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# ENTROPY GENERATION FOR OSCILLATORY FLOW INSIDE THERMAL-LAG TYPE STIRLING ENGINE / NUMERICAL ANALYSIS

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## ABSTRACT

The present paper investigates the heat characteristics of oscillatory piston-driven flow inside thermal-lag type Stirling engine. The geometry consists of a cylinder partially filled with a porous metal structure called regenerator, heated at the lateral wall on one side and cooled on the other side. Brinkman-Forchheimer-Lapwood extended Darcy model is assumed to simulate heat transfer within the regenerator.

A numerical model is used to evaluate average entropy generation rate in the regenerator depending on its characteristics (form factor  $L_r/D_r$ , porosity and material) and on the oscillatory flow characteristics (working fluid, rotational engine speed, hot end temperature and initial pressure). The output power of the thermal lag Stirling engine is estimated for different working conditions. Results show that, the two main contributors to entropy generation in the regenerator are: entropy due to heat transfer (imperfection loss, internal conduction loss) and entropy due to viscous friction. Regenerator design leading to minimum entropy generation was investigated.

## NOMENCLATURE

### Variables

A	Gas cross-section (m <sup>2</sup> )
A <sub>wet</sub>	wetted area (m <sup>2</sup> )
L	Length (m)
D	Diameter (m)
x	Piston position (m)
P	Pressure (Pa)
V	Volume (m <sup>3</sup> )
u	Velocity (m.s <sup>-1</sup> )
W	output power (W)
M	Mass (Kg)
$\dot{m}$	Mass flow rate (kg.s <sup>-1</sup> )
t	Time (s)
T	Temperature (K)
h	Heat transfer coefficient (W.m <sup>-2</sup> .K <sup>-1</sup> )
c <sub>v</sub>	Specific heat capacity at constant volume (J.kg <sup>-1</sup> .K <sup>-1</sup> )
c <sub>p</sub>	Specific heat capacity at constant pressure (J.kg <sup>-1</sup> .K <sup>-1</sup> )
COP	Coefficient of performance
dp	Pore diameter of the porous matrix
df	Solid wire diameter.
f	Friction factor
$\mu$	Dynamic viscosity (Pa.s)
Re	Reynolds number

Pr	Prandtl number
St	Stanton number
K	permeability (m <sup>2</sup> )
$\phi$	Porosity of the regenerator
$\lambda$	Thermal conductivity (W m <sup>-1</sup> K <sup>-1</sup> )
NUT	Number of heat transfer unit
$\epsilon_r$	Regenerator efficiency
$\rho_w$	Density of matrix solid material (kg m <sup>-3</sup> )

### **Indices**

w	Regenerator wall
r	Regenerator
g	Gas
0	Initial or starting value
in	Input
out	Output

## **INTRODUCTION**

In recent years, the interest in Stirling engine has increased due to its ability to use any external heat source including solar energy, fossil fuels and biomass. Numerous studies examine the Stirling engine performances and tested many cogeneration systems using Stirling technology. The Stirling engine operates in a closed thermodynamic regenerative cycle with the same working fluid repeatedly compressed and expanded at different temperature levels. So there is a net conversion of heat to work. There are three different configurations of the Stirling engine: Alpha, Beta and Gamma configurations. They have the same thermodynamic cycle but different mechanical design characteristics.

The Thermal Lag Stirling Engine type looks like Beta configuration with a fixed regenerator and only one moving piston. The Thermal Lag Engine (TLE) was proposed by Tailer in 1993[1] and its mechanical simplicity makes it an appealing technology to access renewable sources of energy or recover waste heat. This engine has a single moving part, the piston, which delivers power and acts as the prime mover of the working fluid. The TLE consists of a hot space, a cold space and a porous-medium stack is fixed in the cylinder as a static regenerative heater. This engine is still not well understood [2]. Despite its mechanical simplicity, only few researchers investigate the thermal-lag Stirling engines performances; therefore, physical understanding of the thermal-lag phenomenon is insufficient [3].

Some simulations of thermal lag type Stirling engine were developed. These simulations provided an edge to the developers, as it could provide an accurate analysis of the performance of the engine before actually manufacturing it. Altamirano et al. (2014) [2] elaborate a two control volume model in order to study Thermal Lag Stirling Engine performances (increasing its power output and efficiency). The discussion shows that the engine performances is good when the hot heat exchanger has the largest possible heat transfer capabilities, whilst the cold heat transfer should be limited to a maximum value within the cycle. They showed that the

regeneration in the hot control volume, due to insufficient cooling, is an undesired effect and should be avoided. They concluded that the engine performance can be improved by enhancing and optimizing the heat transfer in the piston chamber. Cheng and Yang, (2013) [4] made a thermal lag Stirling engine prototype that deliver only 15 W of power and also developed a thermodynamic model for predicting transient variations of the thermodynamic properties in different working spaces of the engine. Effects of geometrical and operating parameters, such as heating and cooling temperatures, volumes of the chambers, thermal resistances, stroke of piston, and bore size on indicated power output and thermal efficiency were also evaluated. Cheng et al. (2013) [5] developed a dynamic model incorporated with thermodynamic model to study the thermal-lag Stirling engine start process. They found that geometric parameters, such as bore size, stroke, and volume of working spaces, also determine the operating mode of the engine, And that the optimal rotational engine speed leading to maximum shaft power is significantly influenced by the geometrical parameters. Yang et al. (2016) [3] studied numerically and experimentally nonlinear instability of thermal-lag phenomenon in the thermal-lag Stirling engine. They clarify the dependence of the thermal lag angles on the influential geometrical and physical parameters. They study the effects of the thermal-lag angles on the indicated work output.

The analysis of the energy use and the entropy generation becomes one of the primary objectives in designing a thermal system. In fact, the study of entropy generation, or thermodynamics second law analysis, is the gateway for optimization studies in thermal equipments and systems. The aim of this study is to develop an appropriate physical and numerical model for thermal lag type Stirling engine and to determine minimum entropy generation rates inside the regenerator taking into consideration the effect of working conditions (temperature ratio between hot and cold source, initial pressure and rotational engine speed, working fluid) and the effect of regenerator parameters (geometry, porosity and material). Average entropy generation rates in regenerator due to viscous friction and due to heat transfer are obtained from the computational fluid dynamics analysis.

### **Presentation of a typical thermal lag Stirling engine**

The Thermal Lag Stirling Engine (TLE) consists of a cylinder with a piston. Heated from the outside (heater) and cooled from the outside (cooler). The cylinder is partially filled with a porous medium through which is crossed twice by a compressible gas. This porous matrix called regenerator is characterized by its porosity  $\epsilon$  and its permeability K.

The Stirling engine uses a working fluid (air, helium, hydrogen ...) contained in a closed enclosure, heated by an external heat source, in which variations in volumes induce cyclic changes in pressure and temperature of the fluid. When the fluid is expanded, it exerts pressure on the engine piston in order to obtain a translational movement which is subsequently transformed into a rotational movement by means of a drive

system. The Stirling engine works in closed cycle (Figure 1). The working fluid trapped in the machine undergoes four processes. During the first process (1→2), the gas is compressed adiabatically until its temperature is  $T_H$ . During the second process (2→3), the gas is compressed isothermally at  $T_H$  while it discharged energy  $Q_H$  to the hot reservoir by heat transfer. During the third process (3→4), the gas expands adiabatically until its temperature decreases to  $T_L$ . And finally, during the process (4→1), the gas expands isothermally at  $T_C$  while receiving energy  $Q_C$  from the cold reservoir by heat transfer.

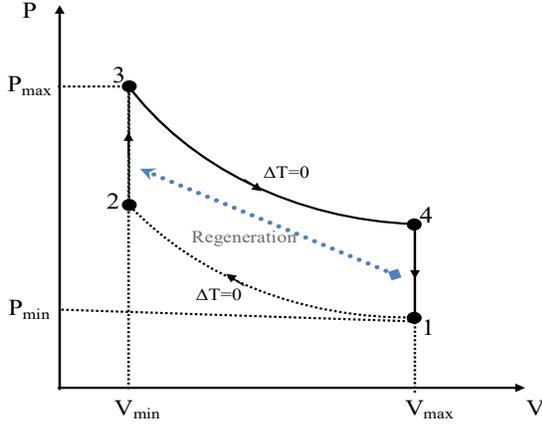


Figure 1. Stirling cycle

The thermal lag engine is characterized by a low output power and efficiency compared to other configurations of Stirling engines. Cheng et al. [4,5] investigated the same engine presented in this paper. The studied thermal lag Stirling engine deliver 15 W of power and working under parameters presented in table 1. Its geometrical parameters are illustrated in Figure 2. and values used for modeling are listed in Table 2.

Table 1. Operating parameters

Parameter	Value	Specification(s)
$\omega$ (rpm)	500	Rotational engine speed
$T_{wh}$ (K)	1000	Wall temperature of the hot chamber
$T_{wk}$ (K)	300	Wall temperature of the cold chamber
$P_i$ (bar)	1	Initial pressure
Working fluid	air	-

Table 2. Geometrical parameters [4, 5]

Parameter(s)	Value	Specification(s)
$r$ (m)	0.025	Piston stroke
$d$ (m)	0.045	Piston diameter
$L_r$ (m)	0.0585	Regenerator length
$L_d$ (m)	0.0031	Compression space dead volume length

$V_1$ (m <sup>3</sup> )	$3010^{-6}$	Fixed hot volume
$V_2$ (m <sup>3</sup> )	$79.52 \cdot 10^{-6}$	Compression space swept Volume
$\epsilon$	0.85	Regenerator porosity
Regenerator material	Stainless steel	-

## NUMERICAL MODEL

The present model describes the dynamic behavior of the Stirling engine. In this model the engine is devised into hot chamber, cold chamber and the regenerator. The hot chamber and the regenerator have a fixed volumes and the cold chamber has a variable volume. This model is based on the following considerations:

- The working fluid is air and it is assumed to be ideal gas.
- Thermo-physical properties of the gas are constant.
- the physical properties of the regenerator are uniform and constant.
- Losses by leakage of mass are neglected.
- The motion of the power piston is considered to be periodic and it is prescribed as follow:

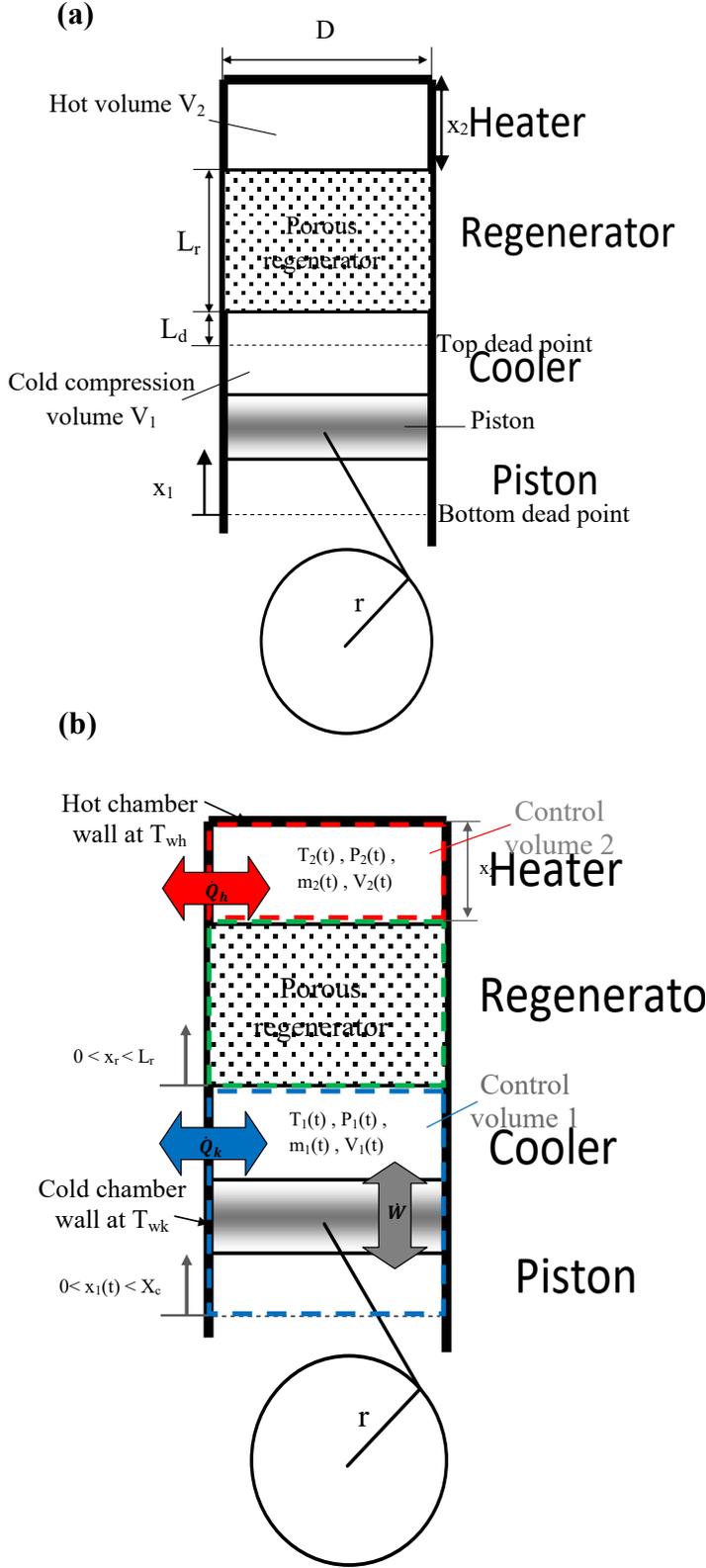
$$x_1(t) = L_d + X_c \sin(2\pi ft) \quad (1)$$

Where  $f$  is the frequency,  $L_d$  and  $X_c$  are respectively the length of dead space and the length of swept space in the compression cylinder.

- Hot and cold chambers walls are at a constant temperature so any heat transfer to or from the walls is considered to be with a cold reservoir for expansion chamber and with a hot reservoir for compression chamber.

Temporal evolution of the piston velocity is expressed as follow:

$$u = \frac{dx_1}{dt} \quad (2)$$



**Figure 2.** Schematic of the thermal-lag Stirling engine (a) geometric parameters (b) control volumes

### Hot and cold spaces temperatures

Energy balance equations in hot and cold chambers are written as follow:

For the cold chamber:

$$\underbrace{c_v m_1 \frac{dT_1}{dt}}_{(1)} = \underbrace{h_1 \pi D x_1 (T_{wk} - T_1)}_{(2)} + \underbrace{c_v \dot{m}_{in,1} (T_{in,1} - T_1) - c_v \dot{m}_{out,1} (T_{out,1} - T_1)}_{(3)} - \underbrace{P_1 \frac{dV_1}{dt}}_{(4)} \quad (3)$$

For the hot chamber:

$$\underbrace{c_v m_2 \frac{dT_2}{dt}}_{(1)} = \underbrace{h_2 \pi D x_2 (T_{wh} - T_2)}_{(2)} + \underbrace{c_v \dot{m}_{in,2} (T_{in,2} - T_2) - c_v \dot{m}_{out,2} (T_{out,2} - T_2)}_{(3)} \quad (4)$$

Where subscripts 1 and 2 denote respectively cold and hot chambers.  $T_1$  and  $T_2$  are the air temperatures,  $T_{wk}$  and  $T_{wh}$  are the wall temperatures,  $m_1$  and  $m_2$  are mass of air,  $h_1$  and  $h_2$  are convective heat transfer coefficients (appendix A) respectively in cold and hot chambers.  $C_v$  is the heat capacity per unit mass of air at constant volume.  $D$  is the piston diameter,  $x_1$  and  $x_2$  is the distance from the regenerator. And  $\dot{m}_{in}$  and  $\dot{m}_{out}$  are respectively mass flow rates into and out of the chamber.

Term (1) represent the rate of change of the temperature of the air in a chamber with respect to time  $t$ . Term (2) is the heat transfer from the walls of the chamber to the air. Term (3) represents the difference of the enthalpy between the incoming flow and the outgoing flow through the control volume. And Term (4) is the work done by the air on the piston.

The mass flow rate of the air in cold and hot chambers is not constant during a cycle  $\dot{m}_{in}$  and  $\dot{m}_{out}$  depends on the nature of the flow. The mass flow rate in cold and hot chambers is expressed as follow:

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m} = \rho u \frac{\pi D^2}{4} \quad (5)$$

Where  $D$  is the cylinder diameter,  $u$  is the velocity of the piston.

The mass of working fluid in cold and hot chambers at every time step can be calculated by:

$$m_1(t + \Delta t) = m_1(t) + \dot{m}_1(t) \Delta t \quad (6)$$

$$m_2(t + \Delta t) = m_2(t) + \dot{m}_2(t) \Delta t \quad (7)$$

### Temporal evolution of pressure

Pressures respectively in cold and hot chambers  $P_1$  and  $P_2$  are calculated when assuming ideal gas behaviour for the air as follow:

$$P_1 = \frac{4m_1 RT_1}{\pi D^2 x_1} \quad (8)$$

$$P_2 = \frac{4m_2 RT_2}{\pi D^2 x_2}$$

The pressure in cold and hot chambers at every time step can be calculated as follow:

$$P_1(t + \Delta t) = P_{av,1}(t) - \Delta P(t)/2 \quad (9)$$

$$P_2(t + \Delta t) = P_{av,2}(t) + \Delta P(t)/2 \quad (10)$$

where  $P_{av}$  is the average pressure in working spaces and  $\Delta P$  is the pressure drop due to viscous force in regenerator.

The temporal evolution of pressure drop is assumed when considering the Darcy law as follow:

$$\Delta P(t) = \frac{\dot{Q}(t)L_r\mu}{K.A} \quad (11)$$

Where  $K$  is the permeability of the regenerator and it is assumed using the following correlation:

$$K = \frac{d_p^2 \phi^3}{150(1 - \phi)^2} \quad (12)$$

#### Pressure along the regenerator

An analytical solution of the pressure along the porous media is proposed using the following relation:

$$\frac{d^2 P}{dx^2} = \frac{\phi \cdot \mu \cdot c_t}{K} \frac{dP}{dt} \quad (13)$$

When considering the following limit conditions:

$$P(x = 0, t) = P_1(t) \quad (14)$$

$$P(x = L_r, t) = P_2(t) \quad (15)$$

Using the boundary conditions of equations (14) and (15), we have:

$$P(x, t) = c_1 x^2 + c_2 x + c_3 \quad (16)$$

$$c_1 = \frac{\phi \cdot \mu \cdot c_t}{2 \cdot K} \frac{\Delta P(t)}{\Delta T(t)}$$

$$c_2 = \frac{\phi \cdot \mu \cdot c_t}{2 \cdot K} \frac{\Delta P(t)}{\Delta T(t)} L_r + \frac{\Delta P(t)}{L_r}$$

$$c_3 = P_1(t)$$

$c_1$ ,  $c_2$  and  $c_3$  are constants of integration.

#### Regenerator interface temperature

The interface temperatures  $T_{in}$  and  $T_{out}$  depend on the flow direction and can be calculated using the definition of the effectiveness of the regenerator as follow:

$$\text{If } \dot{m}_{in,1} > 0 \quad \text{Then} \quad (17)$$

$$T_{in,1}(t + \Delta t) = T_1(t) - \varepsilon_r(T_2(t) - T_1(t))$$

$$\text{If } \dot{m}_{in,2} < 0 \quad \text{Then} \quad (18)$$

$$T_{in,2}(t + \Delta t) = T_2(t) + \varepsilon_r(T_2(t) - T_1(t))$$

$$\text{If } \dot{m}_{out,1} < 0 \quad \text{Then} \quad T_{out,1}(t) = T_1(t) \quad (19)$$

$$\text{If } \dot{m}_{out,2} > 0 \quad \text{Then} \quad T_{out,2}(t) = T_2(t) \quad (20)$$

#### Regenerator equations

The regenerator is a porous cylinder of length  $L_r$  and diameter  $D_r$ . Energy balance equation in the regenerator is written as:

$$\rho c_p \frac{\pi D_r^2}{4} \frac{\partial T_r}{\partial t} + \dot{m} c_p \frac{\partial T_r}{\partial x_r} = h_r \pi D_r (T_{w,r} - T_r) \quad (21)$$

Where  $T_r$  is the temperature of air inside regenerator,  $T_{w,r}$  is the temperature of the solid matrix in the regenerator,  $\rho$  is the density of air in the regenerator,  $c_p$  is the specific heat per unit mass of air at constant pressure and  $h_r$  is the convective heat transfer coefficient in the regenerator (appendix A). The equation (21) is solved using the method of characteristics.

The temperature of the solid matrix in the regenerator  $T_w$  is obtained when solving the following equation:

$$M_w c_w \frac{dT_w}{dt} = - \int_0^{L_r} h_r \pi D_r (T_w - T_r) dx_r \quad (22)$$

$M_w$  is the mass of the regenerative matrix,  $c_w$  is the specific heat per unit mass of the regenerator matrix.  $x_r$  is the location along the regenerator from the cold chamber to the hot chamber. The integral of equation (22) is solved using Euler method.

The mass flow rate through the regenerator can be obtained from the following equation [31]:

$$\dot{m} = \sqrt{\frac{\rho A_r^2 D_h (P_2 - P_1)}{L_r f}} \quad \text{if } P_2 \geq P_1 \quad (23)$$

$$\dot{m} = - \sqrt{\frac{\rho A_r^2 D_h (P_2 - P_1)}{L_r f}} \quad \text{if } P_1 > P_2 \quad (24)$$

Where  $A$  is the regenerator cross section,  $f$  and  $D_h$  are respectively the friction factor and the hydraulic diameter and are calculated using the following correlations:

$$f = 54 + 1.43 \text{Re}^{0.78} \quad (25)$$

$$D_h = d_f \frac{\phi}{1 - \phi} \quad (26)$$

#### Entropy generation

Exergy or entropy generation analysis, as a tool of applied thermodynamics, is becoming a standard practice for analyzing energy conversion systems and for identifying the efficiency of a component in a system. Sources of irreversibility are responsible for entropy generation such as heat and/or mass transfer. One part of this entropy generation is due to heat transfer in the direction of finite temperature gradients, and the other part takes place due to fluid friction irreversibility. Following Bejan (1982), the dimensional volumetric rate of entropy generation for the regenerator can be written as:

$$S'''_{gen} = \underbrace{\frac{k}{T_0^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 \right]}_{(1)} + \underbrace{\frac{1}{T_0} \left[ \frac{\mu u^2}{K_0} + \mu_{eff} \left( \frac{\partial u}{\partial x} \right)^2 \right]}_{(2)} \quad (27)$$

where  $T_0$  is the reference temperature. In the above equation, the first term represents the entropy generated in the radial direction by heat transfer, the second term accounts for axial conduction, and the last 2 terms are the fluid friction contribution in porous medium. For porous media the last term

become important compared with second term. As a usual practice, the last term is set to zero for region with porous medium and second term set to zero for region without porous medium.

Engine performances

Heat transfer and indicated work are evaluated using the following expressions:

$$\dot{Q}_h = \int_0^T h_2 \pi D x_2 (T_{wh} - T_2) dt \tag{28}$$

$$\dot{Q}_k = \int_0^T h_1 \pi D x_1 (T_{wk} - T_1) dt \tag{29}$$

$$\dot{W} = \int_0^T P_1 \frac{dV_1}{dt} dt \tag{30}$$

Thus thermal efficiency is:

$$\eta = \frac{\dot{W}}{\dot{Q}_h} \tag{31}$$

**NUMERICAL RESULTS AND DISCUSSIONS**

Performances

The thermal lag Stirling engine studied here presents lower output power. Its performances are studied in reference [4]. Table 3 show a comparison between numerical model results and Cheng et al. numerical results under the same working conditions.

**Table 3.** Thermal lag Stirling engine performances working with air at 500 rpm when  $T_{wh} = 1000$  K,  $T_{wk} = 300$  K,  $\phi = 0.85$ ,  $L_r/D_r = 1.3$  and the regenerator material is stainless steel

	Present results	Cheng et al. numerical results [4]
Output power (W)	17.25	17.2
Efficiency (%)	12.7	10.5

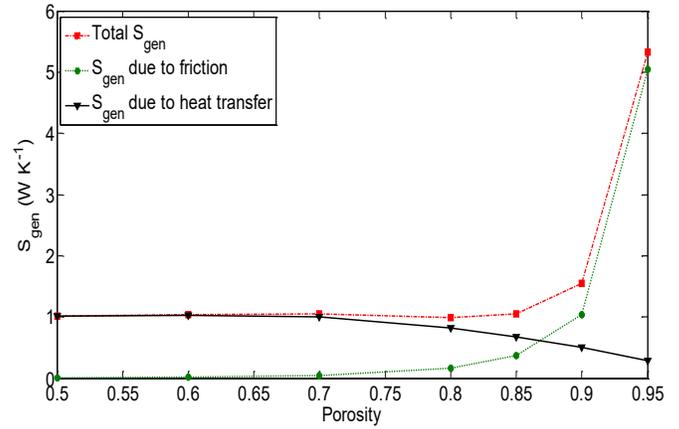
Regenerator porosity effect

The Stirling engine performances are influenced by the regenerator characteristics and its ability to accommodate the high heat flow. Figure 3 (a) presents the evolution of average entropy generation rate in the regenerator. The regenerator porosity increase leads to an increase of the average entropy generation rate due to viscous friction and to a decrease of the average entropy generation rate due to heat transfer. Consequently the average of the total entropy generation rate evolutions presents a minimum about  $1.3 \text{ WK}^{-1}$  when the porosity of the regenerator is around 0.85 for a stainless-steel regenerator material. Figure 3 (b) showed that a regenerator of copper matrix presents more entropy generation rate than a regenerator of Stainless steel material. Thus, the use of a

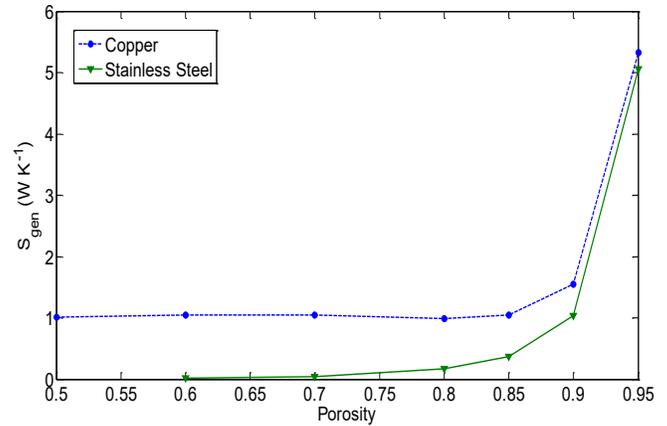
stainless steel regenerator with a porosity of 0.85 contribute to an amelioration of Stirling engine performances.

Figure 4 presents the evolution of average entropy generation rate in regenerator depending on the porosity for three different working fluids. It is shown that the minimum entropy generation is recorded when using helium as working fluid. For this small engine, helium has more advantages than air and nitrogen. This is due to low pressure drop and high specific heat of the helium. In fact, the thermal lag Stirling engine has less loss when using helium as the working gas.

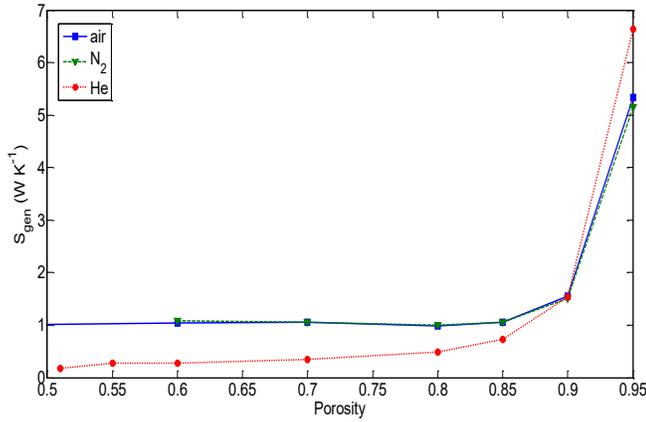
(a)



(b)



**Figure 3.** Average entropy generation rate evolution versus porosity ( $N = 500$  rpm,  $T_{wh} = 1000$  K,  $T_{wk} = 300$  K,  $L_r/D_r = 1.3$ )



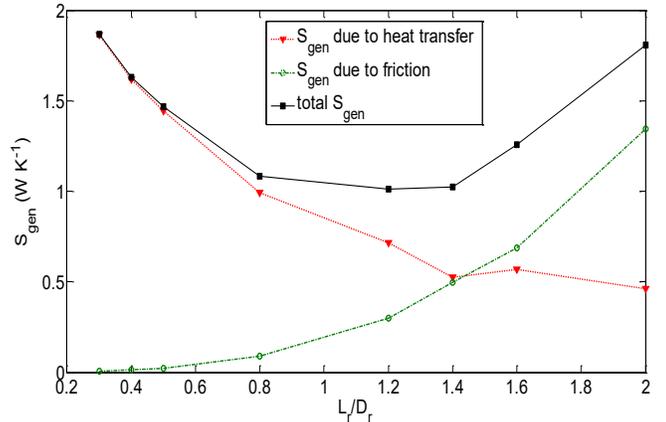
**Figure 4.** Average entropy generation rate versus regenerator porosity for three different working fluids ( $N=500$  rpm,  $T_{wh}=1000$  K,  $T_{wk}=300$  K,  $L_r/D_r=1.3$ , regenerator material is stainless steel)

#### Regenerator form factor effect

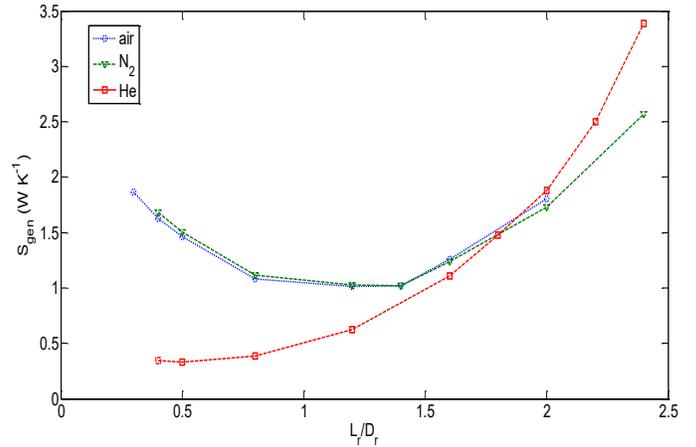
Figure 5 presents the evolution of average entropy generation rates due to heat transfer and due to viscous friction in the regenerator. According to Figure 4, the average entropy generation rate due to viscous friction increases with the form factor  $L_r/D_r$ . However, entropy generation rate due to heat transfer increases. As a conclusion, the average of the total entropy generation rate in regenerator evolution reaches a minimum about  $1\text{WK}^{-1}$  when  $L_r/D_r=1.3$ .

The form factor is defined as the ratio of regenerator length to its diameter  $L_r/D_r$ , the evolution of the average of the total entropy generation rate versus the form factor presents a minimum. In fact, when  $L_r/D_r$  is less than 1.3, the internal conduction loss is more important than the viscous friction loss in the regenerator. However when  $L_r/D_r$  is greater than 1.3, the viscous friction loss becomes more important than the internal conduction loss in the regenerator. There is a compromise between these two losses.

Figure 6 presents the evolution of the total entropy generation rate for different working fluids. It can be seen that the minimum entropy generation is obtained at different form factors. The regenerator form factor depends on the engine's working fluid. For helium, air and nitrogen, form factors are respectively 0.5, 1.3 and 1.4.



**Figure 5.** Average entropy generation rate versus form factor (air,  $N=500$ rpm,  $T_{wh}=1000$ K,  $T_{wk}=300$ K,  $\phi=0.85$ , regenerator material is stainless steel)



**Figure 6.** Average of the total entropy generation rate versus form factor for different working fluids ( $N = 500$  rpm,  $T_{wh} = 1000$  K,  $T_{wk} = 300$  K,  $\phi = 0.85$ , regenerator material is stainless steel)

#### Hot end temperature effect

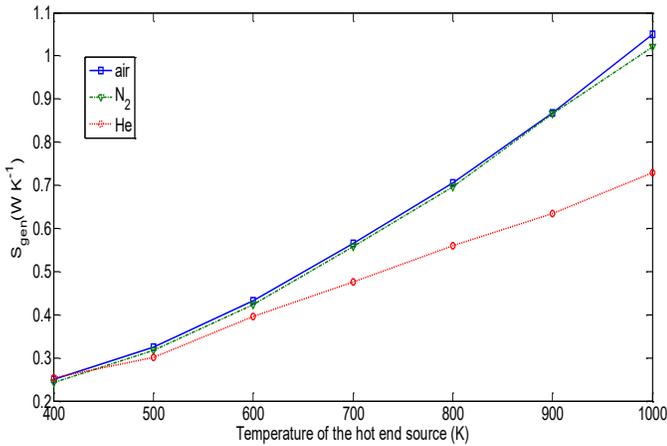
The hot end temperature determines the amount of heat absorbed by the working fluid. The increase of the hot end temperature will bring additional amount of heat to the engine. The increase in the temperature gradient between both heat sources causes the increase of the temperature difference between the two ends of the regenerator, which leads to an increase in the internal conduction loss in the regenerator. Thus, the average total entropy generation rate in the regenerator increases with the hot end temperature increase as shown in Figure 7. When comparing entropy generation for different working fluids. It is shown that the minimum entropy generation is recorded for helium compared to air and nitrogen working fluids.

As shown in table 3, proprieties of air and nitrogen are very close. Thus the average entropy generation rate for air and

nitrogen are nearly the same. So, helium provides the greatest amount of heat exchanged and air provides the smallest one. However, helium presents risks of gas leakage especially for high pressure. Helium is thus more advantageous over other gases.

**Table 3.** Proprieties of Stirling engine working fluids

Working fluid	Thermal conductivity (W.kg <sup>-1</sup> .K <sup>-1</sup> )	Specific heat (kJ.kg <sup>-1</sup> K <sup>-1</sup> )	Dynamic viscosity (x10 <sup>-5</sup> Pa.s)
Helium	0.1513	5.19	1.90
Nitrogen	0.02583	1.04	1.78
Air	0.0239	1.01	1.83

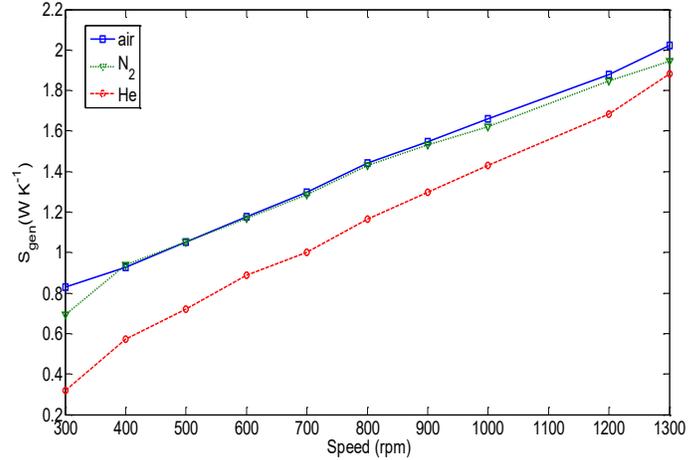


**Figure 7.** Average entropy generation rate evolution versus hot source temperature for different working fluids ( $N = 500$  rpm,  $T_{wk} = 300K$ ,  $\phi = 0.85$ ,  $L_r/D_r = 1.3$ , regenerator material is stainless steel)

Rotational engine speed effect

The rotational engine speed is a critical parameter for the Stirling engine performances. The numerical investigation of the average entropy generation rate in the regenerator showed that the average entropy generation rate increases with rotational engine speed. The increase of rotational engine speed reduces heat loss by external conduction through engine’s walls, reduces heat exchange time and increases viscous friction loss inside each heat exchanger. Results show that fluid friction irreversibility dominates over heat transfer irreversibility for high rotational engine speed.

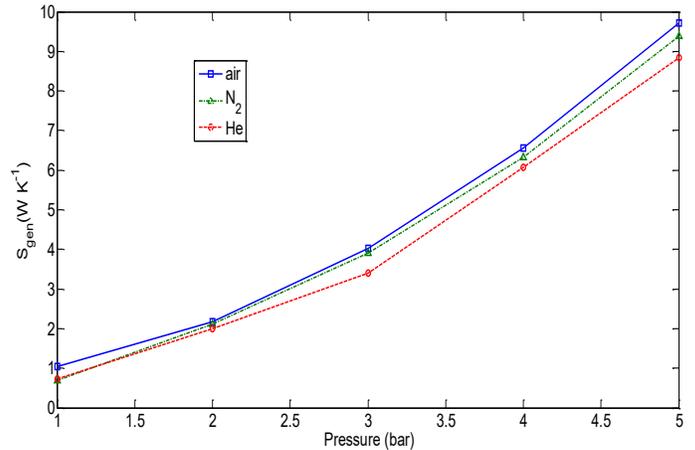
For different working fluids, as shown in Figure 8, it is clearly seen that helium presents the minimum of entropy generation compared to air and nitrogen. Among the three working gases, helium was found to have the highest heat capacity and thermal conductivity and the lowest density compared to air and nitrogen.



**Figure 8.** Average entropy generation rate versus rotational engine speed for different working fluids ( $T_{wh} = 1000$  K,  $T_{wk}=300K$ ,  $\phi = 0.85$ ,  $L_r/D_r = 1.3$ , regenerator material is stainless steel).

Initial pressure effect

The initial pressure is imposed to the engine before it starts operating. It represents the amount of working gas flowing within the engine. This mass is proportional to the increase of the initial pressure. The average of the total entropy generation rate in the regenerator is estimated respectively for air, nitrogen and helium for different initial filling pressure (Figure 9). Results show that average entropy generation rate increases with initial pressure. It is concluded that fluid friction irreversibility dominates over heat transfer irreversibility for high initial pressure.



**Figure 9.** Average entropy generation rate versus initial pressure for different working fluids ( $N = 500$  rpm,  $T_{wh} = 1000K$ ,  $T_{wk} = 300K$ ,  $\phi = 0.85$ ,  $L_r/D_r = 1.3$ , regenerator material is stainless steel)

**CONCLUSIONS**

Like all thermodynamic systems, entropy generation is generated from the irreversibility owing to heat transfer with

finite temperature gradients and the friction of fluid flow. Average entropy generation rate in the Stirling engine regenerator is associated with a number of parameters including the characteristics of the regenerator (geometry, porosity and material), the working conditions (temperature ratio between hot and cold source, initial pressure and rotational engine speed) and the thermo-physical properties of the working fluid. To determine the performance of the engine at different working conditions, an entropy analysis is performed. Our result shows that:

-The best regenerator qualities are those corresponding to the minimum entropy generation ( $\phi = 0.85$ ,  $L_r/D_r = 1.3$  and stainless steel as matrix material).

-The average Entropy generation rate in the regenerator is reduced when using Helium as working fluid.

- The average Entropy generation rate in the regenerator increases with rotational engine speed, hot end temperature and initial pressure.

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## APPENDIX A

### HEAT TRANSFER COEFFICIENT CALCULATION

The heat transfer coefficient in the regenerator is calculated as follow:

$$h_r = St c_p \left( \frac{\dot{m}}{A_r} \right) \quad (24)$$

where St is the Stanton number and it is calculated using the following correlation:

$$St = 0.46 Re^{-0.4} \left( \frac{1}{Pr} \right) \quad (25)$$

The heat transfer coefficients in cold and hot chambers are calculated as follow:

$$h_{1,2} = \frac{f \mu C_p}{2 D_{c,e} Pr} \quad (19)$$

where the friction factor f for circular tubes is calculated as follows:

$$\text{If } (Re < 2000) \text{ then } f = 16 \quad (20)$$

$$\text{If } (2000 < Re < 4000) \text{ then } f = 7.343 \cdot 10^{-0.4} Re^{1.3142} \quad (21)$$

$$\text{If } (Re > 4000) \text{ then } f = 0.0791 Re^{0.75} \quad (22)$$