



HAL
open science

Logistic network modeling and optimization: An approach based on $(\max,+)$ algebra and coloured Petri nets

Yassine Idel Mahjoub, El Houcine Chakir El Alaoui, Ahmed Nait-Sidi-Moh

► **To cite this version:**

Yassine Idel Mahjoub, El Houcine Chakir El Alaoui, Ahmed Nait-Sidi-Moh. Logistic network modeling and optimization: An approach based on $(\max,+)$ algebra and coloured Petri nets. Computers & Industrial Engineering, 2021, 158, pp.107341. 10.1016/j.cie.2021.107341 . hal-03527989

HAL Id: hal-03527989

<https://uphf.hal.science/hal-03527989v1>

Submitted on 13 Jun 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

Logistic network modeling and optimization: An approach based on (max,+) algebra and coloured Petri nets

Yassine Idel Mahjoub^a, El houcine Chakir El-Alaoui^b, Ahmed Nait-Sidi-Moh^c

^aUPHF, INSA HdF, UMR 8201-LAMIH, 59313, Valenciennes, France

^bCadi Ayyad University-FSTG, LSET laboratory, 40000, Marrakesh, Morocco

^cUniversity of Picardie Jules Verne-INSSET, LTI laboratory, 02100 Saint-Quentin, France

Abstract

The present article deals with supply chain management considered as a Discrete Event System (DES). We introduce a new modeling approach based on linear and non-stationary (max, +) equations obtained from a Timed Coloured Petri Nets (TCPN) describing the studied system's behavior. Our contribution does not lie only on the modeling of the dynamics associated with vehicles and their timetables at different sites (e.g., suppliers, warehouses, customers), but also on the evaluation of loading, unloading, and delivery times of products from suppliers to customers, taking into account their appropriate characteristics (e.g., number, nature, destination). Moreover, a control approach is performed to optimize the number of vehicles to deploy on the network by ensuring product storage times below a given threshold at some specific sites considered strategic (e.g., delivery hubs). The developed models can be used as a decision-making system for logistic companies to result in efficient supply chain management. The developed models are tested and validated on several configurations and scenarios to demonstrate our approach's applicability.

Keywords: Modeling, Performance evaluation, Control, Coloured Petri nets, (Max, +) algebra, Supply Chain, Logistics, Transportation, Delivery.

This paper is a extension of the research work presented at the CIE49 (best paper award).

1. Introduction

Nowadays, the importance of supply chain management (SCM) continues to evolve [1]. In fact, with the development of the economy, the transportation of production materials and products is frequent.

E-mail addresses: yassine.idelmahjoub@uphf.fr (Yassine Idel Mahjoub), e.chakirelalaoui@uca.ma (El houcine Chakir El-Alaoui), ahmed.nait@u-picardie.fr (Ahmed Nait-Sidi-Moh).

As a result, the transportation and delivery cost becomes one of the main parts of companies costs [2]. Indeed, it is necessary to ensure that these companies can perform their key activities and fulfill customer orders under specific quality requirements [3]. For such reason, several works and researches have been realized to manage and control logistic networks to reduce companies costs (e.g., transport, storage, and delivery) and reduce gas emissions, and ensure industrial performances [4,1,5].

With analogy to other Discrete Event Systems (DES) classes such as production systems and computer systems, logistic networks (LN) can be considered as a dynamic DES governed by different phenomena, including synchronization, parallelism, competition, and resource sharing. The diversity and the complexity of these phenomena make the study of these classes of DES more complicated and challenging.

To cope with these complicated issues, great efforts have been devoted to modeling these phenomena to solve supply chain problems. In the literature, various methods and models have been proposed for modeling logistic networks. These models can be grouped into three categories: deterministic models [6] where all the parameters are known, stochastic models [7,4] where at least one parameter is unknown but follows a probabilistic distribution, and models based on simulation [8,9]. First of all, deterministic models are insufficient when dealing with the dynamic and stochastic characteristics of the supply chain system, which are due, for example, to demand fluctuations, traffic conditions, etc. Accordingly, several studies explore the stochastic behavior of logistic networks [10,11,12,13]. These studies mostly involve metaheuristic optimization algorithms such as genetic algorithms (GA) [10], evolutionary programming (EP) [11], and genetic programming (GP) [14]. These models do not provide reliable (and in some cases optimal) solutions and states of the studied system at every moment. Furthermore, simulation models (e.g., spreadsheet simulation, discrete-event simulation, business games) do not provide a formal (i.e., mathematical equations with formal syntax and semantics) representation of the studied system and, more importantly, a ‘closed-form’ solution. Instead, the simulation analysts experiment with different input values and model structures to see what happens to the output.

For all these reasons, there is an increasing demand for formal models that produce suitable solutions to model and control these systems, taking into account their stochastic behavior. Petri nets [15,16] combined with dioid algebra [17,18] provide a relevant approach that fulfills this aim [19,20] by providing precise graphical models for the analysis and the simulation of the studied system and mathematical equations that describe its dynamical evolution.

This paper presents a formal approach for modeling and controlling logistic networks. The proposed approach is based on TCPN and $(\max, +)$ algebra. The developed supply chain models explicitly incorporate transport operations of vehicles and goods delivery from suppliers to customers. This paper’s originality lies firstly in combining TCPN and $(\max, +)$ algebra to provide formal, precise, and reliable solutions for management of logistic issues in a supply chain. It provides both appropriate graphical

models that are more faithful to reality and mathematical equations describing logistic networks' dynamical evolution. A second contribution consists of expressing the TCPN model into non-stationary (max, +) models to formally evaluate the studied system metrics. The difficulty lies in solving and expressing all the encountered conflicts, related to resources sharing and situations of choice, in the TCPN model. For this reason, mathematical functions, called routing functions, are explicitly introduced and defended. Afterwards, some quantitative and qualitative properties of the system are evaluated based on developed models. We mainly focused on the departure/arrival of vehicles from/to various network sites (e.g., supplier, customer, logistic hub) and loading, unloading, and storage times of each product at every network site, especially within logistic hubs. The influence of vehicles' limited capacity on transportation and delivery of goods is also addressed in this paper. Finally, the timetabling problem is considered to minimize logistics companies' costs (i.e., by optimizing the number of vehicles deployed on the network) while maintaining product storage time below a given threshold. We call, in what follows ST this minimum Storage Time representing a storage time threshold.

The developed study in this paper presents three main advantages. First, we consider more realistic constraints (e.g., vehicles' finite capacity, network structure, limited vehicle fleet). Second, the developed models are modular and easily extendable for more enormous logistic networks (i.e., worldwide logistic network). Finally, the proposed approach provides efficient and reliable solutions for both customers and logistics operators/companies. The developed models are validated on several scenarios and configurations of the studied logistic network. This paper extends and generalizes the research work presented in CIE49 while considering more large networks instead of small ones. All the proposed models and algorithms in the initial work are now extended and generalized.

The remainder of this paper is organized as follows. Section 2 presents the studied logistic network, its TCPN model and its mathematical modeling using (max, +) algebra. The control approach for the optimal management of the studied network is presented in section 3. An illustrative numerical example is presented in section 4. The last section concludes the paper and presents some perspectives.

2. Logistic network description and modeling

2.1. Description of the studied supply chain

The present section is dedicated to describing the studied logistic network, composed of suppliers, warehouses, and customers, as illustrated in Figure 1. More precisely, this network contains several paths denoted P_i ($i > 0$). In fact, a path P_i is defined by its first supplier and a final customer (denoted S_1^i and $S_{p_i}^i$ respectively), passing by various sites in between namely warehouses (or logistic hubs) ($S_{w_i}^i$) and intermediate suppliers ($S_2^i, \dots, S_{p_i-1}^i$). Two examples of paths are provided in Figure 1. The

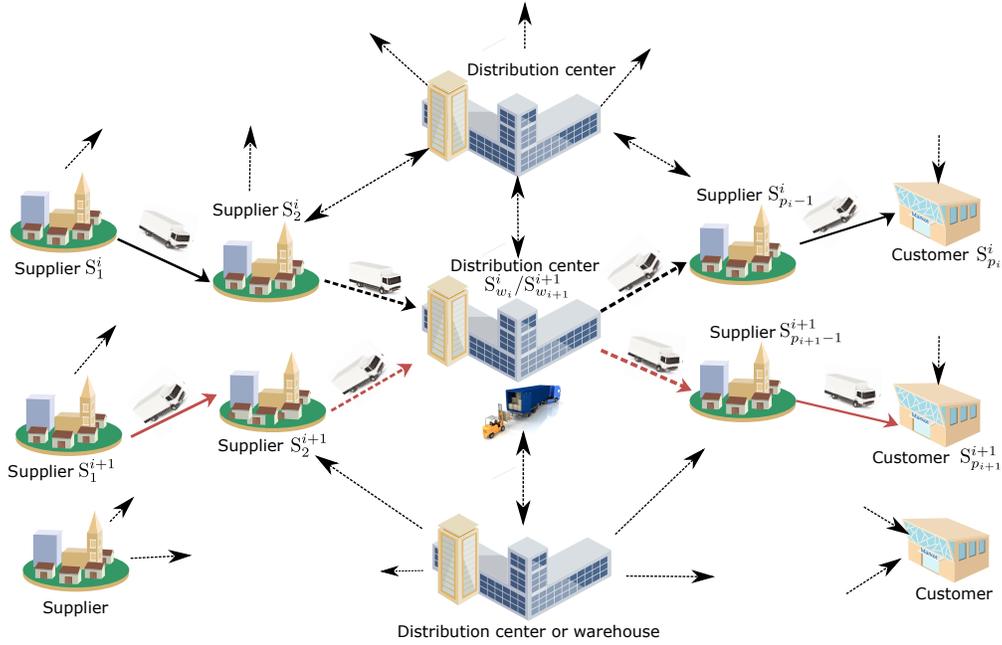


Figure 1: The studied logistic network

first path P_1 is illustrated by black arrows, connecting supplier S_1^i to customer $S_{p_i}^i$ and the second one P_2 is illustrated by red arrows connecting supplier S_1^{i+1} to customer $S_{p_{i+1}}^{i+1}$. Every path is supposed to be served by a fixed number of vehicles n_i for a given period. We note that vehicles are characterized by their limited capacity $Cp_{j_i}^i$ (with $j_i \in 1, \dots, n_i$), which refers to the maximum number of goods (products or pallets) that can be loaded inside. In the present contribution, we suppose that all products have the same size. Moreover, for every path in the studied supply chain, we define τ_2^i (resp. $\tau_4^i, \dots, \tau_{2p_i-2}^i$) as the travel time between the network sites S_1^i (resp. $S_2^i, \dots, S_{p_i-1}^i$) and S_2^i (resp. $S_3^i, \dots, S_{p_i}^i$). We also define τ_1^i (resp. $\tau_3^i, \dots, \tau_{2p_i-1}^i$) as the stop time of vehicles at S_1^i (resp. $S_2^i, \dots, S_{p_i}^i$) supposed fixed in this work, which include the mean time needed to load/unload a product to/from a vehicle, denoted δ . Furthermore, we assume that customers' orders are given randomly, and that every product's destination is predetermined. Knowing the product destination enables to evaluate the suitable loading, unloading and storage times of each product at every site. For instance, and based on the provided network presented in Figure 1, in order to send a product from the supplier S_1^i to the customer $S_{p_{i+1}}^{i+1}$, it will be loaded and transported by vehicles on P_i , placed in the warehouse $S_{w_i}^i$, and then will be transported by vehicles on P_{i+1} to be finally delivered to its destination ($S_{p_{i+1}}^{i+1}$). We precise that the proposed approach can be easily extended to a global network with several path and logistic hubs. In addition, the structure (or paths) of the studied network can be changed at any moment. It is worth noting that in the best of our knowledge, through the realized state of the art, we are the first researchers who developed this new methodology approach combining TCPN and (max,

+) algebra to model, analyze, and control a supply chain seen as a DES. To argue and support this affirmation, a literature review was carried out. The adopted method follows the guidelines proposed in²¹. It consists of four steps: (1) determine inclusion/exclusion criteria, (2) select databases, (3) define keywords and construct database queries, and finally (4) select pertinent literature and report the results. These steps are detailed in what follows.

Step 1. First of all, we consider a set of well-defined inclusion and exclusion criteria described below.

- Inclusion criteria: (i) papers investigating our research problem: logistics modeling and control, (ii) High-quality articles published in peer-reviewed academic journals, and (iii) papers published over the past fifty years (i.e., from 1970 to 2020).

- Exclusion criteria: papers that are written in a language other than English (i.e., logistics studies' dominant language).

Step 2. In this step, we select databases: First of all, Google Scholar and WoS, one of the famous and important databases, were used to search for a set of interest publications. To avoid missing any publication from peer-reviewed academic journals, the previous set was completed with publications from other databases of international editors, namely ScienceDirect, IEEEXplore, Taylor & Francis, Wiley Online Library, and Springer.

Step 3. In this step, we define keywords and construct database queries: this task started with searching publications having the primary terms (or topic): Logistics, modeling, control, Petri nets, and (max, +) algebra. Afterwards, a refinement searching in which correlated keywords identified from the first review were added, namely models/methods, supply chain management, optimization.

Step 4. By applying the criteria mentioned above, no article has been identified. Therefore, we could confirm that TCPN and (max, +) algebra are combined for the first time to model and control a supply chain.

The TCPN model of the studied supply chain is developed in the following section.

2.2. The TCPN model of the studied supply chain

The proposed graphical model describes, in a modular way, the supply chain presented in Figure 1. In fact, we develop models for the path P_i (with $i > 0$). For more details about the basic concepts of TCPN, the reader may refer to [22, 16]. TCPN add another dimension to tokens compared to basic Petri nets. Tokens now can represent different objects by introducing the colour concept. For example, we can use different tokens to represent vehicles ($\langle ji \rangle$ in Figure 2) or sites ($\langle si \rangle$ in Figure 2). These different tokens can then be used to determine which of the multiple validated transitions can be fired. For example, in Figure 2, the transition $x^{i,2}$ will be fired after the firing of $x^{i,1}$ plus the time associated with P_1^i (i.e., τ_1^i).

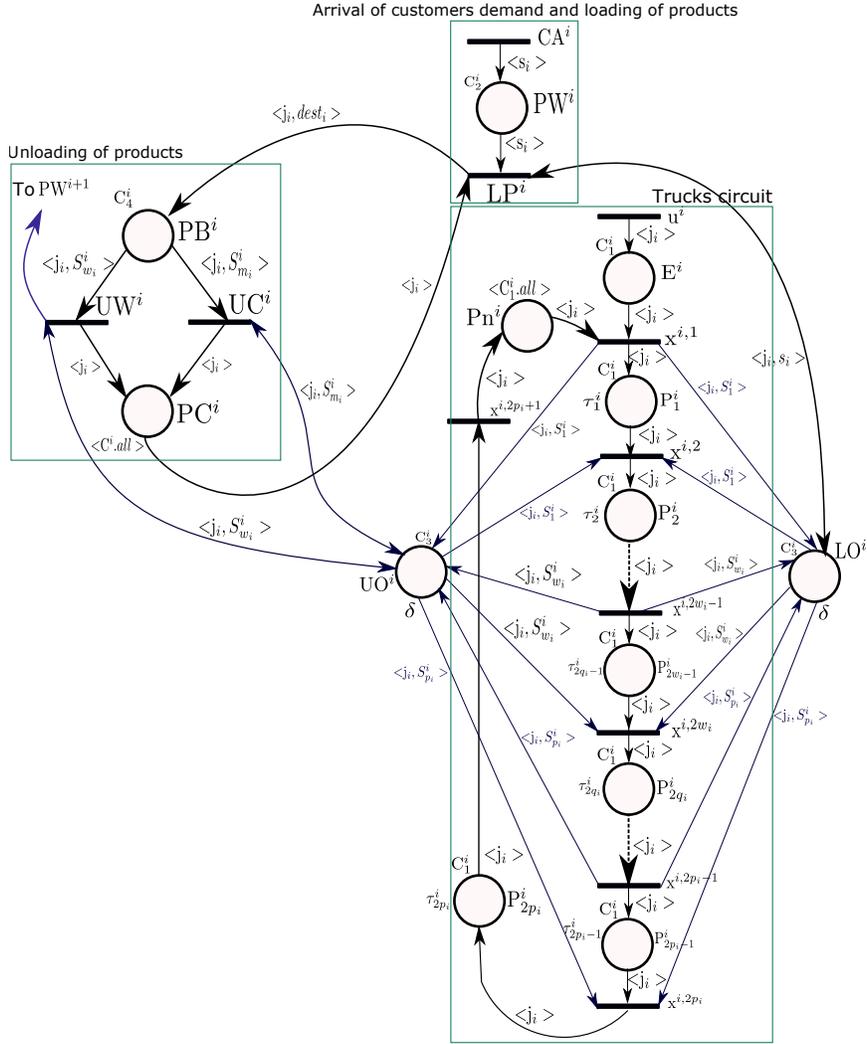


Figure 2: TCPN model of the supply chain

To illustrate various processes, the global TCPN model describing the behaviour of the studied logistic network is divided into three main sub-models: (i) vehicles circuit (ii) arrival of customers demand and loading of products, (iii) unloading of the products within logistic hubs or delivery to final customer sites. The various notations used on the TCPN model are presented in Table 1. In what follows, we develop each sub-model of the TCPN model of Figure 2.

2.2.1. Sub-model 1: Vehicles circuits

Every path P_i is served by a finite number of vehicles j_i ($j_i \in \{1, \dots, n_i\}$) supposed to be initially at their departure site S_1^i which corresponds to the first supplier (modeled by tokens with various colors in the place Pn^i). The transition u^i models authorization for vehicles to transport and deliver some products according to customer demand. The firing of $x^{i,1}$ models the vehicle circuit beginning. It

Table 1: Elements of the TCPN model

Transitions	
u^i	Authorization departure given to a vehicle;
$x^{i,2m_i}$	Departure of a vehicle from the site $S_{m_i}^i$;
$x^{i,2m_i+1}$	Arrival of a vehicle to site $S_{m_i+1}^i$;
CA^i	Arrival of a customer demand;
LP^i	Loading of products;
UC^i	Unloading of products;
UW^i	Unloading of products in a logistic hub;
Places	
E^i	Authorization to begin a circuit;
$P_{2m_i-1}^i$	Vehicle waiting at the site $S_{m_i}^i$;
$P_{2m_i}^i$	Moving vehicle to site $S_{m_i+1}^i$;
Pn^i	Vehicle ready to begin a new circuit;
PW^i	Products ready to be transported;
PB^i	Products on board;
PC^i	Vehicles capacities;
LO^i	Authorization order to load the products;
UO^i	Authorization order to unload the products;
Variables and color domains	
j_i in C_1^i with C_1^i is $1, \dots, n_i$ $s_i, dest_i$ in C_2^i with C_2^i is $S_1^i, \dots, S_{p_i}^i$; C_3^i is $< C_1^i, C_2^i >$; C_5^i is $< C_1^i, C_4^i >$; C^i is $1^{C_{p_i}^i} \cup \dots \cup n_i^{C_{p_i}^i}$; 	

waits τ_1^i time units before leaving the departure site (by firing $x^{i,2}$). This time corresponds to the load operation of products to be transported to different sites. Indeed, the firing of $x^{i,1}$ puts simultaneously a token $\langle j_i \rangle$ in the place P_1^i (which models vehicle waiting), a second token $\langle j_i, S_1^i \rangle$ in the place LO^i (which models the authorization order for products to be loaded) and finally a third token $\langle j_i, S_1^i \rangle$ in the place UO^i (which models the authorization order for products to be unloaded). After that, the vehicle j_i leaves its departure site and moves to the next one (by adding a token $\langle j_i \rangle$ in P_2^i). τ_2^i units of time later, the vehicle j_i arrives at the second site S_2^i (by firing $x^{i,3}$ by the token $\langle j_i \rangle$). The same operation is performed in the other sites ($S_3^i, \dots, S_{p_i}^i$). After delivering the final customer, the vehicle comes back to the starting site S_1^i (by adding a token in $P_{2p_i}^i$). Two options are possible on the return way of a vehicle, coming back directly to the departure point or considering inverse logistic while passing by different actors already visited in the sens First supplier - Final customer.

This inverse logistic is out of the scope of this paper. So we suppose that each vehicle comes back to its departure site after accomplishing a delivery task to the final customer.

2.2.2. Sub-model 2: Arrival of customers demand and loading of products

In all cases, customers' demands are managed by firing the transition CA^i . Moreover, when a customer $S_{c_i}^i$ ($c_i \in \{2, \dots, p_i\}$) demands a product or (a lot of products) from a supplier $S_{m_i}^i$ ($m_i \in \{1, \dots, p_i - 1\}$ with $m_i \neq c_i$), the transition CA^i is fired and a token with color $\langle S_{m_i}^i \rangle$ is putted in the place PW^i . A token in this place models a product to be transported. The product waiting in $S_{m_i}^i$ get loaded on the vehicle j_i (by firing the transition LP^i) when this vehicle arrives at this site with free space inside (tokens $\langle j_i \rangle$ in PC^i). If a vehicle is full (without free space), not yet loaded products have to wait for the next vehicle. Finally, product loading/unloading is supposed to follow the FIFO (First In First Out) rule. This means that demands are managed by starting with the first ones.

2.2.3. Sub-model 3: Unloading of products

Every product has a destination modeled by $\langle dest_i \rangle$ ($\langle dest_i \rangle$ is the color that indicates the customer that demands this product). The place PB^i models the products onboard. The transition UC^i is fired by a token $\langle j_i, S_{m_i}^i \rangle$ when a vehicle j_i arrives at the site $S_{m_i}^i$ to deliver a product. However, if a client from a path P_{i+1} ($r \neq i$) ordered a product from a supplier of path P_i , the vehicle j_i has to unloaded the product at the logistic hub or warehouse (by firing UW^i) so that a vehicle (of path P_{i+1}) j_{i+1} can transport it to the desired customer. After unloading the products from a vehicle j_i (by firing UC^i or UW^i), the free space within this vehicle increases (by adding tokens $\langle j_i \rangle$ in the place PC^i).

A formal verification and validation of the proposed TCPN models are carried out using the software CPN Tools (i.e., one of the most efficient tools for editing, simulating, and analyzing colored Petri nets¹⁶). The formal verification is based on the state space of the TCPN model, generated automatically by CPN Tools. This allows the verification of both quantitative and qualitative (e.g., liveness and deadlock-free) properties.

The proposed TCPN model presents three main advantages. First, the model enlightens the state of vehicles (e.g., moving, number of products onboard, free space inside) and the various sites (e.g., number of products to be transported). Finally, the arrival of customers' demands, loading/unloading of products, and vehicles' finite capacity are also represented.

In the next section, (max, +) models representing the previous TCPN model's dynamic behavior are given to analyze and evaluate logistics behavior quantitatively.

Afterwards, we used a theorem (proven by [Baceli et al. 1992]) for the optimal control in (max, +) algebra using JIT (Just In Time) approach. Our optimal command is mainly based on this theorem, our state equations, and the chosen criteria.

2.3. (Max, +) description of the supply chain behavior

This section proposes a mathematical formulation of the logistic network (described by a TCPN model) to formally evaluate the various transitions' firings. The objective is to obtain a linear analytic description of the studied network characterized by synchronization and conflicts issued from resources sharing, and choice situations. To this end, we use the powerful modeling and analysis ability of (max, +) algebra. This latter allows to obtain linear and non-stationary equations representing the logistic network's behavior.

The (max, +) equations of each sub-model of Figure 2 are presented: (i) vehicles circuit (ii) customers demand arrival and products loading (iii) and unloading of the products. As it is known, (max, +) algebra is dedicated to the computation of the occurrence dates of events (called "daters") [20]. Since we have modeled the logistic network with TCPN, the concept of color must be integrated within (max, +) equations. So, the associated dater with a transition x fired by a token of color y is denoted $x_{<y>}(k)$ and represents the date of the k^{th} firing of x by a token with color y ($<y>$). As already mentioned, we recall that one of the most contributions of this paper is combining TCPN models with (max, +) equations while integrating the color concepts into mathematical equations for managing and controlling the logistic network. For more details concerning dioid algebra, especially (max, +) algebra, the reader may refer to [23].

2.3.1. (Max, +) model of vehicle circuit

In the following, we express the dynamic behavior of vehicles (Figure 2), in terms of daters, by mathematical equations in the usual algebra and then by (max, +) equations. So, the k^{th} firing of each transition of the proposed TCPN model describing vehicles circuit is given by the following system. $\forall k > 1$

$$\left\{ \begin{array}{l} x_{<j_i>}^{i,1}(k) = \max(u_{<j_i>}^i(k), x_{<j_i>}^{i,2p_i+1}(k-1)) \\ x_{<j_i>}^{i,2}(k) = \max(x_{<j_i>}^{i,1}(k) + \tau_1^i, LP_{<j_i, S_1^i>}^i(k_{1,j_i}^i) + \delta, UC_{<j_i, S_1^i>}^i(o_{1,j_i}^i) + \delta) \\ \dots\dots \\ x_{<j_i>}^{i,2w_i-1}(k) = x_{<j_i>}^{i,2w_i-2}(k) + \tau_{2(q_i-1)}^i \\ x_{<j_i>}^{i,2w_i}(k) = \max(x_{<j_i>}^{i,2w_i-1}(k) + \tau_{2w_i-1}^i, LP_{<j_i, S_{w_i}^i>}^i(k_{w_i,j_i}^i) + \delta, UC_{<j_i, S_{w_i}^i>}^i(o_{w_i,j_i}^i) + \delta, UW_{<j_i, S_{w_i}^i>}^i(o_{p_i+1,j_i}^i) + \delta) \\ \dots\dots \\ x_{<j_i>}^{i,2p_i+1}(k) = x_{<j_i>}^{i,2p_i}(k) + \tau_{2p_i}^i \end{array} \right. \quad (1)$$

- For the first equation of (1): the vehicle j_i starts its k^{th} turn (k^{th} firing of x_1^i by $<j_i>$) after both completing his $(k-1)^{th}$ turn ($(k-1)^{th}$ firing of $x_{2p_i+1}^i$ by $<j_i>$), and when a departure permission is given due to a customer demand (k^{th} firing of u^i by $<j_i>$).

- For the second equation of (1): A vehicle j_i leaves the departure site (the first supplier) S_1^i for the k^{th} time (k^{th} firing of $x^{i,2}$ by $\langle j_i \rangle$) after waiting τ_1^i time units in this site so that products can get loaded (firing of LP^i by $\langle j_i, S_1^i \rangle$) and unloaded (firing of UC^i by $\langle j_i, S_1^i \rangle$) from the vehicle. The parameter $k_{1,j_i}^i(k)$ (resp. $o_{1,j_i}^i(k)$) are counters that represents the number of firings of LP^i (resp. UC^i) by $\langle j_i, S_1^i \rangle$ (resp. $\langle j_i, S_1^i \rangle$) before the k^{th} departure of the vehicle j_i from the site S_1^i [20,24]. The other equations are performed in the same way.

To make the system (1) in matrix form, we define vehicles state vector $X_{\langle j_i \rangle}^i(k)$ and the input vector $U_{\langle j_i \rangle}^i(k)$:

$$\begin{cases} X_{\langle j_i \rangle}^i(k) = [x_{\langle j_i \rangle}^{i,1}(k), x_{\langle j_i \rangle}^{i,2}(k), \dots, x_{\langle j_i \rangle}^{i,2p_i}(k), x_{\langle j_i \rangle}^{i,2p_i+1}(k)]^T; \\ U_{\langle j_i \rangle}^i(k) = u_{\langle j_i \rangle}^i(k). \end{cases}$$

We assume that every vehicle j_i leaves a station after loading and unloading all the concerned products. Considering this hypothesis and using (max, +) notations (The operator \oplus represents the (max,+)-addition, and \otimes represents the (max, +)-multiplication), the system (1) can be easily written as follow:

$$X_{\langle j_i \rangle}^i(k) = A_1^i \otimes X_{\langle j_i \rangle}^i(k) \oplus A_2^i \otimes X_{\langle j_i \rangle}^i(k-1) \oplus B_1^i \otimes U_{\langle j_i \rangle}^i(k) \quad (2)$$

$$\text{with: } A_1^i = \begin{bmatrix} \varepsilon & \varepsilon & \dots & \varepsilon & \varepsilon \\ \tau_1^i & \varepsilon & \dots & \varepsilon & \varepsilon \\ \varepsilon & \tau_2^i & \dots & \varepsilon & \varepsilon \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varepsilon & \varepsilon & \varepsilon & \tau_{2p_i}^i & \varepsilon \end{bmatrix}, A_2^i = \begin{bmatrix} \varepsilon & \dots & \varepsilon & e \\ \varepsilon & \dots & \varepsilon & \varepsilon \\ \varepsilon & \dots & \varepsilon & \varepsilon \\ \vdots & \ddots & \vdots & \vdots \\ \varepsilon & \dots & \varepsilon & \varepsilon \end{bmatrix} \text{ and } B_1^i = \begin{bmatrix} e \\ \varepsilon \\ \varepsilon \\ \vdots \\ \varepsilon \end{bmatrix}$$

The matrix A_1^i is nilpotent, so the implicit equation (2) can then be rewritten as follows:

$$X_{\langle j_i \rangle}^i(k) = A^i \otimes X_{\langle j_i \rangle}^i(k-1) \oplus B^i \otimes U_{\langle j_i \rangle}^i(k) \quad (3)$$

with: $A^i = A_1^i * \otimes A_2^i$, and $B^i = A_1^i * \otimes B_1^i$. We recall that $A_1^i *$ is the Kleene star of A_1^i ($A_1^i * = \bigoplus_{k \geq 0} (A_1^i)^k$). From equation (3), we can evaluate vehicles timetables (the arrival and departure times of vehicles to/from sites) for a given period.

In the next section, we present the (max, +) model that describes loading dates of products.

2.3.2. (Max, +) model for goods loading

To model loading of products, we need to find the different relations between the firings of the transitions LP^i , CA^i and UW^{i+1} by the various colors. To this end, we introduce some functions called routing functions to express the firing dates of the transition related to the loading of products (LP^i). More precisely, these routing functions' role is to manage and solve the conflicts related to product loading, according to their destination and the availability of free space inside the vehicle.

The following equations allow evaluating loading times of products within each site (at the supplier site or logistic hub). For $m=1,\dots,p_i$ and $j_i=1,\dots,n_i$, $\forall k \geq 1$,

$$LP^i_{\langle j_i, S_{m_i}^i \rangle}(k) = \begin{cases} CA^i_{\langle S_m^i \rangle}(\alpha^i_{m_i, j_i}) \oplus LP^i_{\langle j_i, S_{m_i}^i \rangle}(k-1) \cdot \delta \oplus x^i_{\langle j_i \rangle}{}^{i, 2m_i-1}(\beta^i_{m_i, j_i}) & \text{if } m_i \neq w_i \\ CA^i_{\langle S_{w_i}^i \rangle}(\alpha^i_{w_i, j_i}) \oplus UW^{i+1}_{\langle S_{w_i}^i \rangle}(\delta^i_{j_i}) \oplus LP^i_{\langle j_i, S_{w_i}^i \rangle}(k-1) \cdot \delta \oplus x^i_{\langle j_i \rangle}{}^{i, 2w_i-1}(\beta^i_{w_i, j_i}) & \text{if } m_i = w_i. \end{cases} \quad (4)$$

with: $\delta^i_{j_i}$ depends on k and represents the routing functions and $\beta^i_{m_i, j_i}{}^{th}(k)$ represents the $\beta^i_{m_i, j_i}{}^{th}(k)$ turn of the vehicle that will transport the k^{th} product to its destination. Indeed, in case of suppliers and customers site ($m_i \neq w_i$), the k^{th} firing of the transition LP^i by $\langle j_i, S_{m_i}^i \rangle$ occurs after the $\alpha^i_{m_i, j_i}{}^{th}(k)$ firing of the transition CA^i by $\langle S_{m_i}^i \rangle$ and $\beta^i_{m_i, j_i}{}^{th}(k)$ firing of the transition $x^i_{\langle j_i \rangle}{}^{i, 2m_i-1}$ by $\langle j_i \rangle$. Besides, in the logistic hubs (or distribution centers, warehouses), products may come from other suppliers served by vehicles of different paths. For this reason, the k^{th} firing of the transition LP^i by $\langle j_i, S_{w_i}^i \rangle$ occurs after $\beta^i_{w_i, j_i}{}^{th}(k)$ firing of the transition $x^i_{\langle j_i \rangle}{}^{i, 2w_i-1}$ by $\langle j_i \rangle$ and either the $\alpha^i_{w_i, j_i}{}^{th}(k)$ firing of the transition CA^i by $\langle S_{w_i}^i \rangle$ or the $\delta^i_{j_i}{}^{th}(k)$ firing of the transition UW^{i+1} by $\langle S_{w_i}^i \rangle$ (equation (4)). The mathematical equations of these routing functions are presented and elaborated with more details in [20]. In the next section, the (max, +) model for evaluating products unloading times is presented.

2.3.3. (Max, +) model for goods unloading

Similarly to the equation (4), the (max, +) equations describing product's unloading time are given by: $\forall k > 1$,

$$UC^i_{\langle j_i, S_{m_i}^i \rangle}(k) = x^i_{m_i, j_i}(k) \oplus x^i_{\langle j_i \rangle}{}^{i, 2m_i-1}(\xi^i_{p_i^2+m_i, j_i}) \otimes \delta \oplus UC^i_{\langle j_i, S_{m_i}^i \rangle}(k-1) \otimes \delta \quad (5)$$

where $x^i_{m_i, j_i}(k)$ is given by the following equations: $\forall k > 1$,

$$x^i_{m_i, j_i}(k) = LP^i_{\langle j_i, S_{j_i}^i \rangle}(\xi^i_{p_i(m_i-1)+1, j_i} \oplus \dots \oplus LP^i_{\langle j_i, S_{j_i}^i \rangle}(\xi^i_{p_i(m_i-1)+d, j_i}) \quad (6)$$

The k^{th} product (for which the destination is site $S_{m_i}^i$) is unloaded from the vehicle j_i when this vehicle, in its $(\xi^i_{p_i^2+m_i, j_i})^{th}$ turn, arrives at this site. The routing functions ξ^i_{u, j_i} ($u \in 1, \dots, p_i(p_i+1)$) are proposed to express the dates of UC^i . These functions depend essentially on the product's destination. An algorithm to compute these routing functions is developed.

As a summary, after verifying and validating the TCPN model, we express the firing dates of TCPN transitions with linear and non-stationary mathematical equations in (max, +) algebra. However, the complexity of the studied logistic network, characterized mainly by conflicts and synchronization, makes this phase quite tricky. Accordingly, we introduce some routing functions to solve these problems

and present the studied network with state equations in $(\max, +)$ algebra. In the next section, an optimal control of the studied system is proposed. This control approach is based on a theorem (proven by¹⁷) for the optimal control in $(\max, +)$ algebra using JIT (Just In Time) paradigm.

3. Optimal control of the studied supply chain

The problem addressed in this work concerns the problem of timetable computation. Our goal is to optimize the number of vehicles deployed on the network to keep the products' storage time in the various sites (e.g., particularly warehouses or distribution centers) below a pre-defined parameter noted ST. This considerably reduces the transportation companies' costs and their environmental impact (e.g., gas emissions). Therefore, a control approach based on the techniques developed in the context of dioid residuation theory is proposed to adjust or regulate timetables to respect the specific ST.

From a control point of view, our system is characterized by the input vector $U_{<j_i>}^i$ which represents departure time of vehicles from their departure sites and the output vector $Y_{<j_i>}^i$ which corresponds to the vector of the arrival times of vehicles j_i at sites considered as strategic (at which the quality of service, presented by ST, must be respected more particularly).

3.1. Synthesis of the optimal control

A relevant goal for the control of such systems is to delay as much as possible the input events occurrences (i.e., to compute the greatest control vector $U_{<j_i>}^i$) while ensuring performances imposed by a given specification (the specification corresponds here to the proposed constraints (ST)). This problem corresponds to the just in time control problem, which commonly aims at supplying the right quantity at the desired time. In this paper, the desired time is denoted $Yd_{j_i}^i$ and corresponds to vehicles arrival dates to sites where the service level must be respected (ST).

We aim to synthesize a command $U_{<j_i>}^i$ which allows the output $Y_{<j_i>}^i$ to occur at the desired moment ($Yd_{j_i}^i$).

$$Y_{<j_i>}^i(k) = C^i(k) \otimes X_{<j_i>}^i(k) \quad (7)$$

with : $C^i(k) \in \mathbb{R}_{max}^{(1) \times (2p_i+1)}$, $C^i(k) = [\varepsilon, \dots, e, \varepsilon, \dots, \varepsilon]$ with $C^i(k)_{2v-1} = e$ (S_v^i is the desired station).

The just in time control method consists of determining the input-output relation (or the transfer matrix) of the system presented by (3). First, considering the earliest functioning rule of the system

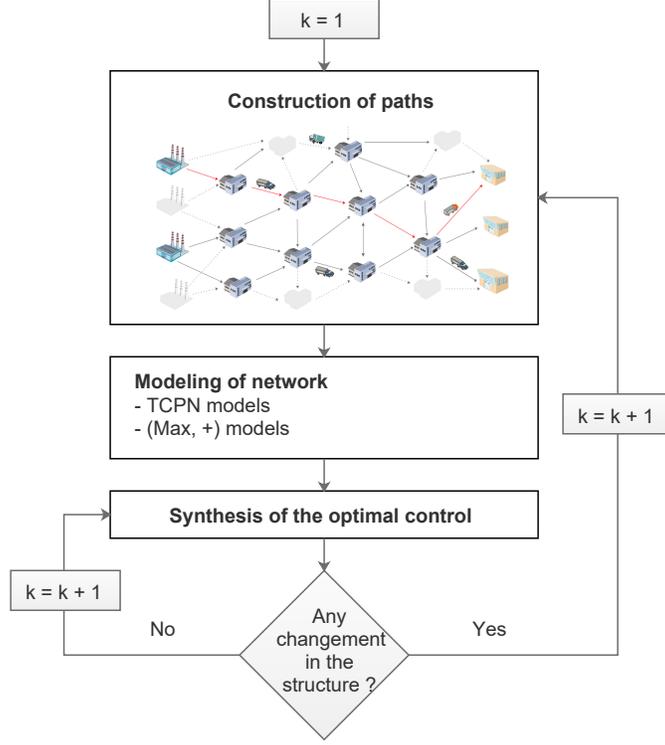


Figure 3: The methodology adopted to study the global network with changes of paths

(²⁵), we obtain from (3) the following equation:

$$X_{<j_i>}^i(k) = A^i * B^i U_{<j_i>}^i(k) \quad (8)$$

Therefore, the input-output relationship of the system presented by (3) is given by:

$$H = C^i A^i * B^i \quad (9)$$

Furthermore, the control is optimal in the sense that it optimizes the chosen criteria ((i) minimize the number of vehicles deployed and (i) keep products storage time below ST). In our case, the optimal command is denoted $U_{<j_i>}^i \text{ opt}$ and it optimizes the criteria mentioned above. According to the residuation theory, the expression of the $U_{opt}^{l,j}$ command is given by:

$$U_{<j_i>}^i \text{ opt} = \bigoplus_{U_{<j_i>}^i | H U_{<j_i>}^i \leq Y d_{j_i}^i} U_{<j_i>}^i = Y d_{j_i}^i \setminus H = Y d_{j_i}^i \setminus C^i A^i * B^i \quad (10)$$

The choice of the vehicle to deploy on the network depends mainly on the desired output $Y d_{j_i}^i$.

This output parameter should respect the constraints related to products storage threshold, and it is chosen as follows: as a first step, we choose among the $(n_i)^{th}$ vehicle that be used with respect to the ST parameter. Therefore, $Yd_{j_i}^i$ is given according to the availability of these vehicles. However, if none of the n_i (initially $n_i=1$) vehicles meet this requirement, we add a new vehicle on the desired path. Every vehicle added is injected either in the departure site or in a given site where the ST is solicited. We precise that developed models are executed each time the structure change (figure 3).

4. The case study

As mentioned before, the proposed approach may be applied to a massive network with various sites (e.g., suppliers, warehouses, customers). However, to simplify the approach, we present a simple case study to show the proposed models' effectiveness. More precisely, we present a simple supply chain (figure 4). The data of the considered network is presented in what follows: number of sites is eight, the number of vehicles is four, and the capacity of these vehicles is 20 pallets. The waiting and traveling times (with hour as time unit) of vehicles are presented in figure 4.

After a formal validation of the proposed TCPN models using CPN Tools, we implement the (max, +) algebra models in the Scilab environment. Indeed, the modularity of the models suggests using efficient software such as Scilab, which allows to model systems with a large number of variables. The resulting simulation model is compact, simple to implement, and keeps the model's modularity feature while reproducing its structure. The adopted methodology is validated on several scenarios and configuration of the studied system (i.e., from small to large scale network), and the simulation results are reported in what follows.

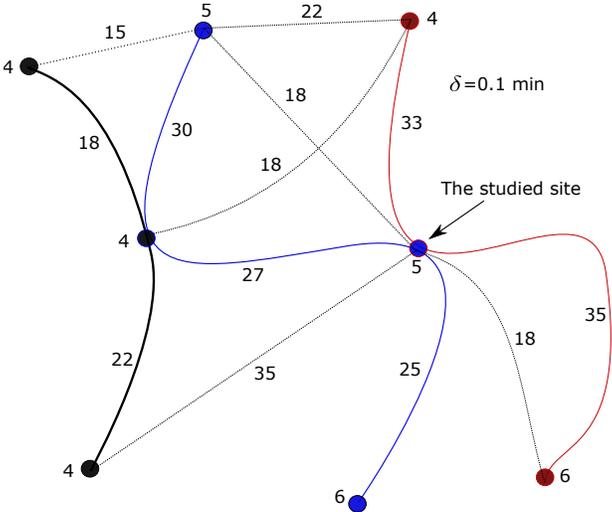


Figure 4: The studied network

For the considered supply chain, Figure 5-(a) reports random arrival of customers demands (sales orders), storage times of the products at the studied site (logistic hub mentioned in figure 4), in case of the initial planning (scenario with no control). For instance, in Figure 5-(a), the storage time of the first product is 44.49 H. Figure 5-(b) presents the two considered scenarios: scenario 1 (in which we suppose that we do not have any control and do not apply any constraint on vehicles circulation (figure 5-(a)) and scenario 2 (in which we set the ST to 15 H at the studied warehouse (figure 4)). The obtained results for the second scenario confirm that products storage time is always below the chosen ST and does not exceed 15H. Moreover, to respect this constraint, five vehicles are used instead of just four (scenario 1).

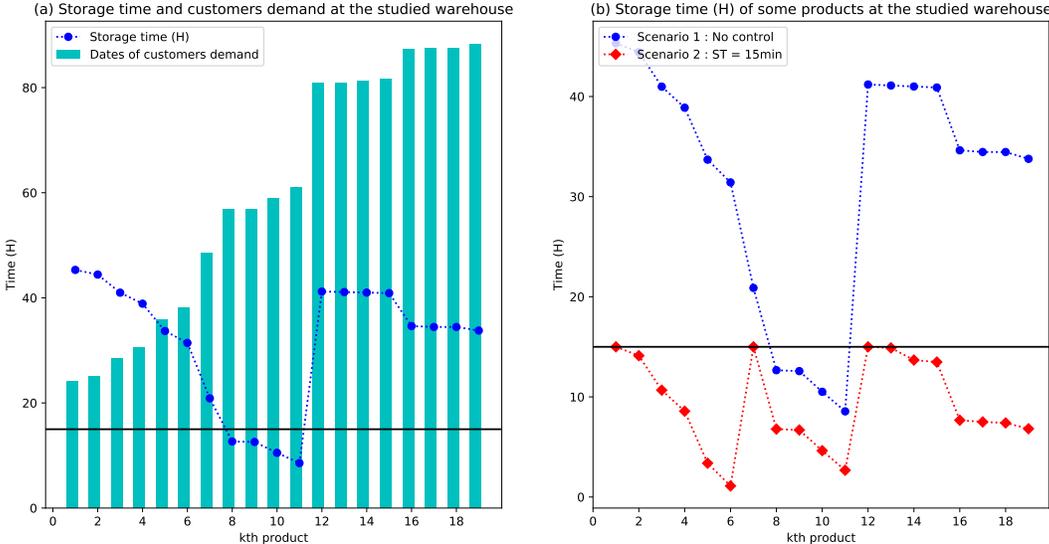


Figure 5: Products storage times under two scenarios

Moreover, in some sites (such as a warehouse on the same path as the studied one) where the control is not considered, the results show that after a given time (transitional regime), storage time of some products remains below a given limit because this storage time is impacted by the control in the previous warehouse. Thus, all the supply chain components are infected by the control of one warehouse, as illustrated in Figure 6.

Furthermore, Figure 6 presents the optimal number of vehicles used to satisfy the scenario where the $ST=15H$ is solicited at the studied warehouse (figure 4). We precise that the optimal number of vehicles is too high in peak periods (time of the day during which customers' demands are at their highest levels) compared to off-peak periods. For instance, in the period of [210 min, 250 min], the number of vehicles to deploy (in order to respect $ST=15H$) is six. The proposed models illustrate the ability to describe the influence of vehicles' capacity on delivery dates of products. As a result, the

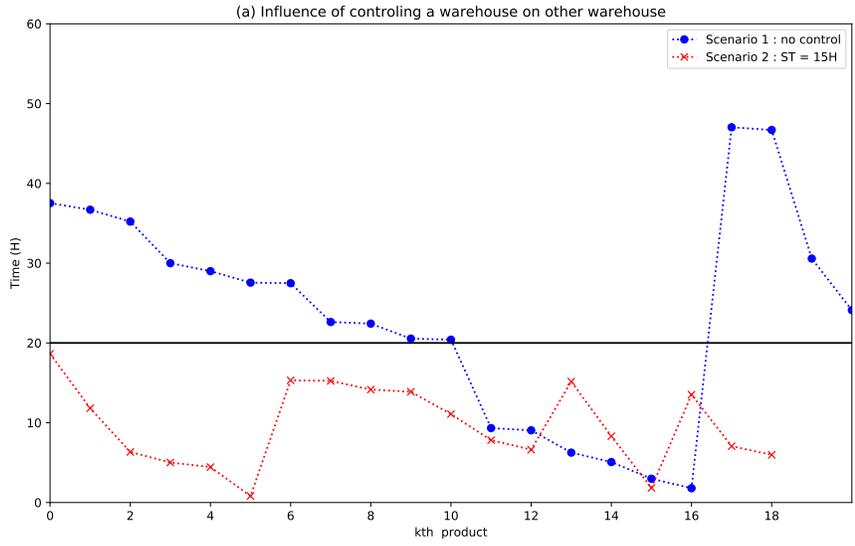


Figure 6: Influence of controlling a warehouse on other warehouse on the same path

delivery dates of these products increase over time. Based on these observations, one can conclude that the used vehicles are not enough to transport simultaneously all the asked products from customers' demands. Thus, in this case, the transportation company should either choose a vehicle with a very high capacity or increase the number of vehicles circulating in the network. This issue will be addressed in a future work to find a compromise between the capacity and the number of vehicles to deploy in the studied supply chain.

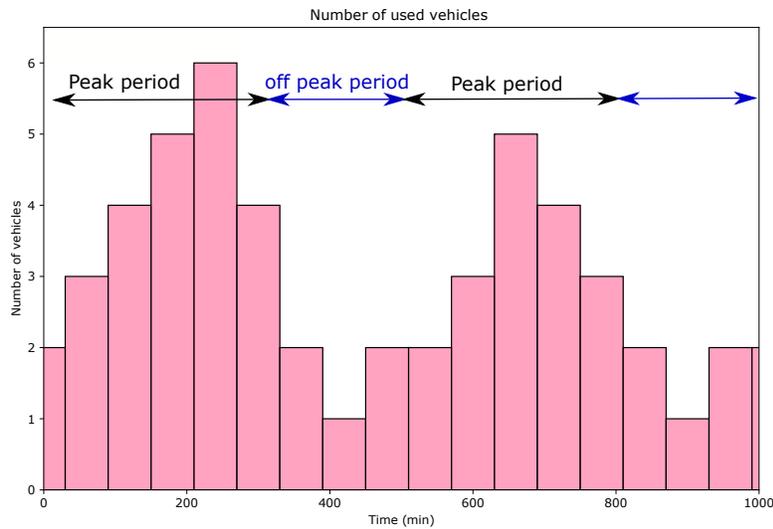


Figure 7: Number of used vehicles in the network

5. Conclusions

This paper introduces a modeling approach based on TCPN and $(\max,+)$ algebra to describe logistic network performances in terms of transportation and delivery of products. The proposed models are able to provide a sufficiently accurate and valid formal representation of the supply chain. To show the models efficiency, a case study describing vehicles' timetables at different sites and localities and products' loading/unloading times is presented. Moreover, the influence of vehicles' limited capacity is studied to show the effects of this constraint on the delivery dates of products. Finally, a control approach, based on $(\max,+)$ theory, to minimize transportation costs in terms of the number of vehicles used in the network is proposed. Further research may introduce more complex modeling features, such as unexpected events like accidents and delays.

References

1. A. Amiri, Designing a distribution network in a supply chain system: Formulation and efficient solution procedure, *European Journal of Operational Research* 171: 567-576 (2006).
2. M. Song, W. Shuhong, F. Ron, Transportation, iceberg costs and the adjustment of industrial structure in china, *Transportation Research Part D: Transport and Environment* 32(10): 278-286 (2014).
3. C. Simon, R. Pietro, G. Mihalis, Supply chain management: an analytical framework for critical literature review, *European Journal of Purchasing and Supply Management* 6 : 67:83 (2000).
4. B. S. Borges, M. G. G. Ferreira, N. E. Moura, C. J. Alcides, F. F. Antônio, G. Jéssica, G. J. B. S. O. de Andrade, Multi-criteria analysis model to evaluate transport systems: An application in florianópolis, brazil, *Transportation Research Part A* (2016) 1–13.
5. M. Melo, S. Nickel, F. S. da Gama, Facility location and supply chain management – a review, *European Journal of Operational Research* 196: 401:412 (2009).
6. L. Macchion, A. Moretto, F. Caniato, P. Danese, A. Vinelli, Static supply chain complexity and sustainability practices: A multitier examination, *Corporate Social Responsibility and Environmental Management* 27 (6) (2020) 2679–2691.
7. A. Toloioe, M. Maity, A. K. Sinha, A two-stage stochastic mixed-integer program for reliable supply chain network design under uncertain disruptions and demand, *Computers & Industrial Engineering* 148 (2020) 106722.

8. P. Hoffa-Dabrowska, K. Grzybowska, Simulation modeling of the sustainable supply chain, *Sustainability* 12 (15) (2020) 6007.
9. G. Maity, S. K. Roy, Solving a multi-objective transportation problem with nonlinear cost and multi-choice demand, *International Journal of Management Science and Engineering Management* 11 (1) (2016) 62–70.
10. X. Gao, G. M. Lee, Moment-based rental prediction for bicycle-sharing transportation systems using a hybrid genetic algorithm and machine learning, *Computers & Industrial Engineering* 128 (2019) 60–69.
11. L. Sun, L. Lin, H. Li, M. Gen, Large scale flexible scheduling optimization by a distributed evolutionary algorithm, *Computers & Industrial Engineering* 128 (2019) 894–904.
12. H. Hu, X. Li, Y. Zhang, C. Shang, S. Zhang, Multi-objective location-routing model for hazardous material logistics with traffic restriction constraint in inter-city roads, *Computers & Industrial Engineering* 128 (2019) 861–876.
13. J. L. Adler, V. J. Blue, A cooperative multi-agent transportation management and route guidance system, *Transportation Research Part C: Emerging Technologies* (2002) 433–454, V. 10.
14. H. Gholizadeh, H. Fazlollahtabar, Robust optimization and modified genetic algorithm for a closed loop green supply chain under uncertainty: Case study in melting industry, *Computers & Industrial Engineering* 147 (2020) 106653.
15. M. DIAZ, *Les réseaux de petri - modèles fondamentaux*, Hermès science publications (2001).
16. K. Jensen, G. Rozenberg, *High-level Petri Nets Theory and Application*, Springer, Verlag Berlin Heidelberg, 1991.
17. F. Baccelli, G. Cohen, G. J. Olsder, J.-P. Quadrat, *Synchronization and linearity: An Algebra for Discrete Event Systems*, J. Wiley, New York, 1992.
18. Y. Idel Mahjoub, A. Nait-Sidi-Moh, E. E. Chakir, Modeling a bus network for passengers transportation management using colored petri nets and $(\max, +)$ algebra, In *proceedings of Ambient Systems, Networks and Technologies (ANT)* 109C (2017) 576–583.
19. A. Nait-Sidi-Moh, M.-A. Manier, A. E. Moudni, Spectral analysis for performance evaluation in a bus network, In *European journal of operational research* 193 (1) (2009) 289–302.
20. Y. Idel Mahjoub, E. E.-A. Chakir, A. Nait-Sidi-Moh, Modeling and developing a conflict-aware scheduling in urban transportation networks, *Future Generation Computer Systems* (2018).

21. C. F. Durach, J. Kembro, A. Wieland, A new paradigm for systematic literature reviews in supply chain management, *Journal of Supply Chain Management* 53 (4) (2017) 67–85.
22. T. Murata, Petri nets: Properties, analysis and applications, *Proceedings of the IEEE* 77 (4) (1989) 541 – 580.
23. S. Gaubert, Systemes dynamiques à événements discrets, Lecture notes, ENSMP, Automation and Master degree ATS Orsay (2000) (Inria, Rocquencourt, France).
24. Y. Idel Mahjoub, A. Nait-Sidi-Moh, E. E. A. Chakir, A. Tajer, Petri nets conflicts resolution for performance evaluation and control of urban bus networks: a (max, +)-based approach, *Transportmetrica A: Transport Science* (2018) 1–30.
25. L. Houssin, S. Lahaye, J. Boimond, Control of (max,+) algebra linear systems minimizing delays, *Discrete Event Dynamic Systems* 23 (3) (2013) 261–276.