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# Exact analysis of signal-to-noise ratio for SAC-OCDMA system with direct detection

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## A B S T R A C T

In this paper, an exact signal-to-noise-ratio (SNR) analysis is carried out for spectral amplitude coding optical code division multiple access (SAC-OCDMA) system with spectral direct detection (SDD). Unlike to the early calculated SNR expression based on SAC-OCDMA with balanced detection decoder scheme, which cannot be applied to SAC-OCDMA system with SDD; exact SNR formulas are derived and compared for two SDD schemes. We also mathematically clarify the justification to use the proposed approaches. Based on these approaches, the SNR formulas of SAC-OCDMA system with both SDD schemes, for various zero cross correlation (ZCC) codes, are accurately determined.

### Keywords:

Signal-to-noise-ratio (SNR)  
Spectral amplitude coding optical code division multiple access (SAC-OCDMA)  
Zero cross-correlation (ZCC) code  
Multi access interference (MAI)  
Phase induced intensity noise (PIIN)

## 1. Introduction

SAC-OCDMA systems are receiving more attention because of their ability to eliminate the multi access interference (MAI) by using code with fixed in-phase cross correlation [1–3]. SAC-OCDMA detection technique can be implemented either using only a single input to the receiver, SDD, or two inputs to the receiver, balanced detection [1,3,4]. Note that the SDD technique will reduce the number of filters used at the receiver part [2]. In SAC-OCDMA system the effective SNR is limited by various types of noises like phase induced intensity noise (PIIN), thermal noise, shot noise, and so forth. The PIIN is closely related to the MAI due to the overlapping of spectral from the different users [5–10], which is ignored when we use ZCC code since these codes eliminate MAI. In recent years, based on shot and thermal noise, some studies have been made to determine the SNR of SAC-OCDMA system with SDD [2,7,10]; these methods of analysis give an incorrect SNR expressions, this is due to the bad adaptation of SNR expression determined in the case of balanced detection to SDD and to non-understanding of the SDD principle. Unlike to these, in this paper, we give an exact analysis of SNR for SAC-OCDMA system with SDD. This paper is organized as follows. Section 2 is devoted to the transmitter and receiver structures of SAC-OCDMA system with SDD. In Section 3, we derive exact SNR expressions for two SDD schemes. In Section 4, some numerical results are presented. Finally, conclusions are drawn in Section 5.

## 2. Spectral direct detection schemes

Fig. 1a and b illustrates the block diagram of SAC-OCDMA system with spectral SDD technique. Unlike the balanced detection technique, which required two branches of inputs to the receiver [4], only one pair of decoder and detector is

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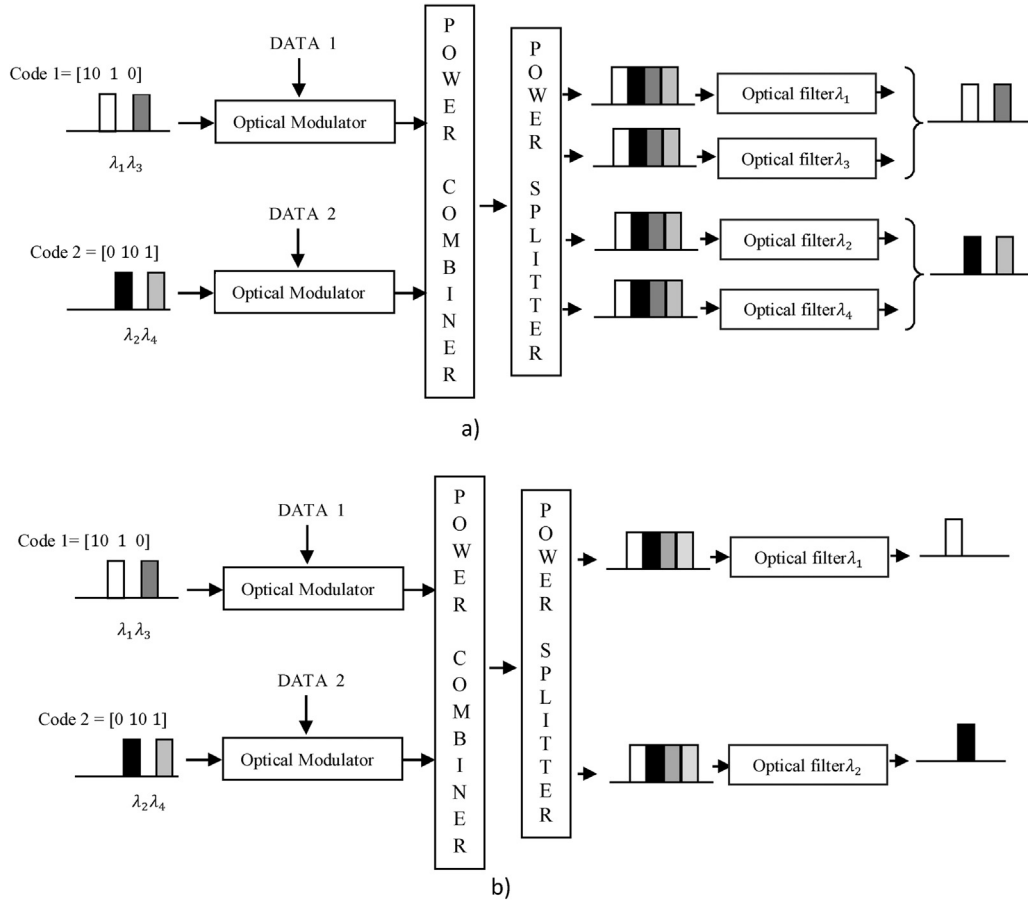


Fig. 1. Implementation of SAC-OCDMA with SDD technique a) Scheme 1, b) Scheme 2.

required [4]. Codes which possess non-overlapping spectra such as ZCC [2,5–10] code can generally be supported by this detection scheme. Due to the properties of ZCC codes, two SDD schemes can be used, the first is based on the detection of the all spectral signature code (Fig. 1a) and in the other, only one wavelength, from the code, is detected (Fig. 1b).

The first detection technique makes it possible to detect all the power emitted by the desired user (the total power multiply by the weight and divided by length of the code). On the other hand, the second technique is based on the detection of a part of the power emitted by the desired user (The total power divided by length of the code), which makes the first technique better in terms of BER. The second technique remains simple and less expensive because only one optical filter is required at the reception.

### 3. New theoretical analysis of SNR

In this section, we analyze the SNR of SAC-OCDMA system with two SDD schemes (see Fig. 1a and b). More specifically, we mathematically derive exact expressions of SNR for the two SDD schemes. In the following analysis, we have considered only the effects of both shot and thermal noises; the effect of the receiver's dark current is neglected and PIIN noise is ignored due to the zero cross correlation condition and no overlapping of spectra from different users.

Hereafter,  $C_K(i)$  denotes the  $i^{\text{th}}$  element of the  $K^{\text{th}}$  code sequence and the code properties for ZCC code of code length  $L$ , can be written as [5–8]:

$$\sum_{i=1}^L C_K(i) \cdot C_l(i) = \begin{cases} w \text{ for } l = k \\ 0 \text{ for } l \neq k \end{cases} \quad (1)$$

The SNR is the average signal power divided by average power of all noise sources.

$$SNR = \frac{i_k^2}{\sigma^2} \quad (2)$$

Where  $\sigma^2$  is defined as the variance of noise source, given by:

$$\sigma^2 = \sigma_{sh}^2 + \sigma_{th}^2 \quad (3)$$

$\sigma_{sh}^2$  is shot noise and  $\sigma_{th}^2$  is thermal noise in photodiode.  $i_k$  is the photocurrent, it can be expressed as [1,2]:

$$i_k = \Re \int_0^{+\infty} r(v) dv \quad (4)$$

Where  $\Re = \eta \cdot e/h \cdot \nu_0$  is the responsivity of the photo-detectors,  $\eta$  is the quantum efficiency,  $e$  is the electron charge, and  $h$  is Planck's constant,  $\nu_0$  is the central frequency of the original broad-band optical pulse and  $r(v)$  is the power spectral density (PSD) of the received optical signals which can be written as:

$$r(v) = \frac{P_{sr}}{\Delta v} \sum_{k=1}^K d_k \sum_{i=1}^L C_k(i) \cdot \pi(i) \quad (5)$$

The  $P_{sr}$  is the effective power of broadband source at the receiver and  $\pi(i)$  is defined by:

$$\pi(i) = u \left[ v - \nu_0 - \frac{\Delta v}{2L} (-L + 2i - 2) \right] - u \left[ v - \nu_0 - \frac{\Delta v}{2L} (-L + 2i) \right] \quad (6)$$

Therefore, the PSD at the photodiode (see Fig. 1a) of the  $l$ -th received during one bit period can be written as

$$r(v) = \frac{P_{sr}}{\Delta v} \sum_{k=1}^K d_k \sum_{i=1}^L C_k(i) \cdot C_l(i) \cdot \pi(i) \quad (7)$$

$$r(v) = \frac{P_{sr}}{\Delta v} \left[ d_1 \sum_{i=1}^L C_1(i) \cdot C_l(i) + \dots + d_l \sum_{i=1}^L C_l(i) \cdot C_l(i) + \dots + d_K \sum_{i=1}^L C_K(i) \cdot C_l(i) \right] \cdot \pi(i) \quad (8)$$

When all users are transmitting bit "1", from Eq. (1),  $r(v)$  can be expressed as

$$r(v) = \frac{P_{sr}}{\Delta v} \left[ \sum_{i=1}^L C_l(i) \cdot C_l(i) \right] \cdot \pi(i) \quad (9)$$

When we substitute Eqs. (9) and (4) instead of  $r(v)$ , the photocurrent can be expressed as

$$i_k = \Re \int_0^{+\infty} \frac{P_{sr}}{\Delta v} \left[ \sum_{i=1}^L C_l(i) \cdot C_l(i) \right] \cdot \pi(i) \cdot dv = \Re \cdot \frac{P_{sr}}{\Delta v} \cdot \sum_{i=1}^L C_l(i) \cdot C_l(i) \int_{v-\nu_0-\frac{\Delta v}{2L}(-L+2i-2)}^{v-\nu_0-\frac{\Delta v}{2L}(-L+2i)} \pi(i) dv \quad (10)$$

$$i_k = \Re \cdot P_{sr} \frac{W}{L} \quad (11)$$

In the analysis of the system (Fig. 1a and b), the variance of photocurrent results respectively from the shot noise and thermal noise can be written as [1,4]:

$$\sigma^2 = 2 \cdot e \cdot B \cdot i_k + \frac{4k_b T_n B}{R_l} = 2 \cdot e \cdot B \cdot \Re \cdot P_{sr} \frac{W}{L} + \frac{4k_b T_n B}{R_l} \quad (12)$$

Where

$B$  Noise-equivalent electrical bandwidth of the receiver;

$T_n$  Absolute receiver noise temperature;

$K_b$  Boltzmann constant;

$R_l$  Receiver load resistor.

Combining Eqs. (2), (11) and (12), we get

$$SNR = \frac{(\Re \cdot P_{sr} \cdot \frac{W}{L})^2}{2 \cdot e \cdot B \cdot \Re \cdot P_{sr} \cdot \frac{W}{L} + \frac{4K_b T_n B}{R_l}} \quad (13)$$

In the case of Fig. 1b, the PSD at the photodiode,  $r(v)$  of the  $l$ -th received during one-bit period can be written as

$$r(v) = \frac{P_{sr}}{\Delta v} \sum_{k=1}^K d_k \sum_{i=1}^L C_k(i) \cdot C_{ld}(i) \cdot \pi(i) \quad (14)$$

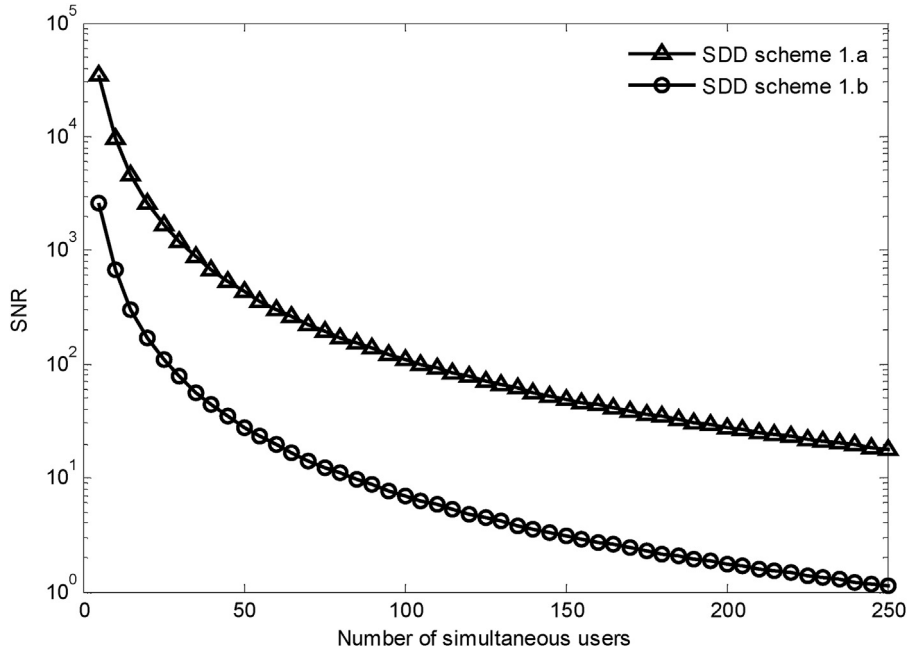


Fig. 2. SNR versus number of simultaneous users when  $P_{sr} = -10$  dBm.

Where  $C_{ld}(i)$  is the decoding code of the  $l$ -th received, it consist of the  $C_l(i)$  code with only one wavelength. Then Eq. (1) becomes

$$\sum_{i=1}^L C_K(i) \cdot C_{ld}(i) = \begin{cases} 1 & \text{for } l = k \\ 0 & \text{for } l \neq k \end{cases} \quad (15)$$

Using Eqs. (10) and (15), the photocurrent is given by

$$i_k = \mathfrak{R} \cdot \frac{P_{sr}}{L} \quad (16)$$

Then, the average SNR can be rewritten as

$$SNR = \frac{\left(R \frac{P_{sr}}{L}\right)^2}{2 \cdot e \cdot B \cdot \mathfrak{R} \frac{P_{sr}}{L} + \frac{4k_b T_n B}{R_l}} \quad (17)$$

#### 4. Performances analysis and numerical results

Bit Error Rate (BER) Expressions can be estimated, from SNR, using Gaussian approximation [1] as

$$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{SNR}{8}}$$

In the case of Fig. 1a, the BER can be expressed as

$$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{\left(\mathfrak{R} \cdot P_{sr} \cdot \frac{W}{L}\right)^2}{8 \times \left(2 \cdot e \cdot B \cdot \mathfrak{R} \cdot P_{sr} \cdot \frac{W}{L} + \frac{4k_b T_n B}{R_l}\right)}} \quad (18)$$

And in the case of Fig. 1b, the BER can be expressed as

$$BER = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{\left(\mathfrak{R} \cdot P_{sr} \cdot \frac{1}{L}\right)^2}{8 \times \left(2 \cdot e \cdot B \cdot \mathfrak{R} \cdot P_{sr} \cdot \frac{1}{L} + \frac{4k_b T_n B}{R_l}\right)}} \quad (19)$$

Figs. 2 and 3 illustrate respectively the SNR and the BER, against the number of active users for SAC-OCDMA with the two SDD schemes employing ZCC code in [6]. The parameters used in our analysis are listed in Table 1.

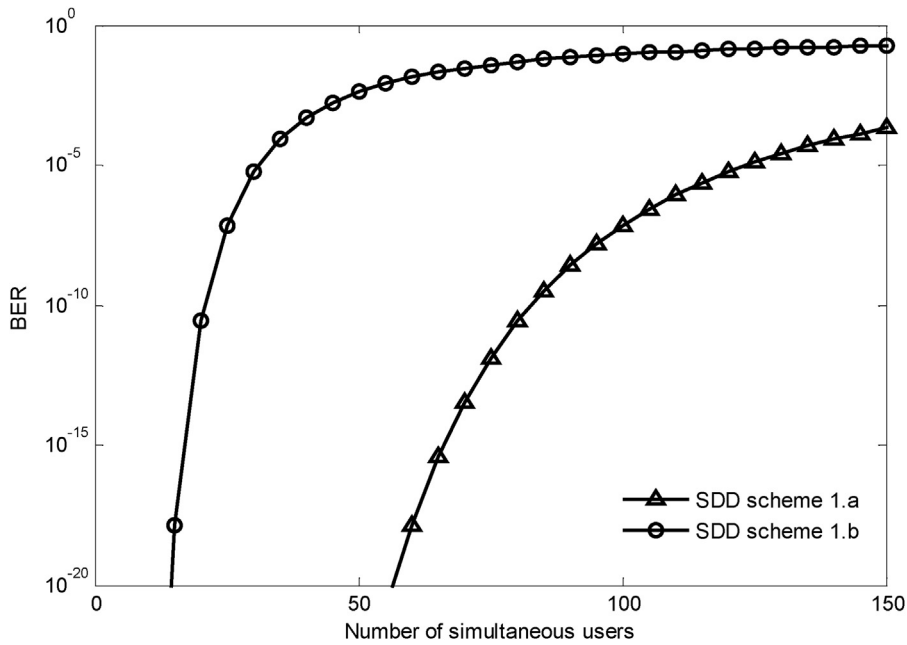


Fig. 3. BER versus number of simultaneous users when  $P_{sr} = -10$  dBm.

Table 1

Parameters used in calculation.

$\eta$	Photo-detector quantum efficiency	0.6
$\nu_0$	Operating wavelength	1550 nm
$P_{sr}$	Effective power	-10 dBm
$B$	Electrical bandwidth	311 Mhz
$T_n$	Receiver noise temperature	300 K
$R_l$	Receiverloadresistor	1030 $\Omega$

Table 2

Exact SNR Expression of SAC-OCDMA system with SDD using various ZCC code.

Codes	Code length $L$	Exact SNR expressions	
		SDD Scheme 1	SDD Scheme 2
ZCC [2,5,8]	$2^m \times L_B$ $L_B = w \cdot (w + 1)$ $m$ is the mapping process	$\frac{\left(\eta \frac{P_{sr}}{2^m \times (w+1)}\right)^2}{2 \cdot e \cdot B \cdot \eta \cdot P_{sr} \cdot \frac{1}{2^m \times (w+1)} + \frac{4k_b T_n B}{R_l}}$	$\frac{\left(\eta \frac{P_{sr}}{2^m \times L_B}\right)^2}{2 \cdot e \cdot B \cdot \eta \cdot P_{sr} \cdot \frac{1}{2^m \times L_B} + \frac{4k_b T_n B}{R_l}}$
ZCC [12]	$K \times w$ $K = w+2$	$\frac{\left(\eta \frac{P_{sr}}{K}\right)^2}{2 \cdot e \cdot B \cdot \eta \cdot \left(\frac{P_{sr}}{K}\right) \cdot \frac{1}{K} + \frac{4k_b T_n B}{R_l}}$	$\frac{\left(\eta \frac{P_{sr}}{L}\right)^2}{2 \cdot e \cdot B \cdot \eta \cdot \left(\frac{P_{sr}}{L}\right) \cdot \frac{1}{L} + \frac{4k_b T_n B}{R_l}}$
ZCC [6,9-11], MD [7]	$K \times w$	$\frac{\left(\eta \frac{P_{sr}}{K}\right)^2}{2 \cdot e \cdot B \cdot \eta \cdot P_{sr} \cdot \frac{1}{K} + \frac{4k_b T_n B}{R_l}}$	$\frac{\left(\eta \frac{P_{sr}}{L}\right)^2}{2 \cdot e \cdot B \cdot \eta \cdot P_{sr} \cdot \frac{1}{L} + \frac{4k_b T_n B}{R_l}}$

Figs. 2 and 3 show respectively the SAC-OCDMA performance in terms of SNR and BER for different SDD schemes. It is clear from Figs. 2 and 3 that an increase in code length ( $L = K \times w$ ) causes worst performances. This is because a large value of length causes a low power in transmitter part fixed weight ( $w$ ).

According to Fig. 2, the numerical results show that for 5 and 30 users; the all wavelengths detection technique (Fig. 1a) outperforms the one wavelength detection technique (Fig. 1b) respectively about 25 and 27 dB.

Based on the Eqs. (18) and (19), the SNR expression of any codes can be obtained only by the substitution of the relationship between the number of users  $K$ , weight  $w$  and the code length  $L$ . In Table 2 different exact SNR expressions are calculated for SAC-OCDMA system with SDD using various ZCC code.

## 5. Conclusion

Exact SNR formulas of SAC-OCDMA systems with two spectral direct detection schemes are derived. The performances of SAC-OCDMA system with SDD has been analyzed using ZCC codes and taking into account effects the shot and thermal

noises sources. BER performances of SAC-OCDMA system with both SDD schemes have been compared. Based on the SNR formulas, the performance of SAC-OCDMA system with SDD for different ZCC codes is accurately determined.

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