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# Analysis of the unsteady behavior of an electrical heater using the lock-in technique: Comparison of theory and experiments

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## H I G H L I G H T S

- Definition of a characteristic dynamic parameter of an electrical heater.
- Experiments confirm the evolution of this value versus the angular frequency.
- Experimental results are obtained with a disturbance amplitude lower than 0.5 °C.

## A B S T R A C T

In the recent years, the lock-in technique has been tested in thermal engineering. Due to its high sensitivity for signal analysis, it has great potentialities for drift detection. Here the aim is more particularly fouling detection. The key point for this technique is the choice of the angular frequency of the exciting signal. It is a tradeoff between getting a high value of the modulus of a complex number, getting a stable value of this modulus in a time as short as possible, and leading to low variations of the temperature of the product passing through the thermal device. The aim of the present study is to show that experimental results obtained using a specialized measurement apparatus are in a good agreement with the theoretical developments for an electrical heater. The first part of the paper is dedicated to a brief presentation of the theory of the lock-in technique for lumped systems. The second part presents the developments for an electrical heater. The third part is dedicated to the presentation of the test rig. Then experimental results are given in the way they have been presented for the theoretical electrical heater. Even if some simplifications have been necessary for the theoretical developments, these experimental results show that the evolution of the modulus is similar to what is expected according to the angular frequency and to the mass flow. A final comparison between experimental results and theoretical computations shows a reasonable agreement for a particular set of parameters.

### Keywords:

Computational methods  
Heat exchangers  
Measurement and instrumentation  
Lock-in  
Experiment  
Heater

## 1. Introduction

The lock-in technique, also called synchronous detection, is widely used in signal processing [1–5] such as analysis of radar signals and thermography. Lock-in detection is used by Xing et al. [6] to obtain a good signal-to-noise ratio for accurate measurement of fine fibers thermal properties by the  $3-\omega$  technique. Its main feature is to be able to retrieve a signal when the amplitude of the signal is comparable to and even lower than the amplitude of the noise. In the very recent years, the lock-in technique has been

tested in thermal engineering on lumped systems, electrical heaters and heat exchangers [7,8]. Due to its high sensitivity for signal analysis, it has great potentialities for drift detection. Here the aim is more particularly fouling detection. The key point for this technique is the choice of the angular frequency of the exciting signal. It is a trade-off between getting a high value of the modulus of the complex number, getting a stable value of this modulus in a time as short as possible, and not disturbing the process (e.g. variations of the outlet temperature of the product passing through the thermal device). To be able to make this choice, it is important to know if what has already been presented is actually happening in practice. So, the aim of the present study is to show that experimental results obtained using a specialized measurement apparatus are in a good agreement with the theoretical developments for an electrical heater.

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The first part of the paper is dedicated to a brief presentation of the theory of the lock-in technique. The second part presents its application to lumped systems. The third part presents the developments for a theoretical cylindrical electrical heater in which the heat is generated by Joule effect. The fourth part is dedicated to the presentation of the test rig, including the measurement devices. The experimental results are given in the way they are presented for the theoretical electrical heater. It is completed by a comparison between experimental results and theoretical computations.

## 2. Basic principle of the lock-in technique

The presentation of the lock-in technique will be focused on the application to thermal devices for clarity reasons. The time signal to be analysed is supposed to be the sum of a constant value and of a purely sinusoidal signal:  $e_s(t) = A_s + B_s \sin(\omega_s t + \varphi_s)$ . Without loss of generality, the measurement noise is considered to be a pure sinusoidal function. This noise is added to the signal to be analysed:

$$e_1(t) = e_s(t) + n(t) = A_s + B_s \sin(\omega_s t + \varphi_s) + B_n \sin(\omega_n t + \varphi_n). \quad (1)$$

This signal is multiplied by a reference signal which is also purely sinusoidal:  $e_2(t) = e_r(t) = B_r \sin(\omega_r t + \varphi_r)$ . The result is a combination of sinusoidal functions:

$$\begin{aligned} V_2(t) = & A_s B_r \sin(\omega_r t + \varphi_r) + \frac{1}{2} B_r B_s \cos((\omega_r - \omega_s)t + \varphi_r - \varphi_s) \\ & - \frac{1}{2} B_r B_s \cos((\omega_r + \omega_s)t + \varphi_r + \varphi_s) + \frac{1}{2} B_r B_n \cos((\omega_r - \omega_n)t \\ & + \varphi_r - \varphi_n) - \frac{1}{2} B_r B_n \cos((\omega_r + \omega_n)t + \varphi_r + \varphi_n) \end{aligned} \quad (2)$$

Noting that the average value of any sinusoidal function over a long duration is zero, the mean value of  $V_2(t)$  over a long enough time is in the specific case characterized by  $\omega_r = \omega_s$ ,

$$\overline{V_2} = \frac{1}{2} B_r B_s \cos(\varphi_r - \varphi_s). \quad (3)$$

This mean value can be obtained either by a simple calculation or by using a low-pass filter having a sufficiently sharp transition band. When using a two-channel lock-in amplifier, a second term is computed using a 90° phase shifter applied to the reference signal. At the output of the low-pass filter of the second channel, the following value is obtained:

$$\overline{V_3} = \frac{1}{2} B_r B_s \cos\left(\varphi_r - \varphi_s - \frac{\pi}{2}\right) = \frac{1}{2} B_r B_s \sin(\varphi_r - \varphi_s). \quad (4)$$

It is possible to consider that these two values are the real and imaginary parts of a complex number  $Z$ . The modulus of this complex number is  $M = 1/2 B_r B_s$ , and its argument is defined by  $\phi = \varphi_r - \varphi_s$ . It is important to note that the preceding calculation can be extended to a more complex noise signal. As the time signal to be analyzed has a finite duration, the noise signal can be represented using a Fourier series [9]:

$$n(t) = \sum_{k=-\infty}^{k=+\infty} B_k \sin(2\pi k f_p t + \varphi_k) \quad (5)$$

where the fundamental frequency  $f_p$  is the reciprocal of the processing time,  $\varphi_k$  is the phase shift of each component, and  $B_k$  is the amplitude of each component. The procedure described in the case

of the sinusoidal noise signal can then be used. More detailed information is available in several publications [10–12].

Note that neither the modulus nor the argument of the complex number obtained by this procedure depends on the constant value  $A_s$ .

It is also very important to note that if the signal to be analyzed, instead of being the sum of a constant value and of a pure sinusoidal function, is the sum of a linear function of time and of a pure sinusoidal function, the complex number obtained using the preceding procedure can be the same under specific conditions, as shown hereafter.

In fact, it is sufficient to show that the average value of the product of a linear function and a sinusoidal function equals zero after a long enough time under specific conditions:

$$\frac{1}{t_p} \int_{t_0}^{t_0+t_p} C_s t B_r \sin(\omega_r t + \varphi_r) dt \stackrel{?}{=} 0$$

When the processing time  $t_p$  is a multiple of the period of the reference signal and when the phase of the reference signal is linked to the angular frequency and the initial time by  $\cos(\omega_r t_0 + \varphi_r) = 0$  the following relations are obtained:

$$\cos(\omega_r (t_0 + t_p) + \varphi_r) = \cos(\omega_r t_0 + \varphi_r) = 0 \quad (6)$$

and

$$\begin{aligned} \int_{t_0}^{t_0+t_p} t \sin(\omega_r t + \varphi_r) dt &= \left[ \frac{1}{\omega_r} t \cos(\omega_r t + \varphi_r) \right]_{t_0}^{t_0+t_p} \\ &+ \int_{t_0}^{t_0+t_p} \frac{1}{\omega_r} \cos(\omega_r t + \varphi_r) dt \\ &= \left[ \frac{1}{\omega_r} t \cos(\omega_r t + \varphi_r) \right]_{t_0}^{t_0+t_p} = 0 \end{aligned} \quad (7)$$

From a practical point of view, these criteria are not an issue. The initial time is usually zero. Hence, it is sufficient to choose  $\varphi_r = 90^\circ$ , which is just an arbitrary choice. The only constraint is to use a processing time that is a multiple of the period of the reference signal.

The way to use the lock-in technique for the thermal analysis of a system is then very simple. It is necessary to excite the system in order to get a response that is similar to the afore mentioned signals. This response is then analyzed using the mathematical procedure that has been just presented. The result is an image of the state or behavior of the device. So, if there is a variation in the state of the thermal device (fouling, leakage, etc.), there will be a variation of the result. This variation is easily detected by e.g. a statistical test.

## 3. Theoretical applications

To facilitate the understanding of the development for the electrical heater, this part presents also the development of the calculations for a lumped system.

### 3.1. Lumped system

The governing equation for a lumped system in contact with a fluid (convection), having an internal source of energy  $\dot{q}$ , is:

$$mc \frac{dT}{dt} = hA(T_f - T) + \dot{q}V \quad (8)$$

if radiation is included in the surface heat transfer coefficient  $h$ ; where  $A$  is the area of the exchanging surface,  $V$  is the volume of the system,  $m$  is the mass of the system,  $c$  is the specific heat of the system material,  $t$  is time,  $T_f$  is the temperature of the convecting fluid, and  $T$  is the temperature of the system, which is assumed to be uniform within the whole system. Note that the fluid temperature is supposed to remain constant.

It is convenient to introduce the difference between the temperature of the system and the temperature of the convecting fluid  $\theta = T - T_f$ , and the characteristic response time  $\tau = \rho c V / hA$ . Then the governing equation is:

$$\frac{d\theta}{dt} + \frac{\theta}{\tau} = \frac{\dot{q}}{\rho c} \quad (9)$$

The analytical solution of this equation for an alternative heat generation  $\dot{q} = \dot{q}_m \cos(\omega_r t)$  has been given in Ref. [7]. It leads to the following expression for the gain:

$$G = \frac{|\theta|}{|\dot{q}|} = \frac{\tau}{\rho c} \frac{1}{\sqrt{1 + (\omega_r \tau)^2}} \quad (10)$$

This expression can be obtained using the procedure that has to be followed for the electrical heater. This procedure is different from what has been presented in Ref. [7] and is much faster. The solution of Eq. (9) in the forced harmonic regime can be written as the real part of a complex temperature

$$\theta = T \exp(i\omega_r t).$$

Its derivative with respect to time is then  $\dot{\theta} = T i \omega_r \exp(i\omega_r t)$ . The heat generation has to be written in the same way  $\dot{q} = \dot{Q} \exp(i\omega_r t)$ . Note that  $T$  and  $\dot{Q}$  are the complex moduli of the temperature and of the heat generation respectively. Introducing these three expression in Eq. (9) leads to

$$\begin{aligned} T(i\omega_r \tau + 1) \exp(i\omega_r t) &= \frac{\dot{Q}\tau}{\rho c} \exp(i\omega_r t) \quad \text{that is equivalent to} \\ T(1 + (\omega_r \tau)^2) &= \frac{\dot{Q}\tau}{\rho c} (1 - i\omega_r \tau) \quad \text{or} \\ \frac{T}{\dot{Q}} &= \frac{\tau}{\rho c} \frac{1}{(1 + (\omega_r \tau)^2)} (1 - i\omega_r \tau) \end{aligned} \quad (11)$$

The gain is then

$$G = \left| \frac{T}{\dot{Q}} \right| = \frac{\tau}{\rho c} \frac{1}{\sqrt{1 + (\omega_r \tau)^2}} \quad (12)$$

which is strictly equivalent to what is given by Eq. (10).

This expression shows that it will be necessary to plot the evolution of the gain, or of the modulus, versus the product of the angular frequency and a representative time. The latter is the response time for a lumped system.

### 3.2. The electrical heater theoretical model

The theoretical electrical heater under consideration in this study is a tube where Joule effect takes place in the tube wall. This kind of circulation heater has been identified as second order

systems using neural networks [13]. The following assumptions are needed to write the governing equations:

- A1: being a single tube, the axial thermal resistance of the tube wall is much higher than the radial thermal resistance of the tube wall
- A2: the temperature difference between the inlet and the outlet of the heater is small (in practice it was lower than 3 °C)
- A3: the heat losses to the surroundings are negligible
- A4: the fluid temperature  $T_f$  depends only on the axial direction (the bulk temperature is representative)

The first two assumptions leads to the fifth one:

- A5: the temperature of the tube wall  $T_w$  is only a function of the axial location (negligible temperature gradient in the radial direction)

Under these assumptions, the local governing equations (per unit length) can be written as follows:

$$hA(T_w - T_f) + M_w c_w \frac{\partial T_w}{\partial t} = J \quad (13)$$

$$hA(T_f - T_w) + M_f c_f \frac{\partial T_f}{\partial t} = \dot{m}_f c_f \frac{\partial T_f}{\partial x} \quad (14)$$

It is then convenient to introduce a residence time per unit length  $\tau = M_f / \dot{m}_f$ , the equivalent of a Number of Transfer Units per unit length  $\alpha = hA / \dot{m}_f c_f$ , the ratio  $\eta = M_w c_w / M_f c_f$ , the temperature differences  $\theta_w = T_w - T_0$  and  $\theta_f = T_f - T_0$  where  $T_0 = T_w(t=0) = T_f(t=0)$ . The governing equations become

$$\frac{\partial \theta_w}{\partial t} = -\frac{\alpha}{\tau \eta} (\theta_w - \theta_f) + \frac{J}{M_w c_w} \quad (15)$$

and

$$\frac{\partial \theta_f}{\partial t} = -\frac{1}{\tau} \frac{\partial \theta_f}{\partial x} + \frac{\alpha}{\tau} (\theta_w - \theta_f) \quad (16)$$

As done for the lumped system, the temperature differences are represented as complex values,  $\theta_w = T_w \exp(i\omega_r t)$ ,  $\theta_f = T_f \exp(i\omega_r t)$ , as well as the heat generation  $\dot{Q} \exp(i\omega_r t)$ , where  $T_f$ ,  $T_w$  and  $\dot{Q}$  are the complex moduli of the fluid temperature, the wall temperature, and the heat flux respectively.

Then, Eq. (15) can be written as  $T_w(i\omega_r + \alpha/\tau \eta) \exp(i\omega_r t) = \alpha/\tau \eta T_f \exp(i\omega_r t) + \dot{Q} \exp(i\omega_r t)$

The product of this equation by  $\alpha/\tau \eta - i\omega_r$  leads to

$$T_w \omega_r^2 + \left( \frac{\alpha}{\tau \eta} \right)^2 = \frac{\alpha}{\tau \eta} T_f \left( \frac{\alpha}{\tau \eta} - i\omega_r \right) + \dot{Q} \left( \frac{\alpha}{\tau \eta} - i\omega_r \right)$$

and finally to

$$T_w = \frac{\alpha}{\eta} T_f \frac{\left( \frac{\alpha}{\eta} - i\omega_r \tau \right)}{\left( (\omega_r \tau)^2 + \left( \frac{\alpha}{\eta} \right)^2 \right)} + \dot{Q} \frac{\left( \frac{\alpha}{\eta} - i\omega_r \tau \right)}{\left( (\omega_r \tau)^2 + \left( \frac{\alpha}{\eta} \right)^2 \right)} \quad (17)$$

In the same manner, Eq. (16) is written:

$$\left[ T_f \left( i\omega_r + \frac{\alpha}{\tau} \right) + \frac{1}{\tau} \frac{dT_f}{dx} \right] = \frac{\alpha}{\tau} T_w \quad (18)$$

It is now convenient to introduce

$$\beta = \frac{i\omega_r\tau + \alpha}{\tau} - \frac{\alpha}{\tau} \frac{\alpha}{\eta} \frac{\left(\frac{\alpha}{\eta} - i\omega_r\tau\right)}{\left((\omega_r\tau)^2 + \left(\frac{\alpha}{\eta}\right)^2\right)} \quad (19)$$

and

$$\gamma = \alpha \frac{\left(\frac{\alpha}{\eta} - i\omega_r\tau\right)}{\left((\omega_r\tau)^2 + \left(\frac{\alpha}{\eta}\right)^2\right)} \quad (20)$$

In that case, Eq. (18) is written as follows:  $[T_f\beta + 1/\tau dT_f/dx] = \gamma \dot{Q}$

The solution of this equation is  $T_f = \gamma \dot{Q}/\beta[1 - \exp(-\beta \tau x)]$  as  $T_f(x = 0) = 0$ . The gain that is looked for is then obtained at the outlet of the heater:  $G = |\gamma/\beta[1 - \exp(-\beta \tau L)]|$  or

$$M = \left| \dot{Q} \right| \left| \frac{\gamma}{\beta} [1 - \exp(-\beta \tau L)] \right| \quad (21)$$

note that this equation shows that the modulus is proportional to the amplitude of the exciting signal. Note also that is case of fouling, i.e. variation of  $\alpha$  through the modification of the convection heat transfer coefficient  $h$ , the two variables  $\beta$  and  $\gamma$  will vary. But the determination of the consecutive variation of the modulus is absolutely not straight forward; but is easily computed when all parameters are known.

In the case of the heater, the representative time is the residence time. It is then possible, for a specific heater, to plot the evolution of the modulus versus the product  $\omega_r\tau$ . This heater is a stainless steel tube of inner diameter 25 mm, 14 mm thick, 500 mm long. Fig. 1 shows this evolution for two mass flow rates (0.092 kg/s and 0.135 kg/s) and for a range of flow rates for two given angular frequencies.

As a side finding, it is interesting to show that the results given by the lock-in technique are in a very good agreement with the results obtained when considering second order systems (Fig. 2).

The aim of the present study is to show that experiments lead to similar evolutions.

#### 4. The test stand and measurements

In the framework of a research work dedicated to fouling detection, a specific test rig has been designed. Fig. 3 shows a

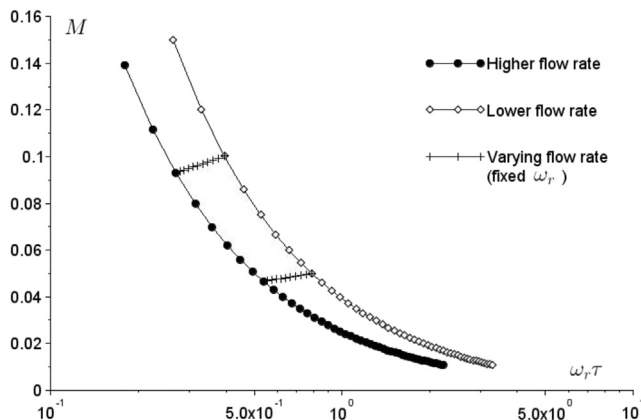


Fig. 1. Evolution of the modulus for a theoretical electrical heater.

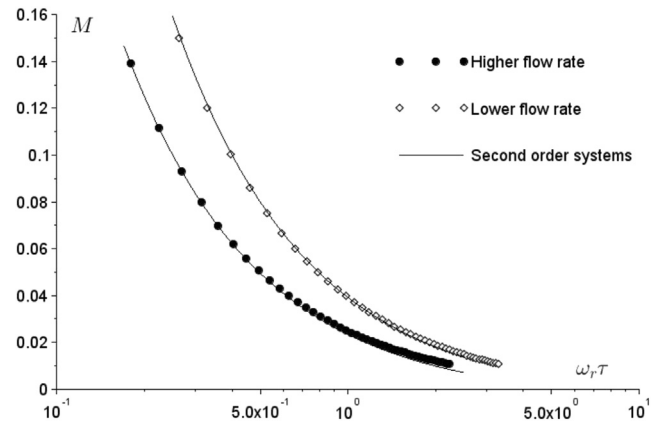


Fig. 2. Evolution of the modulus for a theoretical electrical heater and for second order systems.

schematic of the part of the test rig that includes the electrical heater, and a view of the heater.

It is important to note that the heater is built in a way that it is possible to dismantle it for inspection. Its cross section is shown in Fig. 3. Its length is 500 mm. It is heated by two set of cartridge heaters; one set per half heater. It is made of aluminum. It is then different from the electrical heater used for the theoretical developments.

The flow rate is controlled by the rotational speed of the pump (DC current). As the authors intend to apply the technique in industry, the flow meter is a standard paddle-wheel device. The heat generated by the electrical resistances inserted in the heater is controlled by the power control device. The power delivered is proportional to the voltage at the input of the power control device. To generate an oscillating temperature at the outlet of the heater, an oscillating voltage is sent to the power control device. This voltage is the sum of a constant (controlled) voltage and a sinusoidal voltage generated by the lock-in amplifier. The heat rate is then the sum of a constant power and a pure sinusoidal power. The result is, although the volume of the tank is quite large, an increasing temperature at the inlet of the heater. This increase is very slow and very close to a linear function of time. During the experiments this temperature varied from room temperature to about 50 °C.

Hence, as shown in the previous part of this paper, it is sufficient to study the temperature at the outlet of the heater. The lock-in amplifier used in this study is an SR830 model from Stanford Research System (Fig. 4) [14].

Only one signal, the temperature at the outlet of the heater, is linked to the “signal inputs”. The “CH1 Display” shows the modulus of the complex number computed using the reference signal (Sine output) and the input signal. The “CH2 Display” shows the phase lag between the reference signal and the input signal. The “Ref Display” can show the amplitude, the frequency, or the phase of the reference signal.

The “Analog Outputs” are copies of the CH1 and CH2 displays. They are recorded as well as the reference signal and the input signal by an external recorder (GRAPHTEC DM 3100). This recorder also logs the flow rate and the constant voltage. Whatever the frequency used for the reference signal, the sampling frequency of the recorder is chosen as 1 Hz. This frequency is much higher than the frequency that will be used for the analysis of the heater: the range for the reference signal is [9, 100] mHz.

Fig. 5 shows, the evolution of the modulus of the complex number for this range of the angular frequency and for two flow rates (0.092 kg/s and 0.135 kg/s as done in the theoretical

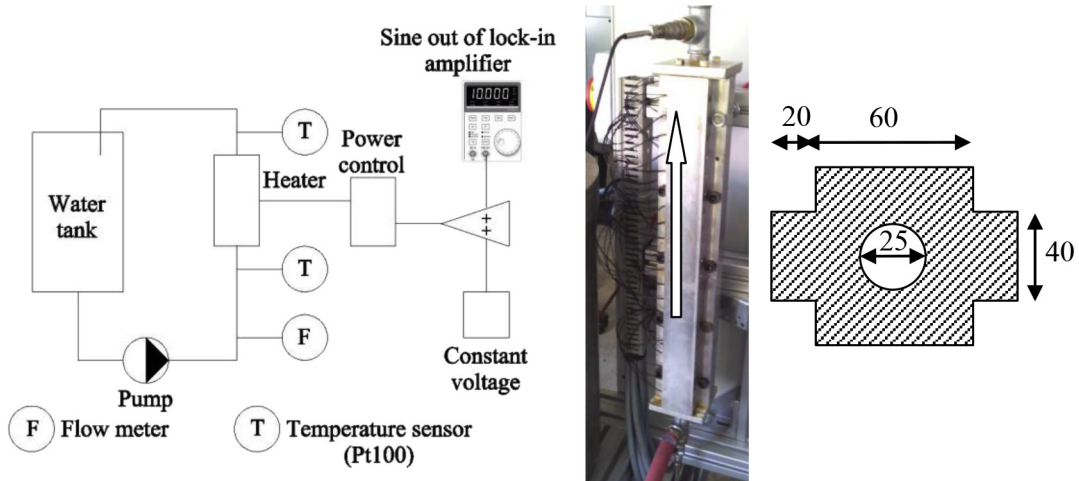


Fig. 3. Schematic representation of the test rig, picture of the electrical heater and schematic cross section of the heater (dimensions in mm).

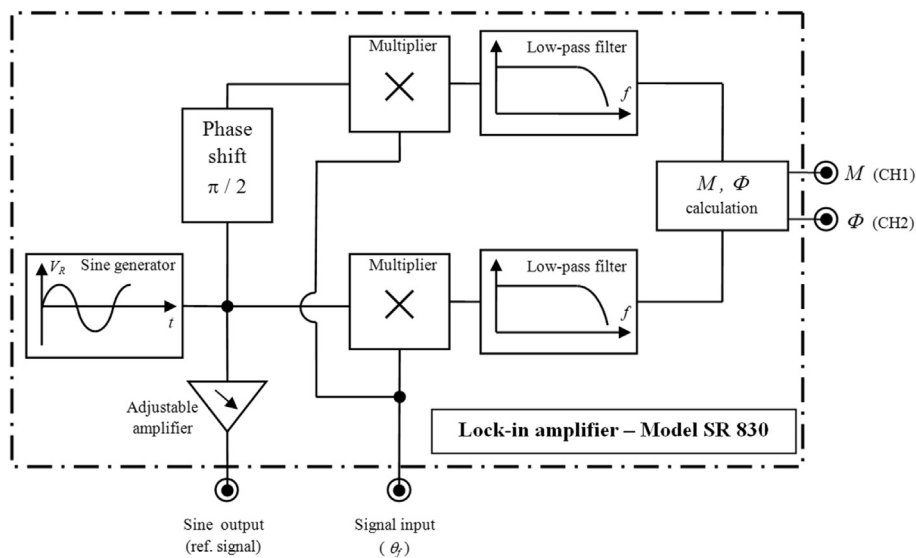


Fig. 4. Simplified block diagram of the SR 830 lock-in amplifier.

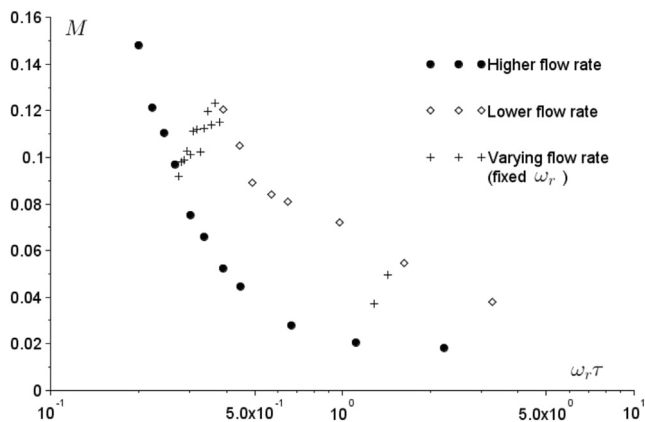


Fig. 5. Evolution of the modulus for the electrical heater.

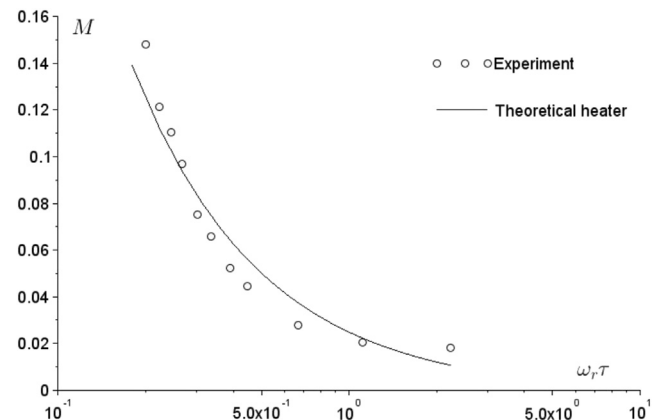


Fig. 6. Comparison of the modulus between experimental values and theoretical results.

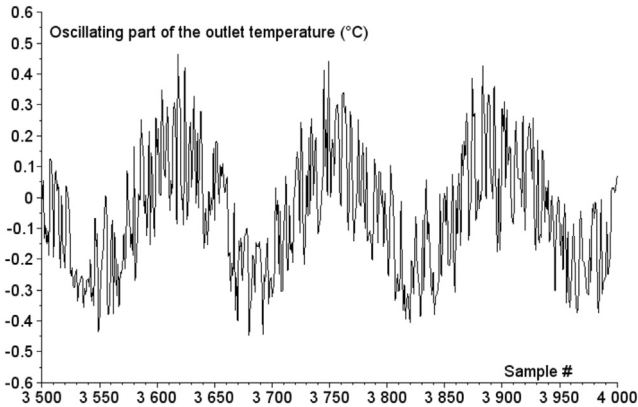


Fig. 7. Oscillating part of the temperature at the outlet of the heater.

development), as well as this evolution for varying flow rates using a constant angular frequency.

It can be observed that the evolution of the modulus is very similar to what is shown in Fig. 1.

It is possible to find a set of characteristics of a theoretical heater that leads to a quite satisfactory comparison between experimental values and theoretical results (Fig. 6). It is reminded here that the experimental results are obtained using an electrical heater that is not a single thin wall tube; this explains the fact that it is not possible to perfectly fit the data.

The discrepancy that is observed when the angular frequency is low is due to the fact that it should be necessary to get a much higher number of samples for the technique to be efficient. But for practical reasons, it is not possible to carry out the experiments for more than 8 h. In the case a very small angular frequency the time needed for the experiment would be counted in days.

It must be noted that all the results presented here have been obtained when the oscillating part of the temperature at the outlet of the heater has an amplitude close to 0.3 °C as shown in Fig. 7. This amplitude is of the usual order of magnitude of the variations due to the control system in an industrial process.

The authors have chosen to use a specialized device to check the applicability of the theoretical developments. Nevertheless, it is important to note that a simpler choice is possible, at least when considering an off line treatment. Fig. 8 shows the comparison of the results obtained by the mathematical analysis of the recorded signals (according to the principles presented in the first part of the

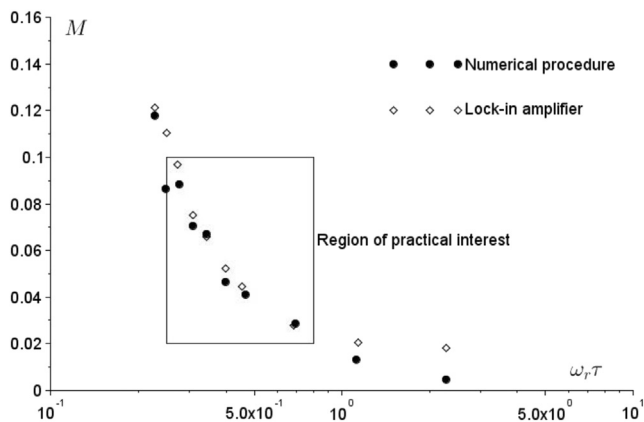


Fig. 8. Comparison of the results obtained by a numerical procedure and by the lock-in amplifier.

paper) and the results computed by the lock-in amplifier. Note that this is only possible if the reference signal is also recorded to be sure to get the exact value of the angular frequency.

It can be noted that the results compare quite well, mostly in the practical range when the value of the modulus is not too low, and that the angular frequency is not too small.

## 5. Conclusions

It has been shown that the lock-in technique applied to an electrical heater leads to experimental results that are very close to the results obtained when considering a simple theoretical device. It can then be concluded that this method is applicable to drift detection such as fouling.

A future application of the lock-in technique will be to heat exchangers. This will have to be linked to a theoretical development. The latter will surely be much more difficult due to the fact that the governing equations are more complicated.

## Acknowledgements

Special thanks are due to Marc Lippert for his help in the design of the test rig, and to Jess Schiffler for his help in the LabVIEW supervising software.

## Nomenclature

$A$	dummy constant when used with a subscript, (–) or convecting area ( $m^2$ ) or convective area per unit length (m)
$B, C$	dummy constants (–)
$c$	specific heat ( $J kg^{-1} K^{-1}$ )
$e_s$	dummy signal to be analyzed (–)
$f$	frequency ( $s^{-1}$ )
$G$	gain (depends on the application)
$h$	convection heat transfer coefficient ( $W m^{-2} K^{-1}$ )
$i$	square root of –1
$J$	heat rate (Joule effect) ( $W m^{-1}$ )
$k$	index (–)
$L$	length of the heater (m)
$M$	modulus of $Z$ or mass per unit length ( $kg m^{-1}$ )
$m$	mass (kg)
$\dot{m}$	mass flow rate ( $kg s^{-1}$ )
$n$	noise function (–)
$\dot{q}$	volumic heat rate ( $W m^{-3}$ )
$\dot{Q}$	complex volumic heat rate ( $W m^{-3}$ )
$T$	temperature (K)
$t$	time (s)
$V$	temporary dummy function (–) or Volume ( $m^3$ )
$Z$	dummy complex number (–)

## Greek symbols

$\alpha$	equivalent to a number of transfer units (–)
$\beta$	defined by eq. (10) ( $s^{-1}$ )
$\phi$	argument of $Z$ (rd)
$\gamma$	defined by eq. (11) (m)
$\eta$	ratio of heat capacities (–)
$\varphi$	phase (rd)
$\theta$	temperature difference (K)
$\rho$	density ( $kg m^{-3}$ )
$\tau$	response time (lumped system) (s) or residence time (heater) ( $s m^{-1}$ )
$\omega$	angular frequency ( $s^{-1}$ )

### Subscripts

$0$	initial time
$f$	fluid
$k$	index $k$
$n$	noise
$p$	processing time
$r$	reference signal
$s$	signal to be analyzed
$w$	tube wall
$\sim$	oscillating

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