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# An augmented system approach for LMI-based control design of constrained Takagi-Sugeno fuzzy systems

Anh-Tu Nguyen<sup>1,\*</sup>, Raymundo Márquez<sup>2</sup>, Antoine Dequidt<sup>3</sup>

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## Abstract

In this paper, we address the control design of a class of constrained Takagi-Sugeno fuzzy systems. These systems are subject to both control input and state constraints. Sufficient design conditions are derived via a judicious use of an equivalent augmentation form of the closed-loop system together with a new parameter-dependent Lyapunov function. The control input nonlinearity is dealt with via a generalized sector condition. These instrumental control tools allow reducing significantly the conservatism of the results while keeping a simple control structure with a low level of numerical complexity. In particular, it has been shown that in many cases the new method can provide better results (in terms of conservatism reduction) than some recent works developed in non-quadratic Lyapunov framework. Several numerical examples are given to illustrate the effectiveness of the proposed method.

*Keywords:* Takagi-Sugeno fuzzy systems, constrained control, system augmentation approach,  $\mathcal{L}_2$ -gain performance, linear matrix inequality (LMI).

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## 1. Introduction

In recent years, control design based on Takagi-Sugeno (T-S) fuzzy models [1] has become one of the most popular and promising research topics in nonlinear model-based control, see [2] for quick overviews. This is due to the fact that T-S fuzzy models can approximate any smooth nonlinear systems with any degree of accuracy [2]. In addition, the nonlinear sector decomposition approach provides a systematic way to derive an *exact* T-S representation of a nonlinear model in a compact set. Moreover, the control technique based on the Parallel Distributed Compensation (PDC) control law provides a systematic and effective design framework to deal with nonlinear systems. Up to now, numerous successful applications of the T-S fuzzy model-based control technique have been reported in the literature [2–4].

For real-world applications, the control input signals and the closed-loop trajectories are usually bounded for physical and/or safety constraints [5]. Notice that state constraints appear *naturally* when the nonlinear sector-decomposition approach is used to derive a T-S representation of a nonlinear system [2, 4]. Such type of system constraints must be *explicitly* taken into account in the control design procedure to obtain a sufficiently large domain of attraction [6]. Control input saturation is ubiquitous in practice, especially for high performance applications; neglecting it may lead to a severe degradation of control performance,

or even unstable behaviors of closed-loop systems [5]. The control problem of T-S fuzzy models with saturated inputs has recently received increasing attention [4, 7–9]. The authors in [9] have proposed an iterative control technique for constrained T-S fuzzy systems which is based on multi-parametric quadratic programming. In [3], a norm-bounded approach is used to handle the input constraints which results in non-saturating controllers with poor performance [5]. Descriptor representation approach [10] has been recently employed to deal with input-saturated T-S systems together with a polytopic representation of the saturation function [11]. Note that the results in [11] are only applicable to a restrictive class of T-S fuzzy models with all linear subsystems that must be *open-loop stable*. Moreover, that method cannot guarantee the closed-loop asymptotic stability even if exogenous disturbances are not involved in the system dynamics. Another polytopic representation of the saturation function proposed in [12] has been employed in [13] for time-delay T-S systems, in [7, 8] for discrete-time T-S systems, and in [4] for a class of switching T-S systems. To reduce the design conservatism, homogeneous polynomial parameter-dependent Lyapunov functions have been exploited in [14] for control design of input-saturated linear parameter varying (LPV) systems. Unfortunately, the method in [14] suffers a *serious* technical problem since the *unknown* time-derivative of the parameter-dependent Lyapunov matrix involved in the development has been completely ignored. Recently, the synthesis of antiwindup (AW) compensators for constrained T-S systems have been also considered. In [15], a two-step AW approach has been employed to deal with time-delay constrained T-S systems. For that, dynamic output feedback controllers must be given *a priori* and the control goal is *only* to design AW compensators to minimize any adverse effect caused by the input saturation. Despite its simple design procedure, two-step

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AW approaches may lead to excessively small operating regions due to the separate design of nominal controllers and AW compensators. To overcome that, a new one-step AW approach for constrained T-S systems has been proposed in [6] where dynamic controllers and AW compensators can be *simultaneously* designed. Notice that state constraints have not been considered in most of existing results in the T-S control framework (except for [4, 6]).

Motivated by the above control issues, this paper presents a new LMI-based control method for constrained T-S fuzzy systems subject to  $\mathcal{L}_2$ -disturbances. The key features of the proposed method can be summarized as follows.

- (i) An augmented system approach is used with a new parameter-dependent Lyapunov function for control design. By this way, slack decision variables can be introduced to reduce significantly the design conservatism.
- (ii) The new method can lead to less conservative results than the most recent fuzzy Lyapunov based approach dealing with the same control context in [16]. Notice that this latter has already outperformed many remarkable results in non-quadratic Lyapunov control framework, e.g. [17, 18]. In addition, our control structure is much simpler and does not require online inversion of parameter-dependent matrices for implementation. Moreover, the proposed design conditions are of lower numerical complexity compared to those in [16]. These outstanding features are of crucial importance for real-time applications.
- (iii) Differently from [11], the proposed method is able to deal with *open-loop unstable* T-S systems. Therefore, it can be applied to a larger class of nonlinear constrained systems. Moreover, it can guarantee the asymptotic stability of the closed-loop system in the absence of disturbances.

The paper is organized as follows. Section 2 describes the control problem and presents useful results for control design. Section 3 is devoted to the theoretical derivation of the new design conditions. Examples illustrating the interests of the new method are given in Section 4. Finally, Section 5 provides some concluding remarks.

*Notations.*  $\Omega_r$  denotes the set  $\{1, 2, \dots, r\}$ .  $I$  denotes the identity matrix of appropriate dimension. For any square matrix  $X$ ,  $\text{He}(X) = X + X^\top$ , and  $X > 0$  means that  $X$  is positive definite.  $X_{(i)}$  denotes the  $i$ th row of matrix  $X$ .  $(*)$  stands for matrix blocks that can be deduced by symmetry. For a matrix  $P > 0$  and a positive scalar  $\rho > 0$ , we denote  $\mathcal{E}(P, \rho) = \{x : x^\top P x \leq \rho\}$  and  $\mathcal{E}(P) \equiv \mathcal{E}(P, 1)$ . The scalar functions  $\eta_i$ ,  $i \in \Omega_r$ , verify the convex sum property if  $\eta_i \geq 0$  and  $\sum_{i=1}^r \eta_i = 1$ . For such functions with any argument  $\theta$ , and matrices  $Y_i$  and  $Z_{ij}$  of appropriate dimensions, we denote  $Y_\theta = \sum_{i=1}^r \eta_i(\theta) Y_i$ ,  $Z_{\theta\theta} = \sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) Z_{ij}$ ,  $Y_\theta^{-1} = \left( \sum_{i=1}^r \eta_i(\theta) Y_i \right)^{-1}$ . The explicit time-dependence of the variables is omitted for convenience.

## 2. Problem Statement and Preliminaries

### 2.1. Control problem statement

Consider the following T-S fuzzy system [2]:

$$\begin{aligned} \dot{x} &= \sum_{i=1}^r \eta_i(\theta) (A_i x + B_i^u \text{sat}(u) + B_i^w w) \\ z &= \sum_{i=1}^r \eta_i(\theta) (C_i x + D_i^w w) \end{aligned} \quad (1)$$

where  $x \in \mathbb{R}^{n_x}$  is the state,  $u \in \mathbb{R}^{n_u}$  is the control input,  $w \in \mathbb{R}^{n_w}$  is the disturbance,  $z \in \mathbb{R}^{n_z}$  is the performance output, and  $\theta \in \mathbb{R}^{n_p}$  is the vector of measured premise variables. The constant matrices  $A_i, B_i^u, B_i^w, C_i, D_i^w$ ,  $i \in \Omega_r$  are of adequate dimensions, and the scalar membership functions (MFs)  $\eta_i(\theta)$  satisfy the convex sum property. The saturation function  $\text{sat}(u) : \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_u}$  in (1) is defined as

$$\text{sat}(u_{(l)}) = \text{sign}(u_{(l)}) \min(|u_{(l)}|, u_{\max(l)}),$$

where the bounds  $u_{\max(l)} > 0$ ,  $l \in \Omega_{n_u}$ , are given. The disturbance  $w$  in (1) belongs to the set of functions

$$\mathcal{W}_\delta = \left\{ w : \mathbb{R}^+ \rightarrow \mathbb{R}^{n_w}, \int_0^\infty w(\tau)^\top w(\tau) d\tau \leq \delta \right\},$$

where the energy bound  $\delta > 0$  is given. In this paper, we consider the following PDC control law:

$$u = \sum_{i=1}^r \eta_i(\theta) K_i x = K_\theta x. \quad (2)$$

Let  $\tilde{x}^\top = [x^\top \quad z^\top \quad u^\top]$  and  $\psi(u) = u - \text{sat}(u)$ . From (1) and (2), the closed-loop system can be strictly rewritten in the following augmented descriptor form:

$$\mathbb{E} \dot{\tilde{x}} = \mathbb{A}_\theta \tilde{x} + \mathbb{B}_\theta^w w - \mathbb{B}_\theta^\psi \psi(u), \quad (3)$$

where

$$\begin{aligned} \mathbb{E} &= \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbb{A}_i = \begin{bmatrix} A_i & 0 & B_i^u \\ C_i & -I & 0 \\ K_i & 0 & -I \end{bmatrix}, \\ \mathbb{B}_i^w &= [B_i^{w\top} \quad D_i^{w\top} \quad 0]^\top, \quad \mathbb{B}_i^\psi = [B_i^{u\top} \quad 0 \quad 0]^\top. \end{aligned}$$

**Remark 1.** There are different ways to choose the augmented state  $\tilde{x}$  of (3) for control design, for instance  $\tilde{x}^\top = [x^\top \quad u^\top]$  could be another possible option. Each choice leads to a corresponding set of design conditions which would be different to those of others. Here, we define  $\tilde{x}^\top = [x^\top \quad z^\top \quad u^\top]$  to *explicitly* consider  $z$  in (3). This conveniently allows the derivation of LMI conditions as shown later. However, the new method can be generalized to other options of  $\tilde{x}$ .

This paper presents a new LMI-based method to design a PDC controller (2) for T-S fuzzy system (1) that satisfies the three properties.

(P1) [State constraints]. Given vectors  $h_k \in \mathbb{R}^{n_x}$ ,  $k \in \Omega_q$ , there exist a matrix  $P > 0$  and a positive scalar  $\rho > 0$  such that, for any  $x(0) \in \mathcal{E}(P, \rho)$  and  $w \in \mathcal{W}_\delta$ , the corresponding solution of (1) remains inside the following polyhedral set:

$$\mathcal{P}_x = \{x \in \mathbb{R}^{n_x} : h_k^\top x \leq 1, \quad k \in \Omega_q\}. \quad (4)$$

(P2) [Local stability]. Given a scalar  $\alpha > 0$ , any closed-loop trajectory of the undisturbed system (i.e.  $w = 0$ ) starting from  $\mathcal{E}(P)$  converges exponentially to the origin with a decay rate less than  $\alpha/2$ .

(P3) [ $\mathcal{L}_2$ -gain performance]. There exists a scalar  $\gamma > 0$  such that the system (1) has a restricted  $\mathcal{L}_2$ -gain less than  $\sqrt{\gamma}$ . That is, for  $x(0) = 0$  and any  $w \in \mathcal{W}_\delta$  compatible with  $x(0) = 0$ , one has  $\|z(t)\|_2 < \sqrt{\gamma}\|w(t)\|_2$ .

Notice that the above closed-loop specifications have been also considered in [6] for the control design of an anti-windup based dynamic output feedback controller of T-S systems subject to persistent disturbances.

## 2.2. Preliminaries

The following preliminary results are useful for the theoretical development in next section.

**Lemma 1.** Given matrices  $K_i, G_i \in \mathbb{R}^{n_u \times n_x}$ ,  $i \in \Omega_r$ , we define the following set:

$$\mathcal{P}_u = \bigcap_{i=1}^r \Phi(K_i - G_i),$$

where

$\Phi(K_i - G_i) = \{x \in \mathbb{R}^{n_x} : |(K_{i(l)} - G_{i(l)})x| \leq u_{\max(l)}\}$ , for  $l \in \Omega_{n_u}$ . If  $x \in \mathcal{P}_u$ , then it follows that

$$\psi(u)^\top \left( \sum_{i=1}^r \eta_i S_i \right)^{-1} \left[ \psi(u) - \sum_{i=1}^r \eta_i G_i x \right] \leq 0,$$

for any positive diagonal matrices  $S_i$ , and any family of functions  $\eta_i$ ,  $i \in \Omega_r$ , satisfying the convex sum property.

Lemma 1 presents an extension of the generalized sector condition to deal with the control input nonlinearity [19]. This extension allows all involved matrices being parameter-dependent which contributes to reduce the conservatism of the proposed method.

**Lemma 2.** [20] Let  $\Upsilon_{ij}$ ,  $i, j \in \Omega_r$  be symmetric matrices of appropriate dimensions and  $\eta_1, \dots, \eta_r$  be a family of scalar functions satisfying the convex sum property. The condition  $\sum_{i=1}^r \sum_{j=1}^r \eta_i \eta_j \Upsilon_{ij} < 0$  holds if

$$\begin{aligned} \Upsilon_{ii} &< 0, \quad i \in \Omega_r, \\ \frac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} &< 0, \quad i, j \in \Omega_r, \quad i \neq j. \end{aligned}$$

Lemma 2 is used to convert a parameter-dependent LMI condition to a finite set of LMIs. Notice that more efficient relaxation results can be found in [21] at the expense of high complexity.

## 3. Main Results

The following theorem provides LMI conditions for designing a PDC controller (2) that solves the control problem considered in Section 2.

**Theorem 1.** Given positive scalars  $\delta$  and  $\alpha$ , assume there exist positive definite matrix  $X \in \mathbb{R}^{n_x \times n_x}$ , positive diagonal matrices  $S_i \in \mathbb{R}^{n_u \times n_u}$ , matrices  $X_{21}^{ij} \in \mathbb{R}^{n_z \times n_x}$ ,  $X_{22}^{ij} \in \mathbb{R}^{n_z \times n_z}$ ,  $X_{23}^{ij} \in \mathbb{R}^{n_z \times n_u}$ ,  $X_{31}^i \in \mathbb{R}^{n_u \times n_x}$ ,  $X_{32}^i \in \mathbb{R}^{n_u \times n_z}$ ,  $X_{33}^i \in \mathbb{R}^{n_u \times n_u}$ ,  $V_i \in \mathbb{R}^{n_u \times n_x}$ ,  $W_i \in \mathbb{R}^{n_u \times n_x}$ ,  $i, j \in \Omega_r$ , and positive scalars  $\rho, \gamma$  such that

$$\begin{bmatrix} X & * \\ V_{i(l)} - W_{i(l)} & u_{\max(l)}^2 \end{bmatrix} \geq 0, \quad i \in \Omega_r, \quad l \in \Omega_{n_u} \quad (5)$$

$$\begin{bmatrix} X & Xh_k \\ * & 1 \end{bmatrix} \geq 0, \quad k \in \Omega_q \quad (6)$$

$$\rho + \gamma\delta \leq 1 \quad (7)$$

$$\Psi_{ii} < 0, \quad i \in \Omega_r \quad (8)$$

$$\frac{2}{r-1} \Psi_{ii} + \Psi_{ij} + \Psi_{ji} < 0, \quad i, j \in \Omega_r, \quad i \neq j \quad (9)$$

where the quantity  $\Psi_{ij}$  is defined in (10).

$$\Psi_{ij} = \begin{bmatrix} \Psi_{ij}^{11} & \Psi_{ij}^{12} & \Psi_{ij}^{13} & \Psi_{ij}^{14} & \Psi_{ij}^{15} & \Psi_{ij}^{16} \\ * & \Psi_{ij}^{22} & \Psi_{ij}^{23} & 0 & 0 & D_i^w \\ * & * & \Psi_{ij}^{33} & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -2S_j & 0 \\ * & * & * & * & * & \Psi_{ij}^{66} \end{bmatrix}, \quad (10)$$

with  $\Psi_{ij}^{11} = \text{He}(A_i X + B_i^u X_{31}^j) + \alpha X$ ,  $\Psi_{ij}^{12} = B_i^u X_{32}^j + X C_i^\top - X_{21}^{ij\top}$ ,  $\Psi_{ij}^{13} = B_i^u X_{33}^j + V_i^\top - X_{31}^{j\top}$ ,  $\Psi_{ij}^{14} = X C_i^\top$ ,  $\Psi_{ij}^{15} = W_i^\top - B_i^u S_j$ ,  $\Psi_{ij}^{16} = B_i^u + X C_i^\top D_j^w$ ,  $\Psi_{ij}^{22} = -\text{He}(X_{22}^{ij})$ ,  $\Psi_{ij}^{23} = -X_{23}^{ij} - X_{32}^{j\top}$ ,  $\Psi_{ij}^{33} = -\text{He}(X_{33}^j)$  and  $\Psi_{ij}^{66} = -\gamma I + D_i^w{}^\top D_j^w$ . Then, the PDC controller (2) with the feedback gains given by

$$K_i = V_i X^{-1}, \quad i \in \Omega_r, \quad (11)$$

solves the design problem stated in Section 2 for the closed-loop T-S fuzzy system (1).

*Proof.* Let us introduce the parameter-dependent matrix

$$\mathbb{X}_{\theta\theta} = \sum_{i=1}^r \sum_{j=1}^r \eta_i(\theta) \eta_j(\theta) \begin{bmatrix} X & 0 & 0 \\ X_{21}^{ij} & X_{22}^{ij} & X_{23}^{ij} \\ X_{31}^i & X_{32}^i & X_{33}^i \end{bmatrix}. \quad (12)$$

Since  $X > 0$ , it follows that

$$\mathbb{X}_{\theta\theta}^\top \mathbb{E}^\top = \mathbb{E} \mathbb{X}_{\theta\theta} = \begin{bmatrix} X & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \geq 0. \quad (13)$$

Notice that if (8) and (9) hold, then by Lemma 2, it follows that  $\Psi_{\theta\theta} < 0$  with  $\Psi_{ij}$  given in (10). This implies that

$\begin{bmatrix} X_{22}^{\theta\theta} & X_{23}^{\theta\theta} \\ X_{32}^{\theta\theta} & X_{33}^{\theta\theta} \end{bmatrix}$  is nonsingular, and so is  $\mathbb{X}_{\theta\theta}$  since  $X > 0$ . Let  $\mathbb{P}_{(\theta)} = \mathbb{X}_{\theta\theta}^{-1}$ . Pre- and post-multiplying (13) with  $\mathbb{P}_{(\theta)}^\top$  and its transpose, it follows that

$$\mathbb{E}^\top \mathbb{P}_{(\theta)} = \mathbb{P}_{(\theta)}^\top \mathbb{E} \geq 0. \quad (14)$$

Consider a candidate Lyapunov function

$$\mathbb{V}(\tilde{x}) = \tilde{x}^\top \mathbb{E}^\top \mathbb{P}_{(\theta)} \tilde{x}. \quad (15)$$

Denote  $P = X^{-1}$ . From the definitions of  $\tilde{x}$  and  $\mathbb{E}$  in (3), and the relation (14), it follows that

$$\mathbb{V}(\tilde{x}) \equiv \mathbb{V}(x) = x^\top P x. \quad (16)$$

**Remark 2.** The time-derivative of the Lyapunov function  $\mathbb{V}(\tilde{x})$  in (15) along the solution of (3) is given by

$$\dot{\mathbb{V}}(\tilde{x}) = \dot{\tilde{x}}^\top \mathbb{E}^\top \mathbb{P}_{(\theta)} \tilde{x} + \tilde{x}^\top \mathbb{E}^\top \mathbb{P}_{(\theta)} \dot{\tilde{x}} + \tilde{x}^\top \mathbb{E}^\top \dot{\mathbb{P}}_{(\theta)} \tilde{x}.$$

Due to the definition of  $\mathbb{E}$  and  $\mathbb{X}_{\theta\theta}$  in (12), it follows that  $\tilde{x}^\top \mathbb{E}^\top \dot{\mathbb{P}}_{(\theta)} \tilde{x} = 0$ . This allows to avoid the *unknown* time-derivatives of the MFs, see also Remark 5.

Let  $G_i = W_i X^{-1}$ . Using Schur complement lemma [22], we can prove that the condition  $\Psi_{\theta\theta} < 0$  is equivalent to

$$\begin{bmatrix} \Gamma_{11}(\theta) & \Gamma_{12}(\theta) & \Gamma_{13}(\theta) & \Gamma_{14}(\theta) & \Gamma_{15}(\theta) \\ * & \Gamma_{22}(\theta) & \Gamma_{23}(\theta) & 0 & D_\theta^w \\ * & * & \Gamma_{33}(\theta) & 0 & 0 \\ * & * & * & -2S_\theta & 0 \\ * & * & * & * & \Gamma_{55}(\theta) \end{bmatrix} < 0, \quad (17)$$

where  $\Gamma_{11}(\theta) = \Psi_{\theta\theta(1,1)} + X C_\theta^\top C_\theta X$ ,  $\Gamma_{12}(\theta) = \Psi_{\theta\theta(1,2)}$ ,  $\Gamma_{13}(\theta) = \Psi_{\theta\theta(1,3)}$ ,  $\Gamma_{14}(\theta) = W_\theta^\top - B_\theta^u S_\theta$ ,  $\Gamma_{15}(\theta) = B_\theta^w + X C_\theta^\top D_\theta^w$ ,  $\Gamma_{22}(\theta) = -\text{He}(X_{22}^{\theta\theta})$ ,  $\Gamma_{23}(\theta) = -X_{23}^{\theta\theta} - X_{32}^{\theta\top}$ ,  $\Gamma_{33}(\theta) = -\text{He}(X_{33}^{\theta\theta})$  and  $\Gamma_{55}(\theta) = -\gamma I + D_\theta^{w\top} D_\theta^w$ . Note that (17) can be rewritten in the form

$$\begin{bmatrix} \Upsilon(\theta) & -\mathbb{B}_\theta^\psi S_\theta + \mathbb{X}_{\theta\theta}^\top \mathbb{G}_\theta^* & \mathbb{B}_\theta^w + \mathbb{X}_{\theta\theta}^\top \mathbb{D}_{\theta\theta}^w \\ * & -2S_\theta & 0 \\ * & * & -\gamma I + D_\theta^{w\top} D_\theta^w \end{bmatrix} < 0 \quad (18)$$

where

$$\Upsilon(\theta) = \text{He}(\mathbb{A}_\theta \mathbb{X}_{\theta\theta}) + \mathbb{X}_{\theta\theta}^\top \mathbb{C}_{\theta\theta}^* \mathbb{X}_{\theta\theta}, \quad \mathbb{G}_\theta^* = \begin{bmatrix} G_\theta & 0 & 0 \end{bmatrix},$$

$$\mathbb{C}_{\theta\theta}^* = \begin{bmatrix} \alpha P + C_\theta^\top C_\theta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbb{D}_{\theta\theta}^w = \begin{bmatrix} C_\theta^\top D_\theta^w \\ 0 \\ 0 \end{bmatrix}.$$

Pre- and post- multiplying (18) with  $\text{diag}(\mathbb{X}_{\theta\theta}^{-\top}, S_\theta^{-1}, I)$  and its transpose yields

$$\begin{bmatrix} \Phi(\theta) & -\mathbb{P}_{(\theta)}^\top \mathbb{B}_\theta^\psi + \mathbb{G}_\theta^{*\top} S_\theta^{-1} & \mathbb{P}_{(\theta)}^\top \mathbb{B}_\theta^w + \mathbb{D}_{\theta\theta}^w \\ * & -2S_\theta^{-1} & 0 \\ * & * & -\gamma I + D_\theta^{w\top} D_\theta^w \end{bmatrix} < 0 \quad (19)$$

with  $\Phi(\theta) = \mathbb{P}_{(\theta)}^\top \mathbb{A}_\theta + \mathbb{A}_\theta^\top \mathbb{P}_{(\theta)} + \mathbb{C}_{\theta\theta}^*$ . Pre- and post-multiplying (19) by  $\begin{bmatrix} \tilde{x}^\top & \psi(u)^\top & w^\top \end{bmatrix}$  and its transpose, after some simple manipulations we obtain:

$$\begin{aligned} \dot{\mathbb{V}}(x) + \alpha x^\top P x + z^\top z - \gamma w^\top w \\ - 2\psi(u)^\top S_\theta^{-1} [\psi(u) - G_\theta x] < 0. \end{aligned} \quad (20)$$

By Lemma 1, (20) implies clearly that

$$\dot{\mathbb{V}}(x) + \alpha x^\top P x + z^\top z - \gamma w^\top w < 0, \quad x \in \mathcal{P}_u. \quad (21)$$

Following similar line as in [6], we can prove that (5) (respectively (6)) implies the inclusion  $\mathcal{E}(P) \subseteq \mathcal{P}_u$  (respectively  $\mathcal{E}(P) \subseteq \mathcal{P}_x$ ). Now, if  $w = 0$ , then it follows from (21) that  $\dot{\mathbb{V}}(x) < -\alpha x^\top P x$ ,  $\forall x \in \mathcal{E}(P)$ , which guarantees (P2). For  $w \neq 0$  and  $w \in \mathcal{W}_\delta$ , integrating both sides of (21) from 0 to  $T_f > 0$  yields

$$\mathbb{V}(x(T_f)) < \mathbb{V}(x(0)) + \gamma \int_0^{T_f} w(\tau)^\top w(\tau) d\tau \quad (22)$$

From (7) and (22), it follows that  $\mathbb{V}(x(T_f)) < \rho + \gamma \delta \leq 1$ ,  $\forall x(0) \in \mathcal{E}(P, \rho)$ . This means that the corresponding solution of (1) is confined in  $\mathcal{E}(P) \subseteq \mathcal{P}_x$ , which proves (P1). Moreover, from (22) and considering  $T_f \rightarrow \infty$ , we obtain  $\int_0^\infty z(\tau)^\top z(\tau) dt < \gamma \int_0^\infty w(\tau)^\top w(\tau) d\tau$ ,  $x(0) = 0$ . This, in turn, proves (P3). Therefore, the proof of Theorem 1 can be now concluded.  $\square$

The following remarks can be stated for Theorem 1.

**Remark 3.** The matrix  $\mathbb{X}_{\theta\theta}$  in (12) related to the Lyapunov function (15) depends on the MFs  $\eta_i(\theta)$ ,  $i \in \Omega_r$ , in a polynomial form. A general form of this matrix is given as follows:

$$\begin{aligned} \mathbb{X}_{(\theta)} &= \begin{bmatrix} X & 0 & 0 \\ X_{21}(\theta) & X_{22}(\theta) & X_{23}(\theta) \\ X_{31}(\theta) & X_{32}(\theta) & X_{33}(\theta) \end{bmatrix} \\ &= \sum_{i_0=1}^r \cdots \sum_{i_N=1}^r \prod_{j=0}^N \eta_{i_j}(\theta) \begin{bmatrix} X & 0 & 0 \\ X_{21}^{i_0 \dots i_N} & X_{22}^{i_0 \dots i_N} & X_{23}^{i_0 \dots i_N} \\ X_{31}^{i_1 \dots i_N} & X_{32}^{i_1 \dots i_N} & X_{33}^{i_1 \dots i_N} \end{bmatrix}. \end{aligned} \quad (23)$$

Then, less conservative results can be achieved by increasing  $N$ . In this paper, a trade-off between relaxation and computational burden has been done by taking  $N = 1$ , but the results for  $N \geq 2$  can be easily generalized from Theorem 1. Then, the relaxation results in [21] can be also applied in these cases.

**Remark 4.** Under the effect of the PDC controller (2), the form of the Lyapunov function (15) (respectively system (3)) and the quadratic Lyapunov function (16) (respectively system (1)) are strictly *equivalent*. However, the use of (15) associated with (3) allows introducing the parameter-dependent matrices  $X_{21}(\theta)$ ,  $X_{22}(\theta)$ ,  $X_{23}(\theta)$ ,  $X_{31}(\theta)$ ,  $X_{32}(\theta)$ ,  $X_{33}(\theta)$  (see (23)) into the control design. This aims to reduce the conservatism of the results [23, 24]. Corollary 1 presents the design conditions *without* using the augmented system approach.

**Corollary 1.** Given positive scalars  $\delta$  and  $\alpha$ , assume there exist positive definite matrix  $X \in \mathbb{R}^{n_x \times n_x}$ , positive diagonal matrices  $S_i \in \mathbb{R}^{n_u \times n_u}$ , matrices  $V_i \in \mathbb{R}^{n_u \times n_x}$ ,  $W_i \in \mathbb{R}^{n_u \times n_x}$ ,  $i \in \Omega_r$ , and positive scalars  $\rho, \gamma$  satisfying (5), (6), (7) and the following conditions:

$$\begin{aligned} \Xi_{ii} &< 0, \quad i \in \Omega_r \\ \frac{2}{r-1} \Xi_{ii} + \Xi_{ij} + \Xi_{ji} &< 0, \quad i, j \in \Omega_r, i \neq j \end{aligned}$$

where the quantity  $\Xi_{ij}$  is defined by

$$\Xi_{ij} = \begin{bmatrix} \Xi_{ij(1,1)} & \Xi_{ij(1,2)} & B_i^w + X C_i^\top D_j^w & X C_i^\top \\ * & -2S_j & 0 & 0 \\ * & * & -\gamma I + D_i^{w\top} D_j^w & 0 \\ * & * & * & -I \end{bmatrix},$$

with  $\Xi_{ij(1,1)} = \text{He}(A_i X + B_i^u V_j) + \alpha X$  and  $\Xi_{ij(1,2)} = W_j^\top - B_i^u S_j$ . Then, the PDC controller (2) with the feedback gains (11) solves the control problem in Section 2.

*Proof.* The result of Corollary 1 is derived from the proof of Theorem 1 by using respectively (1) and (16) in place of (3) and (15) in the development. Thus, this proof is omitted here for brevity.  $\square$

**Remark 5.** The main source of conservatism of the proposed method comes from the fact that  $X$  is not parameter dependent, see (23). Multiple Lyapunov functions [17] may be used to overcome this drawback. However, if this approach is successfully applied to discrete-time T-S systems [7, 8], it is still a challenging research topic for continuous-time cases. This is basically due to the presence of *a priori* unknown time-derivatives of the MFs  $\eta_i(\theta)$ ,  $i \in \Omega_r$ , in the derivation of design conditions [16]. The use of  $\mathbb{X}(\theta)$  in (23) (or (12) in the proof of Theorem 1) allows avoiding this major drawback.

**Remark 6.** For local control purposes, two following convex optimization problems can be formulated [6].

*Optimization problem 1.* To find a controller that satisfies the properties in Section 2 while minimizing the  $\mathcal{L}_2$ -gain, the following optimization can be performed:

$$\min \quad \gamma,$$

subject to  $\gamma > 0$  and LMI conditions in Theorem 1.

*Optimization problem 2.* To find a controller that satisfies the properties in Section 2 while maximizing  $\mathcal{E}(P)$ , the following optimization can be performed [22]:

$$\min \quad \text{trace}(R),$$

subject to the LMIs in Theorem 1 and  $\begin{bmatrix} R & I \\ I & X \end{bmatrix} > 0$ . By Schur complement lemma, this latter leads to  $\tilde{R} > P$ , thus  $\text{trace}(R) > \text{trace}(P)$ . Then, the minimization of  $\text{trace}(R)$  implies clearly the minimization of  $\text{trace}(P)$ .

**Remark 7.** The control problems in *local* context based on non-quadratic Lyapunov approaches usually lead to complex design conditions [16]. The numerical complexity of LMI-based optimizations can be evaluated by the number of scalar decision variables  $\mathcal{N}_{\text{var}}$  and the number of rows of all considered LMI conditions  $\mathcal{N}_{\text{row}}$ . For Theorem 3 in [16], these numbers are given as follows:

$$\begin{aligned} \mathcal{N}_{\text{var}} &= 1 + r(n_x(n_x + 1)/2 + n_u n_x + n_p n_u n_x) \\ &\quad + n_p n_x^2 r^2 (n_x + n_u)(n_x + n_u + 1 + (n_x + 1)r) \\ \mathcal{N}_{\text{row}} &= 1 + r n_x + r^3 (n_x + n_z + n_w + n_p n_x^2 n_u) \\ &\quad + r^2 (n_u n_x + (n_x + n_u) n_p n_x^2) \end{aligned}$$

whereas these numbers corresponding to Theorem 1 are

$$\begin{aligned} \mathcal{N}_{\text{var}} &= 2 + n_x(n_x + 1)/2 + r^2 (n_z n_x + n_z^2 + n_z n_u) \\ &\quad + r(n_u + n_u n_z + n_u^2 + 3n_u n_x) \\ \mathcal{N}_{\text{row}} &= 3 + r n_u (n_x + 1) + r^2 (n_x + 2n_u + 2n_z + n_w) \\ &\quad + q(n_x + 1) \end{aligned}$$

The numerical complexity of these methods is evaluated later. Note also that the control structure in [16] is much more complex than (2), which requires online inversion of a parameter-dependent matrix for implementation. Thus, this approach does not seem suitable for real-time applications, especially for high dimensional T-S systems with an important number of linear subsystems. At last, from the viewpoint of input-saturated control systems, in many cases the new method can lead to less conservative results than those of [16]. These facts are clearly illustrated in Section 4.

## 4. Illustrative Examples

In this section, three examples are given to illustrate the interests of the proposed method. The first example is a physically motivated application while the two others are used to demonstrate the capacity of the new control method in reducing the design conservatism. Note that since all systems considered here are *open-loop unstable*, the method in [11] cannot be applied to these cases. All LMI optimizations are solved with the help of YALMIP toolbox [25] in Matlab and SDPT3 solver.

**Example 1.** Let us the control problem of an inverted pendulum whose dynamics is given as follows [2]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g \sin(x_1) - a m l x_2^2 \sin(2x_1)/2 - a \cos x_1 u}{4l/3 - a m l \cos^2 x_1} + w \end{aligned} \quad (24)$$

where  $x_1$  is the angle of the pendulum from the vertical,  $x_2$  is the angular velocity, and  $u$  is the control force applied to the cart. Physically reasonable parameters for (24) are given by  $M = 8$  [kg],  $m = 2$  [kg],  $2l = 1$  [m],  $g = 9.8$  [m/s<sup>2</sup>] and  $a = 1/(m + M)$ . For the performance output, we take  $z = x_1$ . The disturbance  $w$  represents a force exerted on the pendulum. Suppose that system (24) is constrained by  $u_{\text{max}} = 300$  [N],

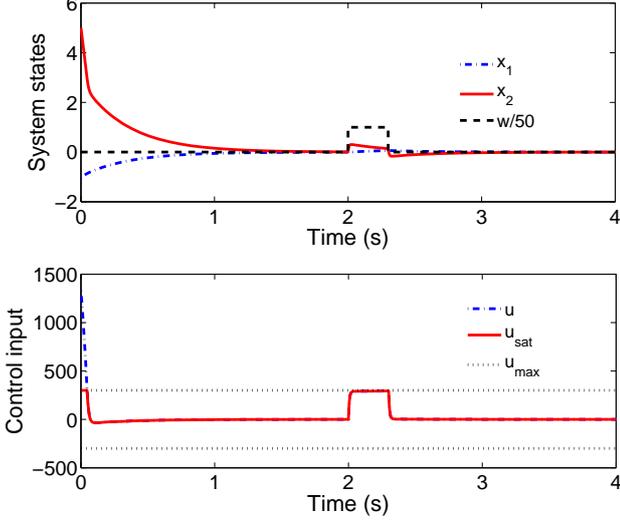


Figure 1: Closed-loop behaviors obtained with the proposed control method corresponding to the initial condition  $x_0^\top = [-\pi/3 \ 5]$ .

$|x_1| \leq \pi/3$  [rad] and  $|x_2| \leq 5$  [rad/s]. Therefore, the vectors characterizing  $\mathcal{P}_x$  defined in (4) are given by

$$\begin{aligned} h_1^\top &= [3/\pi \ 0], & h_2^\top &= [-3/\pi \ 0], \\ h_3^\top &= [1/5 \ 0], & h_4^\top &= [-1/5 \ 0]. \end{aligned}$$

In the light of the results in [2], the following T-S model is used for control design of nonlinear system (24):

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 \\ 17.294 & 0 \end{bmatrix}, & B_1^u &= \begin{bmatrix} 0 \\ -0.177 \end{bmatrix}, & B_1^w &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 1 \\ 12.631 & 0 \end{bmatrix}, & B_2^u &= \begin{bmatrix} 0 \\ -0.078 \end{bmatrix}, & B_2^w &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \end{aligned}$$

with the membership functions

$$\begin{aligned} \eta_1(x_1) &= \frac{\omega_2(x_1) - 1}{\omega_1(x_1)\omega_2(x_1)}, & \eta_2(x_1) &= 1 - \eta_1(x_1), \\ \omega_1(x_1) &= 1 + \exp(-7(x_1 + \pi/4)), \\ \omega_2(x_1) &= 1 + \exp(-7(x_1 - \pi/4)). \end{aligned}$$

Notice that the decay rate  $\alpha$  in (P2) is related to the time performance of the closed-loop system [2]. Solving the Optimization problem 1 with  $\alpha = 0.1$  leads to

$$P = \begin{bmatrix} 1.78 & 0.30 \\ 0.30 & 0.13 \end{bmatrix}, \quad K_1 = \begin{bmatrix} 1155.4 & 444.81 \\ 788.28 & 290.35 \end{bmatrix},$$

and  $\sqrt{\gamma} = 0.12$ . For simulation, the unknown disturbance  $w$  is selected as a constant of 25 happening at  $t = 2$ s with a duration of 0.3 [s]. Then, the closed-loop response corresponding to the initial condition  $x_0^\top = [-\pi/3 \ 5]$  is depicted in Figure 1. Observe that the closed-loop system (24) is stabilized despite an important input saturation at the beginning and the disturbance is well attenuated. Note that the classical PDC controller (without any constraint consideration) in [2] induces *unstable* behaviors under the same simulation conditions, see Figure 2.

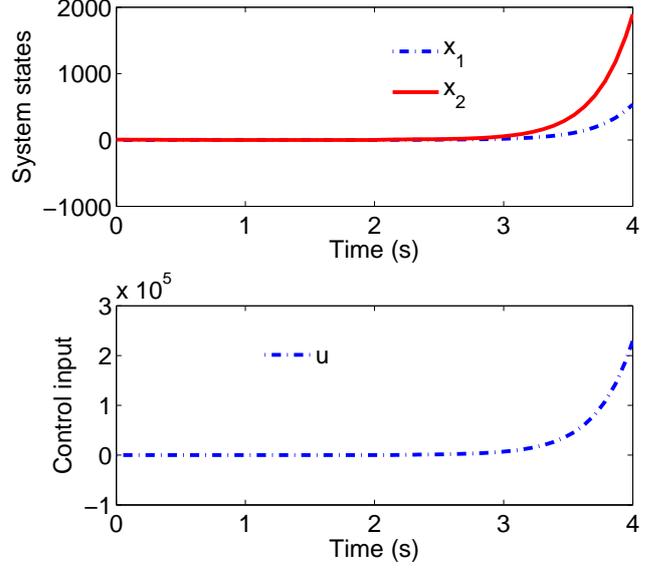


Figure 2: Closed-loop behaviors obtained with the control method in [2] corresponding to the initial condition  $x_0^\top = [-\pi/3 \ 5]$ .

This clearly shows the interest of considering the system constraints into the control design for real-world applications.

For the maximization of  $\mathcal{E}(P)$ , the Optimization problem 2 is performed leading to

$$P = \begin{bmatrix} 0.95 & 0.06 \\ 0.06 & 0.09 \end{bmatrix}, \quad K_1 = \begin{bmatrix} 405.03 & 366.47 \\ 408.11 & 263.04 \end{bmatrix},$$

and an  $\mathcal{L}_2$ -gain of  $\sqrt{\gamma} = 0.183$ . Figure 3 shows that  $\mathcal{S}_1 = \mathcal{E}(P)$  is maximized along  $\mathcal{S}_2 = \mathcal{P}_u \cap \mathcal{P}_x$  which is included inside  $\mathcal{S}_3 = \Phi\left(\sum_{i=1}^r \eta_i(K_i - G_i)\right)$ . Thus, the set inclusion property is clearly verified.

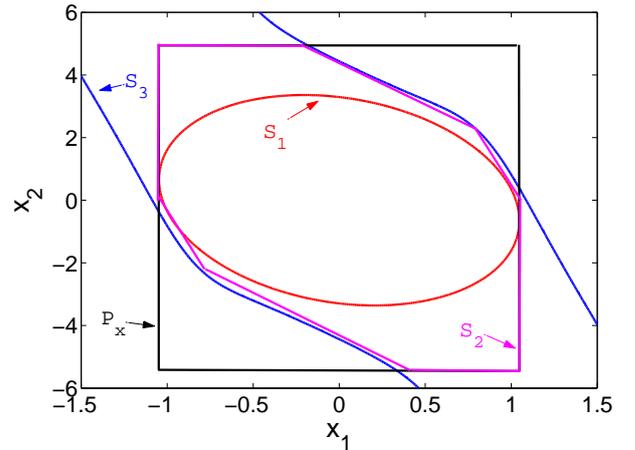


Figure 3: Set inclusion property:  $\mathcal{S}_1 = \mathcal{E}(P)$ ,  $\mathcal{S}_2 = \mathcal{P}_u \cap \mathcal{P}_x$  and  $\mathcal{S}_3 = \Phi\left(\sum_{i=1}^r \eta_i(K_i - G_i)\right)$ .

**Example 2.** Consider the following family of T-S fuzzy systems (1) taken from Example 1 in [16]:

$$A_1 = \begin{bmatrix} 2 & -10 \\ 2 & 0 \end{bmatrix}, B_1^u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_1^w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, D_1^w = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} a & -5 \\ 1 & 2 \end{bmatrix}, B_2^u = \begin{bmatrix} b \\ 2 \end{bmatrix}, B_2^w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, D_2^w = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

for  $a \in [-20, 25]$  and  $b \in [0, 25]$ . The existence of stabilizing controllers for this example is checked using the new quadratic relaxation approach (QRA: Theorem 1) and the non-quadratic approach (NQA: Theorem 3 in [16]) for 806 points of a 31-by-26 parameter grid. For that, both approaches are applied with  $\delta = 0.1$ ,  $\alpha = 0$ ,  $u_{\max} = 10$  and  $\mathcal{P}_x = \{x \in \mathbb{R}^2 : |x_i| \leq \pi/2, i = 1, 2\}$ . Note that the NQA in [16] are only for *undisturbed* T-S systems ( $w = 0$ ) and no constraint guaranteeing an  $\mathcal{L}_2$ -gain has been incorporated therein. Hence, the comparison done in this example between two approaches is not objective. However, this can provide an idea on the conservatism of the new method.

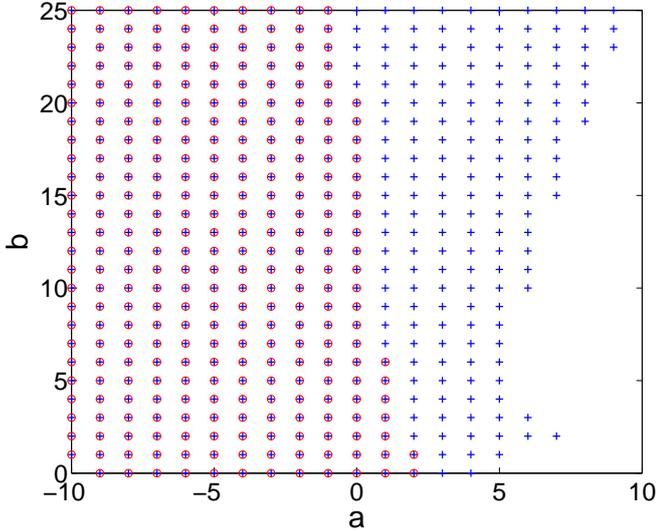


Figure 4: Feasibility spaces obtained with QRA (+) and NQA (o).

Figure 4 shows that the feasibility space given by the proposed method is distinctly larger than that of the NQA. Notice that the results of the NQA have already outperformed those based on a fuzzy Lyapunov function [17], and a line-integral Lyapunov function [18]. For this example, the new method also leads to less computational burden compared to the NQA, see Table 1. The results in Figure 4 and Table 1 confirm Remark 7.

Table 1: Comparison of complexity between different methods

Complexity	Theorem 1	Theorem 3 in [16]
$\mathcal{N}_{\text{row}}$	57	137
$\mathcal{N}_{\text{var}}$	65	255

The outperformance of the proposed method compared to [16] in terms of relaxation is not surprising since in its present state of progress, the NQA in [16] is based on an *unsaturated*

control law and depends strongly on the control bound limits (due to its way to deal with *unknown* time-derivatives of the MFs). This can make the results become *over-conservative*.

**Example 3.** Consider the following family of open-loop unstable T-S fuzzy systems in the form (1) with:

$$A_1 = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix}, B_1^u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} -0.1 \\ -0.4 \end{bmatrix}^\top$$

$$A_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix}, B_2^u = \begin{bmatrix} 8 \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}^\top$$

$$A_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}, B_3^u = \begin{bmatrix} 6-b \\ -1 \end{bmatrix}, C_3 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}^\top \quad (25)$$

$$A_4 = \begin{bmatrix} 0.89 & -5.29 \\ 0.1 & 0 \end{bmatrix}, B_4^u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_4 = \begin{bmatrix} -0.1 \\ -0.4 \end{bmatrix}^\top$$

$$B_1^w = B_3^w = [0.1 \ 0.1]^\top, B_2^w = B_4^w = [0 \ 0]^\top$$

$D_i^w = -0.5$ ,  $i \in \Omega_4$  and  $a, b \in [-4, 4]$ . The family of T-S systems (25) can represent *complex* nonlinear systems since the matrices of all four subsystems are significantly different. This example aims to show the potential of Theorem 1 in terms of relaxation with respect to different values of  $N$  in (23). To this end, the existence of stabilizing controllers for (25) is checked for 289 points of a 17-by-17 square parameter grid. In this study, the following design data are considered for comparison purposes  $\delta = 0.1$ ,  $\alpha = 0$ ,  $u_{\max} = 10$  and  $\mathcal{P}_x = \{x \in \mathbb{R}^2 : |x_1| \leq 2, |x_2| \leq 2\}$ .

The feasibility spaces obtained with the proposed method for three different cases  $N = 1, 2$  and 3 are depicted in Figure 5. As indicated in Remark 3, less conservative results are obtained by increasing  $N$ . Moreover, the feasibility space obtained with the conditions in Corollary 1 is clearly included by that obtained with the new augmented system approach, see Remark 4. It is also important to note that the design conditions using: (1) a fuzzy Lyapunov function in [16]; (2) polytopic representations of the saturation function in [4, 11]; (3) a one-step AW dynamic output feedback control scheme in [6], are all *unfeasible* for any point of the considered parameter grid. The above facts strongly confirm the interest of the new method for its capacity to reduce the design conservatism compared to existing results.

## 5. Concluding Remarks

In this paper, a new method is proposed to design a saturated PDC control law for constrained T-S fuzzy systems subject to  $\mathcal{L}_2$ -disturbances. The control design is cast as an LMI optimization, and then the obtained controller can guarantee several closed-loop properties. The novel idea of this method consists in a judicious use of an equivalent augmented form of the closed-loop system associated with a parameter-dependent Lyapunov function for constrained T-S systems. Several examples are given to demonstrate the effectiveness of the proposed control method. It has been proved that in many cases the new method allows outperforming the most recent results based on non-quadratic approaches dealing with the same local control

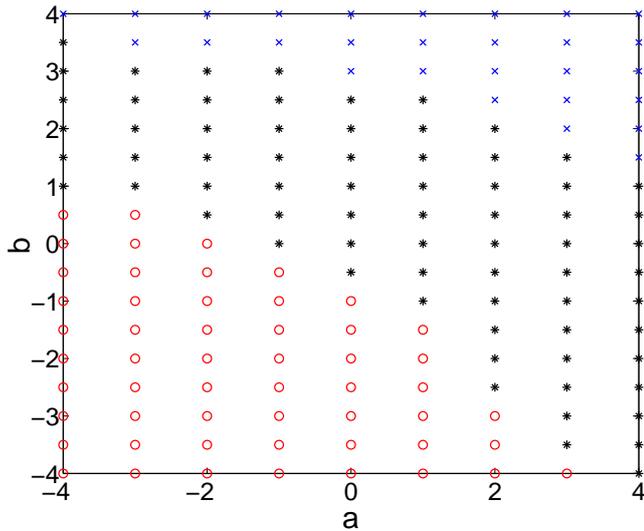


Figure 5: Feasibility spaces obtained with Theorem 1 for  $N = 1$  and Corollary 1 (o);  $N = 2$  (o and \*);  $N = 3$  (o, \* and x).

context. Notice also that the control structure in this work is much simpler than those used for comparison purposes. Therefore, this systematic control method is more suitable for practical uses. Future works focus on the real-time application of the proposed method to the challenging design issue of vehicle shared steering control [26].

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