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Static Output Feedback Design for a Class of Constrained Takagi-Sugeno Fuzzy Systems

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Abstract

This paper addresses the static output feedback (SOF) control problem of Takagi-Sugeno fuzzy systems subject to both control input and state constraints. A new parameter-dependent sector condition is proposed to deal with the input saturation in SOF control context. Based on a judicious use of Finsler's lemma in fuzzy Lyapunov control framework, new design conditions guaranteeing the existence of a stabilizing controller are recast as an LMI-based optimization with additional slack variables. These extra variables offer more flexibility to reduce the design conservatism. In particular, the proposed method requires neither special constraints on the state-space system matrices nor linear matrix equalities, which are hard to be satisfied. Several numerical examples are given to demonstrate the interests of the new control method.

Keywords: Takagi-Sugeno fuzzy systems, static output feedback control, actuator saturation, delayed fuzzy Lyapunov functions, domain of attraction, linear matrix inequality (LMI).

1. Introduction

Over the past two decades, nonlinear control based on Takagi-Sugeno (T-S) fuzzy systems [1] has been extensively studied [2]. The success of T-S fuzzy models comes from their outstanding capacity to approximate any smooth nonlinear system with any preciseness [3]. Especially, if the system nonlinearities are

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bounded in a compact set, the sector nonlinearity approach [3] provides a systematic way to derive an *equivalent* T-S fuzzy representation of the original nonlinear system. As a consequence, a great number of results on stability analysis and control design of T-S fuzzy systems have been reported in the literature [4–8]. In addition, T-S fuzzy-model-based control techniques have been successfully applied to various engineering applications [2, 3, 9]. It is noteworthy that most of T-S fuzzy control methods are based on state feedback control schemes [2]. However, for numerous real-time applications, output feedback control must be used since only partial state information is accessible for control design and implementation [10, 11]. Note also that state-feedback based methods are not easily adapted to output control designs which are more difficult and involved [12]. For these reasons, output feedback control has recently received intensive research efforts [11, 13].

Among all output feedback schemes, static output feedback (SOF) represents the simplest control structure. Note that the design of dynamic controllers can be reformulated as a SOF control problem involving augmented plants in many cases [14]. Despite its practical and theoretical importance, SOF control is still one of the most challenging control topics due to its inherent non-convex characterization [11, 15–17]. Moreover, existing convex SOF design methods are often too restrictive [15, 18]. Up to now, various approaches dealing with SOF control have been proposed which can be classified into two research directions. The first one focuses on developing numerical algorithms to solve non-convex design conditions [19–21]. The second direction consists in getting new sufficient convex conditions as less conservative/restrictive as possible that can be effectively solved with current numerical solvers [15–17, 22]. In this case, some notable works dealing with linear systems can be cited as follows. A state coordinate transformation approach has been proposed in [17] for linear polytopic uncertain systems. Parameter independent slack variables with a lower-triangular structure have been introduced in [18, 23] while a linear parameter dependent Lyapunov function has been employed in [22]. In T-S fuzzy control framework, a special structure of the quadratic Lyapunov matrix together with a coordinate transformation have been used in [24]. Based on the *P-problem* and *W-problem* given in [25], quadratic design conditions involving *linear matrix equalities* are presented in [10, 16, 26]. Recently, to reduce the design conservatism, piecewise quadratic conditions have been proposed in [11] and [27] for T-S fuzzy affine systems and networked T-S fuzzy systems, respectively. Fuzzy Lyapunov functions in conjunction with some convexification techniques have been also exploited in [15].

Actuator saturation is another important control issue [4, 28–30]. This topic is motivated by the facts that control input saturation is ubiquitous in practice, especially for applications requiring a high performance level. The effects of input saturation on the system dynamics may be critical since it can seriously degrade

the control performance, and in some cases may lead to system instability [28]. Although numerous methods are now available to handle saturation effects, most of them have only dealt with linear systems [28]. It should be stressed that this practical control issue has not been completely addressed for T-S fuzzy systems [29, 30]; especially in output feedback control framework [12]. Note that besides the control input limitations, the system states are also usually bounded in engineering applications due to physical and/or safety reasons. Specifically, the validity domain of T-S fuzzy models obtained with the sector nonlinearity approach can be described by state constraints [29]. Hence, an *explicit* consideration of state constraints in the control design is crucial to guarantee the closed-loop stability [4, 29].

Motivated by the practical and theoretical significance of both SOF scheme and constrained T-S fuzzy systems, this paper aims to develop a new input-saturated SOF formulation using LMI framework [31]. The contributions of the proposed method can be summarized as follows.

1. The new method requires neither special structure of Lyapunov matrices as in [24] nor additional *linear matrix equalities*, which are hard to be satisfied, as in [10, 16, 25, 26]. In particular, this method can handle T-S fuzzy systems with multiple output matrices of subsystems. Moreover, these matrices are not *explicitly* required to be of full row rank as in most of existing works [10, 20–22, 24, 26]. Hence, the proposed method can not only lead to less conservative results but also be applied to a larger class of T-S fuzzy systems.
2. A new parameter-dependent sector condition is proposed to deal with the input nonlinearity. This sector condition is especially appropriate for the considered SOF scheme. Together with a judicious use of Finsler’s lemma, this allows for an LMI-based formulation.
3. A delayed fuzzy Lyapunov function (FLF) is exploited to derive the design conditions. This type of FLFs offers more flexibility to decrease the conservatism of the results via slack variables [32].

To the best of our knowledge, the proposed results on SOF design for constrained T-S fuzzy systems using fuzzy Lyapunov control framework have been not observed in any previous work. Furthermore, it will be shown through several examples that the new method provides less conservative results compared to recent related works dealing with *unconstrained* T-S fuzzy systems, i.e. systems without control input and/or state constraints. The paper is organized as follows. The control problem and preliminary results are presented in Section 2. The main results are stated in Section 3. The interests of the proposed method are illustrated via numerical examples in Section 4. Finally, Section 5 concludes the paper.

Notations. Ω_r denotes the set $\{1, 2, \dots, r\}$. For a vector x , x_i denotes its i th element. For a matrix X , $X > 0$ means X is positive definite, $X_{(i)}$ (respectively

$X_{(i,j)}$ denotes its i th row (respectively its element in the i th row and j th column) and $\text{He}(X) = X + X^\top$. I denotes the identity matrix of appropriate dimension. Denote $\Xi = \{\xi \in \mathbb{R}^r : \sum_{i=1}^r \xi_i = 1, \xi_i \geq 0\}$. For a function $\eta : \mathbb{R} \mapsto \Xi$ and matrices X_i, Y_{ij}, Z_{ijk} of appropriate dimensions, we denote $X_\eta = \sum_{i=1}^r \eta_i(t)X_i$, $X_{\eta_d} = \sum_{i=1}^r \eta_i(t-1)X_i$, $Y_{\eta_d\eta} = \sum_{k=1}^r \sum_{j=1}^r \eta_k(t-1)\eta_j(t)Y_{kj}$, $Z_{\eta_d\eta\eta} = \sum_{k=1}^r \sum_{j=1}^r \sum_{i=1}^r \eta_k(t-1)\eta_j(t)\eta_i(t)Z_{kji}$. We assume that $z(-1) = z(0)$ for any variable of interest $z(t)$. Arguments will be omitted when their meaning is straightforward.

2. Problem Formulation and Preliminaries

2.1. System Description

Consider the following T-S fuzzy system with control input saturation:

$$\begin{aligned} x(t+1) &= \sum_{i=1}^r \eta_i(t)(A_i x(t) + B_i \text{sat}(u(t))) \\ y(t) &= \sum_{i=1}^r \eta_i(t)C_i x(t) \end{aligned} \quad (1)$$

where $x(t) \in \mathcal{P}_x \subset \mathbb{R}^{n_x}$ is the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $y(t) \in \mathbb{R}^{n_y}$ is the measured output. The real matrices of adequate dimensions A_i, B_i, C_i are constant. The following assumptions are considered in this paper.

Assumption 1. The domain of validity \mathcal{P}_x of system (1) is described by the polyhedral set

$$\mathcal{P}_x = \{x \in \mathbb{R}^{n_x} : M_{(m)}x \leq 1, \quad m \in \Omega_q\} \quad (2)$$

where the matrix $M \in \mathbb{R}^{q \times n_x}$ is given.

Assumption 2. The scalar membership functions (MFs) $\eta_i(t)$ in (1), for $i \in \Omega_r$, are functions of measurable signals of interest such as system output, measured external disturbances and/or time. It is also assumed that the sector nonlinearity approach [3] is used to derive an *exact* T-S fuzzy representation (1) of a given nonlinear system, then $\eta \in \Xi$ as long as $x \in \mathcal{P}_x$.

Assumption 3. The control input $u(t)$ is subject to a componentwise saturation map whose l th component is defined as $\text{sat}(u_l) = \text{sign}(u_l) \min(|u_l|, \bar{u}_l)$, $l \in \Omega_{n_u}$, where $\bar{u}_l > 0$ denotes the amplitude bound of the l th control input.

In order to motivate the above assumptions, we consider the following example.
Motivating Example. Consider the *nonlinear* system borrowed from [15]:

$$\begin{aligned}
x_1(t+1) &= -x_2(t)x_3(t) - x_3(t) + u_1(t) \\
x_2(t+1) &= x_1(t) \\
x_3(t+1) &= x_2(t) + u_2(t) \\
y(t) &= \begin{bmatrix} (2+x_3(t))x_2(t) & x_3(t) \end{bmatrix}^\top
\end{aligned} \tag{3}$$

where $-1 \leq x_3 \leq 1$. From (3), x_3 can be naturally chosen as the premise variable which is a component of the measured output y . Using the sector nonlinearity approach [3], the nonlinear system (3) can be *exactly* represented in the form (1) for $x \in \mathcal{P}_x = \{x \in \mathbb{R}^3 : |x_3| \leq 1\}$. It should be stressed that, although $|x_3| \leq 1$ is the *primary condition* to obtain such a T-S fuzzy representation, this was not *explicitly* taken into account in [15] and other related works. Moreover, we also consider the limitations of the control inputs, i.e. $|u_1| \leq 0.5$ and $|u_2| \leq 0.5$. As will be shown in Section 4, an explicit consideration of these system constraints in the control design can increase significantly the performance of the closed-loop system, for instance in terms of the size of the guaranteed domain of attraction.

2.2. Control Problem

Let us consider a static output feedback controller of the form

$$u(t) = H_{\eta_d \eta}^{-1} K_{\eta_d \eta} y(t) \tag{4}$$

where the matrices $H_{kj} \in \mathbb{R}^{n_u \times n_u}$ and $K_{kj} \in \mathbb{R}^{n_u \times n_y}$, for $k, j \in \Omega_r$, have to be designed. For initializing the controller, we consider that $\eta_d(0) = \eta(0)$. Note that at each time instant, the computation of the current control input requires the inversion of the matrix $H_{\eta_d \eta}$. From (1) and (4), the closed-loop T-S fuzzy system can be rewritten as follows:

$$\begin{aligned}
x(t+1) &= (A_\eta + B_\eta H_{\eta_d \eta}^{-1} K_{\eta_d \eta} C_\eta) x(t) - B_\eta \psi(u(t)) \\
y(t) &= C_\eta x(t)
\end{aligned} \tag{5}$$

where the dead-zone nonlinearity is defined as $\psi(u) = u - \text{sat}(u)$.

This paper aims to propose a constructive LMI-based control method to design a SOF control law (4) such that the closed-loop system (5) satisfies the following properties.

- **Property 1.** Given a matrix $M \in \mathbb{R}^{q \times n_x}$, see (2). There exist positive definite matrices $P_i \in \mathbb{R}^{n_x \times n_x}$, $i \in \Omega_r$, such that, for any initial state x_0 taken in a set \mathcal{E}_0 depending on those matrices and specified later, the corresponding closed-loop trajectory of (5) remains inside the set \mathcal{P}_x .

- **Property 2.** The set \mathcal{E}_0 is included in the domain of attraction of the closed-loop system (5), namely any closed-loop trajectory starting from \mathcal{E}_0 converges to the origin.

2.3. Preliminary Results

We present hereafter some useful lemmas for the design procedure.

Lemma 1. Given diagonal positive definite matrices $S_{ijk} \in \mathbb{R}^{n_u \times n_u}$, matrices $U_{ijk} \in \mathbb{R}^{n_u \times n_x}$ for $i, j, k \in \Omega_r$, two vectors $\xi, \eta \in \Xi$, and a vector $x \in \mathbb{R}^{n_x}$. If x satisfies

$$\left| \left(S_{\xi\eta\eta}^{-1} U_{\xi\eta\eta} \right)_{(l)} x \right| \leq \bar{u}_l, \quad \forall l \in \Omega_{n_u}, \quad (6)$$

then

$$\psi(u)^\top S_{\xi\eta\eta} \left[u - \psi(u) - S_{\xi\eta\eta}^{-1} U_{\xi\eta\eta} x \right] \geq 0, \quad (7)$$

for any $u \in \mathbb{R}^{n_u}$.

Proof. Note that the left-hand term of (7) is equal to

$$\sum_{l=1}^{n_u} S_{\xi\eta\eta(l,l)} \left[(u_l - \text{sat}(u_l)) (\text{sat}(u_l) - (S_{\xi\eta\eta}^{-1} U_{\xi\eta\eta})_{(l)} x) \right].$$

Since $S_{\xi\eta\eta(l,l)} > 0$, then it suffices to show that

$$(u_l - \text{sat}(u_l)) (\text{sat}(u_l) - (S_{\xi\eta\eta}^{-1} U_{\xi\eta\eta})_{(l)} x) \geq 0. \quad (8)$$

This is obvious if $|u_l| \leq \bar{u}_l$ since then $u_l - \text{sat}(u_l) = 0$. Inequality (8) holds also if $u_l > \bar{u}_l$, since then $u_l - \text{sat}(u_l) > 0$, and from (6)

$$\text{sat}(u_l) - \left(S_{\xi\eta\eta}^{-1} U_{\xi\eta\eta} \right)_{(l)} x = \bar{u}_l - \left(S_{\xi\eta\eta}^{-1} U_{\xi\eta\eta} \right)_{(l)} x \geq 0.$$

The last possible case $u_l < -\bar{u}_l$ can be treated similarly. \square

Remark 1. Lemma 1 presents a powerful tool to deal with the dead-zone non-linearity $\psi(u)$. It is motivated by the results on the generalized sector condition in [33]. However, since the feedback gain matrices H_{kji} , K_{kji} are not *explicitly* involved in (6) and (7), this new version is especially appropriate for the SOF control context. Indeed, it seems hard to obtain an LMI formulation by applying the sector condition in [33] to the considered SOF control problem. As will be seen, this aims for a direct computation of the control gains without using any change of variable. Hence, all feedback gains are parameter-dependent (i.e. $H_{\eta d\eta}$ and $K_{\eta d\eta}$) to improve the closed-loop performance and to reduce the design conservatism.

Lemma 2 (Finsler's Lemma [34]). For two matrices $Q = Q^\top \in \mathbb{R}^{n \times n}$ and $\mathcal{R} \in \mathbb{R}^{m \times n}$ such that $\text{rank}(\mathcal{R}) < n$, the following statements are equivalent:

- i) $x^\top Qx < 0, \forall x \in \{\xi \in \mathbb{R}^n : \xi \neq 0, \mathcal{R}\xi = 0_{m \times 1}\},$
- ii) $\exists \mathcal{F} \in \mathbb{R}^{n \times m}$ such that $Q + \mathcal{F}\mathcal{R} + \mathcal{R}^\top \mathcal{F}^\top < 0.$

Lemma 3. Let Υ_{kji} , for $i, j, k \in \Omega_r$, be symmetric matrices of appropriate dimensions. The condition $\sum_{k=1}^r \sum_{j=1}^r \sum_{i=1}^r \xi_k \eta_j \eta_i \Upsilon_{kji} < 0$ holds for $\xi, \eta \in \Xi$ if

$$\begin{cases} \Upsilon_{kii} < 0, & i, k \in \Omega_r \\ \frac{2}{r-1} \Upsilon_{kii} + \Upsilon_{kij} + \Upsilon_{kji} < 0, & i, j, k \in \Omega_r \text{ and } j > i \end{cases}$$

This result is directly extended from [35]. The relaxation techniques in [10, 36] are also applicable to obtain less conservative results at the price of an increasing computational complexity.

3. Static Output Control of Constrained T-S Fuzzy Systems

A new LMI-based method to design a SOF controller for the T-S fuzzy system (1) is presented. To obtain less conservative results, a delayed fuzzy Lyapunov function is exploited for theoretical developments.

Theorem 1. Given a T-S fuzzy system (1) with its domain of validity \mathcal{P}_x described by (2). If there exist a scalar α , positive definite matrices $P_i \in \mathbb{R}^{n_x \times n_x}$, positive diagonal matrices $S_{kij} \in \mathbb{R}^{n_u \times n_u}$, and matrices $H_{kj} \in \mathbb{R}^{n_u \times n_u}$, $U_{kij} \in \mathbb{R}^{n_u \times n_x}$, $K_{kj} \in \mathbb{R}^{n_u \times n_y}$, $G_{1kj} \in \mathbb{R}^{n_x \times n_x}$, $G_{2kj} \in \mathbb{R}^{n_x \times n_x}$, $G_{3kj} \in \mathbb{R}^{n_u \times n_x}$, $G_{4kj} \in \mathbb{R}^{n_u \times n_x}$, for $i, j, k \in \Omega_r$, such that

$$\begin{bmatrix} P_k & U_{kji(l)}^\top \\ U_{kji(l)} & \alpha \bar{u}_l^2 (2S_{kji(l)} - \alpha) \end{bmatrix} \geq 0, \quad i, j, k \in \Omega_r, l \in \Omega_{n_u} \quad (9)$$

$$\begin{bmatrix} P_i & M_{(m)}^\top \\ M_{(m)} & 1 \end{bmatrix} \geq 0, \quad i \in \Omega_r, \quad m \in \Omega_q \quad (10)$$

$$H_{kj} + H_{kj}^\top > 0, \quad k, j \in \Omega_r \quad (11)$$

$$\begin{cases} \Psi_{kii} < 0, & i, k \in \Omega_r \\ \frac{2}{r-1} \Psi_{kii} + \Psi_{kij} + \Psi_{kji} < 0, & i, j, k \in \Omega_r \text{ with } j > i \end{cases} \quad (12)$$

where the quantity Ψ_{kji} is defined as follows:

$$\Psi_{kji} = \text{He} \begin{bmatrix} \Psi_{kji(1,1)} & -G_{1kj} & \Psi_{kji(3,1)} & -G_{1kj}B_i \\ \Psi_{kji(1,2)} & P_j/2 - G_{2kj} & \Psi_{kji(3,2)} & -G_{2kj}B_i \\ \Psi_{kji(1,3)} & -G_{3kj} & \Psi_{kji(3,3)} & S_{kji} - G_{3kj}B_i \\ \Psi_{kji(1,4)} & -G_{4kj} & \Psi_{kji(3,4)} & -S_{kji} - G_{4kj}B_i \end{bmatrix} \quad (13)$$

with $\Psi_{kji(1,1)} = G_{1kj}A_i - P_k/2$, $\Psi_{kji(1,2)} = G_{2kj}A_i + \mathbb{J}_i K_{kj}C_i$, $\Psi_{kji(1,3)} = G_{3kj}A_i + \alpha K_{kj}C_i$, $\Psi_{kji(1,4)} = -U_{kji} + G_{4kj}A_i$, $\Psi_{kji(3,1)} = G_{1kj}B_i$, $\Psi_{kji(3,2)} = G_{2kj}B_i - \mathbb{J}_i H_{kj}$, $\Psi_{kji(3,3)} = G_{3kj}B_i - \alpha H_{kj}$, $\Psi_{kji(3,4)} = G_{4kj}B_i$ and

$$\mathbb{J}_i = \begin{cases} B_i, & B_i \text{ is column non-full rank} \\ B_i(B_i^\top B_i)^{-1}, & B_i \text{ is column full rank} \end{cases}$$

Then, the SOF controller (4) solves the control problem defined in Section 2.

Proof. Consider the following delayed fuzzy Lyapunov function:

$$\mathbb{V}(t, x) = x^\top \left(\sum_{i=1}^r \eta_i(t-1)P_i \right) x = x^\top P_{\eta_d} x, \quad (14)$$

and let \mathcal{E}_x be the set defined as

$$\mathcal{E}_x = \left\{ x \in \mathbb{R}^{n_x} : \sum_{i=1}^r \eta_i x^\top P_i x \leq 1 \text{ for some } \eta \in \Xi \right\}.$$

Step 1). We show that the SOF control law (4) is well defined. Indeed, for any ξ , $\eta \in \Xi$, it follows from (11) that $\sum_{k=1}^r \sum_{j=1}^r \xi_k \eta_j (H_{kj} + H_{kj}^\top) > 0$. This guarantees the nonsingularity of $H_{\eta_d \eta}$ at any time t .

Step 2). The set \mathcal{E}_x is contained in the set \mathcal{P}_x described by (2).

By convexity and applying Schur complement lemma [31] to (10), it follows that

$$P_\xi - M_{(m)}^\top M_{(m)} \geq 0, \quad \forall \xi \in \Xi, \forall m \in \Omega_q.$$

This guarantees the inclusion $\mathcal{E}_x \subseteq \mathcal{P}_x$.

Step 3). Given any $t > 0$ and any $x \in \mathbb{R}^{n_x}$. If $\mathbb{V}(t, x) \leq 1$, then

i) x is in \mathcal{P}_x , and

ii) the following inequality holds:

$$\left| (S_{\eta_d \eta}^{-1} U_{\eta_d \eta})_{(l)} x \right| \leq \bar{u}_l, \quad l \in \Omega_{n_u}. \quad (15)$$

For point i), as $x^\top P_{\eta_d} x \leq 1$, then $x \in \mathcal{E}_x$. Thus, $x \in \mathcal{P}_x$ according to *Step 2*. For point ii), since $(\beta - \alpha)^2 \geq 0$, it follows that $\beta^2 \geq \alpha(2\beta - \alpha)$ for any $\alpha, \beta \in \mathbb{R}$. Using this inequality with $\beta = S_{kji(l,l)}$, it can be deduced from (9) that

$$\begin{bmatrix} P_k & U_{kji(l)}^\top \\ U_{kji(l)} & \bar{u}_l^2 S_{kji(l,l)}^2 \end{bmatrix} \geq 0, \quad \forall i, j, k \in \Omega_r, \forall l \in \Omega_{n_u}. \quad (16)$$

By convexity and Schur complement lemma, (16) implies that

$$\bar{u}_l^2 P_{\eta_d} - 1/S_{\eta_d \eta \eta(l,l)}^2 U_{\eta_d \eta \eta(l)}^\top U_{\eta_d \eta \eta(l)} \geq 0, \quad \forall l \in \Omega_{n_u}. \quad (17)$$

Since $x \in \mathcal{E}_x$, it follows from (17) that

$$\bar{u}_l^2 \geq \bar{u}_l^2 x^\top P_{\eta_d} x \geq 1/S_{\eta_d \eta \eta(l,l)}^2 x^\top U_{\eta_d \eta \eta(l)}^\top U_{\eta_d \eta \eta(l)} x, \quad \forall l \in \Omega_{n_u},$$

from which (15) can be easily derived.

Step 4). We prove that, for any given $t > 0$ and $x \in \mathbb{R}^{n_x}$ such that $\mathbb{V}(t, x) \leq 1$ and $x \neq 0$, the variation $\Delta \mathbb{V}$ of the Lyapunov function (14) along the solution of (5) is negative, that is

$$\begin{aligned} \Delta \mathbb{V} &= \mathbb{V}(t+1, x(t+1)) - \mathbb{V}(t, x) \\ &= x(t+1)^\top P_\eta x(t+1) - x^\top P_{\eta_d} x < 0. \end{aligned}$$

To this end, we remark that the matrix $\Psi_{\eta_d \eta \eta}$ based on the matrices Ψ_{kji} defined in (13) can be rewritten in the form

$$\Psi_{\eta_d \eta \eta} = \mathcal{Q}_{\eta_d \eta \eta} + \mathcal{F}_{\eta_d \eta \eta} \mathcal{R}(\eta_d, \eta) + \mathcal{R}(\eta_d, \eta)^\top \mathcal{F}_{\eta_d \eta \eta}^\top,$$

where

$$\begin{aligned} \mathcal{Q}_{\eta_d \eta \eta} &= \begin{bmatrix} -P_{\eta_d} & 0 & 0 & -U_{\eta_d \eta \eta}^\top \\ 0 & P_\eta & 0 & 0 \\ 0 & 0 & 0 & S_{\eta_d \eta \eta} \\ -U_{\eta_d \eta \eta} & 0 & S_{\eta_d \eta \eta} & -2S_{\eta_d \eta \eta} \end{bmatrix}, \quad \mathcal{F}_{\eta_d \eta \eta} = \begin{bmatrix} G_{1\eta_d \eta} & 0 \\ G_{2\eta_d \eta} & \mathbb{J}_\eta H_{\eta_d \eta} \\ G_{3\eta_d \eta} & \alpha H_{\eta_d \eta} \\ G_{4\eta_d \eta} & 0 \end{bmatrix}, \\ \mathcal{R}(\eta_d, \eta) &= \begin{bmatrix} A_\eta & -I & B_\eta & -B_\eta \\ H_{\eta_d \eta}^{-1} K_{\eta_d \eta} C_\eta & 0 & -I & 0 \end{bmatrix}. \end{aligned}$$

From (12) and Lemma 3, it follows that

$$\Psi_{\eta_d \eta \eta} < 0. \quad (18)$$

Denoting $\xi(t) = [x^\top \quad x(t+1)^\top \quad u^\top \quad \psi(u)^\top]^\top$, it is easy to show that

$$\Delta\mathbb{V} + \text{He} \left(\psi(u)^\top S_{\eta_d\eta\eta} [u - \psi(u) - S_{\eta_d\eta\eta}^{-1} U_{\eta_d\eta\eta} x] \right) = \xi(t)^\top \mathcal{Q}_{\eta_d\eta\eta} \xi(t). \quad (19)$$

Moreover, $\xi(t)$ satisfies the equality $\mathcal{R}(\eta_d, \eta)\xi(t) = 0$, which is simply a reformulation of the closed-loop system equations (4) and (5). Applying Lemma 2, we can prove that (18) implies

$$\xi(t)^\top \mathcal{Q}_{\eta_d\eta\eta} \xi(t) < 0. \quad (20)$$

Finally, since x satisfies (15) as shown in Step 3, applying Lemma 1 leads to

$$\psi(u)^\top S_{\eta_d\eta\eta} [u - \psi(u) - S_{\eta_d\eta\eta}^{-1} U_{\eta_d\eta\eta} x] \geq 0. \quad (21)$$

The conclusion $\Delta\mathbb{V} < 0$ is then deduced from (19), (20) and (21).

Step 5). Conclusion. Let \mathcal{E}_0 be the set defined by

$$\mathcal{E}_0 = \{x \in \mathbb{R}^{n_x} : \mathbb{V}(0, x) \leq 1\}. \quad (22)$$

Note that $\mathcal{E}_0 \subset \mathcal{E}_x \subset \mathcal{P}_x$. It follows from previous steps that the solution $x(t)$ of (5) for any initial condition $x_0 \in \mathcal{E}_0$ is such that $\mathbb{V}(t, x(t)) \leq 1$, and so, $x(t) \in \mathcal{P}_x$ for any $t \geq 0$. Thus, Property 1 holds. Since it has been proved in Step 4 that $\mathbb{V}(t, x)$ is a Lyapunov function for the closed-loop system (5) on the set \mathcal{E}_x , then Property 2 holds also. \square

Remark 2. It has been shown that the set \mathcal{E}_0 mentioned in Properties 1 and 2 can be defined as in (22). Since it may be difficult to characterize \mathcal{E}_0 for the general case, we consider here the following set of initial conditions (instead of \mathcal{E}_0):

$$\mathcal{E}_{\mathbb{V}} = \{x \in \mathbb{R}^{n_x} : x^\top P_i x \leq 1, \quad \forall i \in \Omega_r\}. \quad (23)$$

Obviously, Properties 1 and 2 still hold in this case since $\mathcal{E}_{\mathbb{V}} \subset \mathcal{E}_0$.

Remark 3. The use of the delayed fuzzy Lyapunov function (14) can lead to less conservative results without increasing the number of LMIs compared to the classical one $V(t, x) = x^\top P_\eta x$ intensively studied in the literature [32]. This is due to the possibility of this new type of FLFs to introduce some slack variables, namely $K_{\eta_d\eta}$, $H_{\eta_d\eta}$, $G_{i\eta_d\eta}$, $i \in \Omega_4$, and especially $S_{\eta_d\eta\eta}$ and $U_{\eta_d\eta\eta}$ (for the SOF control context of input-saturated T-S fuzzy systems) instead of K_η , H_η , $G_{i\eta}$, $i \in \Omega_4$, S_η and U_η . Note also that the proposed method can be extended by using more complex FLF forms such as double-sum Lyapunov function [37], FLFs incorporating the MFs over several past samples [38]; or the control approaches based on κ -sample variation of FLFs [32] to obtain less conservative results at the expense of high computational cost.

Remark 4. By Finsler’s lemma, all complex couplings between Lyapunov matrices, feedback gain matrices and system matrices are avoided. This key point allows an LMI formulation for SOF control without requiring any *linear matrix equality* constraint on Lyapunov matrices [10, 16, 26] and/or rank conditions on system matrices as in most of existing works. Moreover, due to the elimination of these coupling terms, additional performance specifications can be naturally incorporated into the design conditions of Theorem 1, e.g. robustness with respect to norm-bounded uncertainties affecting all matrices (A_i, B_i, C_i) , decay rate, finite \mathcal{L}_2 –induced gain. Note that the control approaches based on the cone complementarity linearization algorithm [19] cannot have these interesting features [17].

Remark 5. Due to the products of α with other decision variables, the conditions in Theorem 1 are a set of LMIs with a line search over α . Since α is a scalar, the design conditions can be effectively solved with some constructive numerical procedures [17]. Here, a line search for α is done with 100 points linearly gridded over a logarithmic scale in $[10^{-4}, 10^4]$.

Remark 6. Since Theorem 1 is developed in local control context, it is thus desirable to achieve the largest domain of attraction. Note that the domain of attraction is here non-convex due to its associate FLF (14). The following optimization is proposed to achieve the control goal:

$$\min \quad \mu \tag{24}$$

subject to $\mu > 0$, $\text{trace}(P_i) \leq \mu$, $i \in \Omega_r$, LMIs (9)-(12) of Theorem 1.

The optimization (24) aims to minimize the largest trace (P_i) , $i \in \Omega_r$, and can be effectively solved with standard LMI numerical solvers.

4. Numerical Examples

This section presents three examples to demonstrate the effectiveness of the new control method. Note that existing works used for comparison purposes can be only applied to *unconstrained* T-S fuzzy systems. All LMI optimizations are conducted in Matlab R2011b with YALMIP toolbox and SeDuMi solver.

Example 1. We revisit the *motivating example* given in (3), which is rewritten as

$$\begin{aligned} x(t+1) &= A(x)x(t) + B \text{sat}(u(t)) \\ y(t) &= C(x)x(t) \end{aligned} \tag{25}$$

where $x^\top = [x_1 \quad x_2 \quad x_3]$, $u^\top = [u_1 \quad u_2]$ and

$$A(x) = \begin{bmatrix} 0 & -x_3 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C(x) = \begin{bmatrix} 0 & 2+x_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note that this nonlinear system is *open-loop unstable* and subject to the following constraints:

$$\mathcal{P}_x = \{x \in \mathbb{R}^3 : |x_3| \leq 1\}, \quad u_{\max} = [0.5 \ 0.5]. \quad (26)$$

Using the sector nonlinearity approach [3] with the premise variable x_3 , the nonlinear system (25) can be *exactly* represented by (1) for $x \in \mathcal{P}_x$ with

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \eta_1(x_3) &= (1 - x_3)/2, \quad \eta_2(x_3) = 1 - \eta_1(x_3). \end{aligned} \quad (27)$$

The condition $|x_3| \leq 1$ is crucial to guarantee that $\eta = [\eta_1 \ \eta_2] \in \Xi$. However, this state constraint is usually neglected in T-S fuzzy control framework. Note also that for this example, SOF design methods proposed in [20, 21, 24] are not applicable since $C_1 \neq C_2$ whereas all design conditions in [10, 18] are *infeasible*. Applying Theorem 1 to (27) leads to the following results:

$$P_1 = \begin{bmatrix} 10.854 & -0.032 & -1.048 \\ -0.032 & 10.469 & -0.031 \\ -1.048 & -0.031 & 12.479 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 10.903 & -0.004 & 0.619 \\ -0.004 & 10.452 & -0.069 \\ 0.619 & -0.069 & 12.443 \end{bmatrix},$$

and

$$\begin{aligned} H_{11} &= \begin{bmatrix} 4.73 & -0.72 \\ -0.94 & 6.23 \end{bmatrix}, \quad K_{11} = \begin{bmatrix} -4.32 & 7.45 \\ -6.33 & -2.65 \end{bmatrix}, \quad K_{12} = \begin{bmatrix} 3.12 & 11.2 \\ -2.66 & 2.51 \end{bmatrix}, \\ H_{21} &= \begin{bmatrix} 4.75 & -0.71 \\ -0.94 & 6.22 \end{bmatrix}, \quad H_{12} = \begin{bmatrix} 7.2 & 1.09 \\ 0.66 & 6.6 \end{bmatrix}, \quad K_{21} = \begin{bmatrix} -4.28 & 7.54 \\ -6.34 & -2.69 \end{bmatrix}, \\ H_{22} &= \begin{bmatrix} 7.17 & 1.1 \\ 0.63 & 6.61 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} 3.06 & 11.42 \\ -2.66 & 2.32 \end{bmatrix}, \end{aligned}$$

with $\alpha = 0.298$. Figure 1 shows the closed-loop trajectory corresponding to the initial condition $x_0^\top = [1 \ -1 \ 1]$. Note that the controller given in [15] induces *unstable* behaviors under the same simulation condition. Remark that the system constraints (26) have not been *explicitly* taken into account in [15]. Hence, the resulting design conditions do not provide any guarantee on the domain of

attraction of (27) in the presence of these constraints. To highlight this, by numerical simulations, we have computed the closed-loop solutions for initial conditions taken on a $20 \times 20 \times 20$ hyper-rectangle grid in the initial condition set $[-5, 5] \times [-5, 5] \times [-1, 1]$. Note that this latter set is included inside the validity domain \mathcal{P}_x defined in (26). It can be verified that, for the proposed method, among the total of 8000 tested trajectories, there are 7267 closed-loop trajectories behave correctly. This is, these trajectories do not leave the validity domain \mathcal{P}_x and converge towards the origin. For the method proposed in [15], only 2587 solutions induce a correct behavior. This fact strongly confirms the interest of considering the system constraints into the control design procedure.

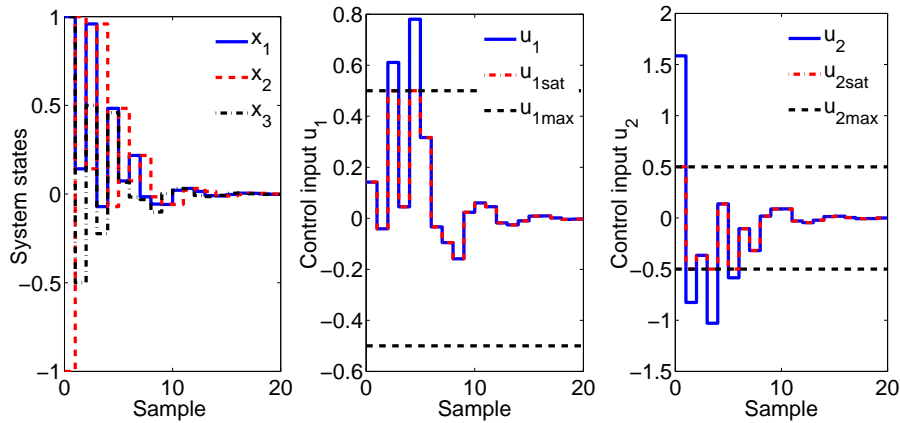


Figure 1: Closed-loop behaviors obtained with the proposed controller for $x_0^\top = [1 \ -1 \ 1]$ in Example 1.

Example 2. Consider a family of *open-loop unstable* T-S fuzzy systems (1) with the following data:

$$A_1 = \begin{bmatrix} 1 & -\delta \\ -1 & -0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 5 + \delta \\ 2\delta \end{bmatrix}, A_2 = \begin{bmatrix} 1 & \delta \\ -1 & -0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 5 - \delta \\ -2\delta \end{bmatrix} \quad (28)$$

$$C_1 = C_2 = [1 \ 0], \eta_1(x_1) = (x_1 + \delta)/(2\delta), \eta_2(x_1) = 1 - \eta_1(x_1)$$

where $\delta > 0$. Note that the validity of both MFs $\eta_1(\cdot)$ and $\eta_2(\cdot)$ in (28) is only guaranteed if $|x_1| \leq \delta$. For simulation purposes, the following state and control input limitations are considered:

$$\mathcal{P}_x = \{x \in \mathbb{R}^2 : |x_1| \leq \delta, \quad |x_2| \leq 1\}, \quad \bar{u} = 2. \quad (29)$$

This example is used as a benchmark to study the design conservatism. Note that Theorem 1 and the methods in [10, 15, 18, 26] can be applied to (28). Table 1

provides the maximal value of δ , denoted by δ^* , for which a stabilizing SOF controller can be computed with each method. Observe that the new method provides less conservative results even if none of the existing works has dealt with the system constraints (29).

Table 1: Computed values of δ^* for several methods.

Conditions	Value of δ^*
Proposed method	0.79
Method in [18]: Conditions (18a) and (38a)	0.74
Method in [18]: Condition (18b)	0.78
Method in [18]: Condition (38b)	0.71
Methods in [10, 15, 26]	Infeasible

Solving the optimization problem (24) for system (28) with $\delta = 0.7$ leads to

$$P_1 = \begin{bmatrix} 2.041 & 0.011 \\ 0.011 & 1.018 \end{bmatrix}, P_2 = \begin{bmatrix} 2.047 & 0.077 \\ 0.077 & 1.012 \end{bmatrix}, \mu = 3.059, \alpha = 0.142. \quad (30)$$

Figure 2 shows the guaranteed region of attraction \mathcal{E}_V of the closed-loop system which is maximized inside \mathcal{P}_x . By means of simulations, it can be checked that the corresponding SOF controller provides stable closed-loop behaviors for $\forall x_0 \in \mathcal{P}_x$.

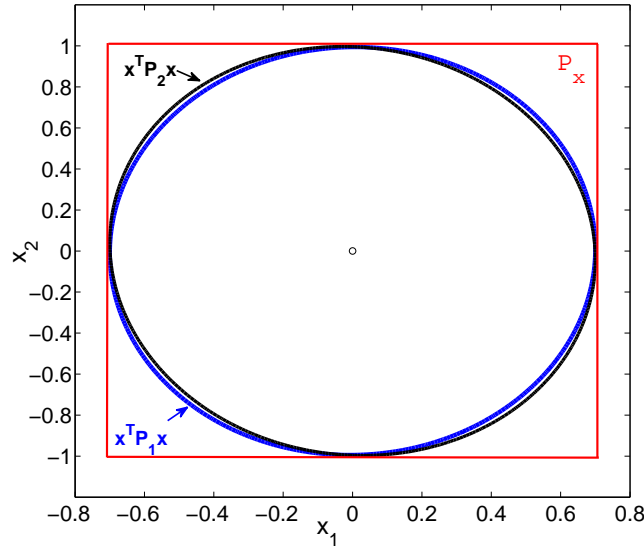


Figure 2: Guaranteed region of attraction \mathcal{E}_V and domain of validity \mathcal{P}_x in Example 2.

Remark 7. Note that the size of \mathcal{E}_\forall defined in (23) is maximized if the intersection size of all the r ellipsoids $\mathcal{E}(P_i) = \{x : x^\top P_i x \leq 1\}$ is maximized. Hence, when solving (24), the Lyapunov matrices $P_i, i \in \Omega_r$, tend to be close, i.e. we approach the quadratic Lyapunov function, see (30) and Figure 2. However, the fuzzy Lyapunov function (14) offers more flexibility to reduce the design conservatism since the quadratic one is only a special case of (14) by imposing $P_i = P, \forall i \in \Omega_r$.

Example 3. Consider the following T-S fuzzy system (1) borrowed from [18]:

$$\begin{aligned} A_1 &= \begin{bmatrix} 1.3 + \delta & -1 \\ 0.2 & 0.6 \end{bmatrix}, A_2 = \begin{bmatrix} 0.8 & -1.5 \\ 0.1 & 0.3 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}, \\ B_2 &= [1 \ 0]^\top, C_1 = [1 \ 1.5], C_2 = [1 \ 1]. \end{aligned} \quad (31)$$

This example aims to study not only the design conservatism but also the *applicability* of some notable methods dealing with SOF control. Since $B_1 \neq B_2$ and $C_1 \neq C_2$, the methods in [20–24] cannot be applied to (31). However, Theorem 1 as well as the methods in [10, 15, 16, 18, 26] can be used for this example to design SOF controllers. The stabilization interval of δ obtained with each control method is given in Table 2.

Note that differently from all above methods, the following constraints are *explicitly* considered for (31) when applying Theorem 1:

$$\mathcal{P}_x = \{x \in \mathbb{R}^3 : |x_i| \leq 5, \forall i \in \Omega_3\} \text{ and } \bar{u} = 3. \quad (32)$$

The results in Table 2 reconfirm that despite the consideration of (32), the proposed method provides less conservative results than existing ones. Note also that if the state/input limitations are increased, then the stabilization interval of δ given by the new method can be still enlarged.

Conditions	Stabilization intervals of δ
Theorem 1	$[-3.18, 0.92]$
Condition (18a) in [18]	$[-2.08, 0.55]$
Condition (18b) in [18]	$[-2.15, 0.87]$
Condition (38a) in [18]	$[-2.08, 0.52]$
Condition (38b) in [18]	$[-2.04, 0.48]$
Methods in [10, 15, 16, 26]	Infeasible
Methods in [20–24]	Non-applicable

5. Concluding Remarks

This paper presents a new SOF control method for T-S fuzzy systems subject to both state and control input constraints. New design conditions are expressed in terms of LMIs with a line search over a scalar variable. In contrast to most of existing results, these conditions do not require any restriction on the system state-space matrices. In addition, matrix equality constraints between system matrices and Lyapunov matrices are also eliminated via Finsler's lemma. It has been shown that the proposed method can lead to less conservative results compared to existing results. For the control design, the premise vector is assumed to be measured. Further research to deal with unmeasured premise variables is under study.

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