B'_k - B_{n,1})^2}{\sum_{n=2^{(m/2)+1}}^{2^m} \exp - (1/2\sigma^2)(B'_k - B_{n,0})^2}, \quad i \in \left\{ \frac{m}{2} + 1, \dots, m \right\} \quad (16)$$

3.2. Simplified LLR for Gray-QAM with square constellation under Gaussian channel

Performing some minor simplifications from Eqs. (8) and (9), we can demonstrate that the LLR $\Lambda(u_{k,i})$ is well approximated by [3]:

$$\begin{aligned} \Lambda(u_{k,1}) &= A'_k \\ \Lambda(u_{k,2}) &= -|\Lambda(u_{k,1})| + 2^{(m/2)-1} \\ &\vdots \\ \Lambda(u_{k,i}) &= -|\Lambda(u_{k,i-1})| + 2^{(m/2)-i+1} \\ &\vdots \\ \Lambda(u_{k,(m/2)}) &= -|\Lambda(u_{k,(m/2)-1})| + 2 \end{aligned} \quad (17)$$

$$\begin{aligned} \Lambda(u_{k,(m/2)+1}) &= B'_k \\ \Lambda(u_{k,(m/2)+2}) &= -|\Lambda(u_{k,1})| + 2^{(m/2)-1} \\ &\vdots \\ \Lambda(u_{k,(m/2)+i}) &= -|\Lambda(u_{k,(m/2)+i-1})| + 2^{(m/2)-i+1} \\ &\vdots \\ \Lambda(u_{k,m}) &= -|\Lambda(u_{k,m-1})| + 2 \end{aligned} \quad (18)$$

3.3. LLR calculation over GF(q)

In the case of the non-binary LDPC code, i.e. over $GF(q)$, the LLR of each symbol $a \in GF(q)$ are computed at the symbol level and not at the bit level.

In the general case of $GF(2^S)$, each symbol a contains S bits.

Let $a = (u_0, u_1, \dots, u_S)$ be a $GF(2^S)$ symbol and $a' = (u'_0, u'_1, \dots, u'_S)$ be the received symbol.

The LLR of a is defined by:

$$\Lambda(a) = \log \left(\frac{\Pr(a'/a)}{\Pr(a'/0)} \right) \quad (19)$$

If we have an independent and identical distribution of errors, the $LLR(X)$ can be written as [14]:

$$\Lambda(a) = \log \left(\frac{\prod_{i=0}^{S-1} \Pr(u'_i/u_i)}{\prod_{i=0}^{S-1} \Pr(u'_i/0)} \right) \quad (20)$$

The previous equation can be simplified to:

$$\Lambda(a) = \sum_{i=0}^{S-1} \log \left(\frac{\Pr(u'_i/u_i)}{\Pr(u'_i/0)} \right) \quad (21)$$

$$\Lambda(a) = \sum_{i=0}^{S-1} \Lambda(u_i) \quad (22)$$

So, Eq. (22) use two simplification for $\Lambda(a)$ and $\Lambda(u_i)$

4. Simulation results

Simulation results are given in terms of Bit Error Rate (BER) versus Signal to Noise Ratio (SNR). They have been simulated on the Additive Wight Gaussian Noise (AWGN) channel. All simulated non-binary LDPC codes have been constructed on $GF(4)$ of rate 1/2 with size (128, 256) (4-ary (128, 256) LDPC code).

Comparisons between simplified demapping algorithm and exact algorithm are committed. We compare, in Fig. 1, the performances between the exact and simplified demapping algorithms of 16-QAM, 64-QAM and 256-QAM associated with 4-ary (128, 256) LDPC Code decoded by EMSA.

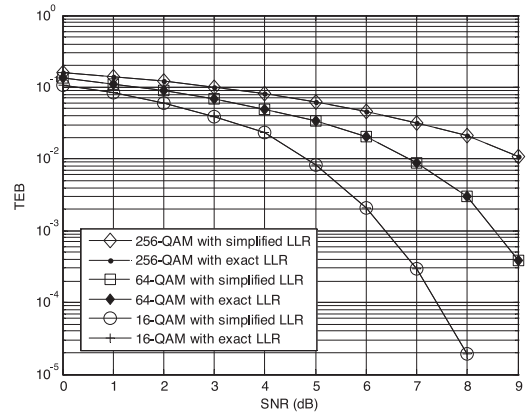


Fig. 1. Performances comparison, under Gaussian channel, of exact and simplified Gray demapping algorithm of 16-QAM, 64-QAM and 256-QAM with 4-ary (128, 256) LDPC code decoded by EMSA.

As seen in Fig. 1, the simplification of demapping algorithm has no performance degradation with square-QAM constellation. These results are generalized with small or high order square-QAM. Thus, we can use high order square-QAM constellation for high spectral efficiency. As a result, the simplification of the soft output demapping algorithm can be achieve a good performance/complexity tradoff.

5. Conclusion

In this paper, we presented a simplified soft output demapping algorithm for Gray-square-QAM constellation using non-binary LDPC code. The simplified algorithm greatly decreases the soft demapping complexity with no degradation performance. Therefore, the simplified algorithm achieves a good performance/complexity tradeoff.

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