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Observer-Based Tracking design using H_∞ criteria : Application to Eco-driving in a Tramway System

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Abstract

This paper investigates an H_∞ Observer-Based Controller design for tracking a tramway system eco-driving trajectory. The model of the tramway system is given in state space form, and the poor manoeuvres of the driver when following a reference trajectory are modelled as disturbances with finite energy that affect the system dynamics. To minimize the impact of poor driver manoeuvres, an H_∞ Observer-Based Tracking Controller (H_∞ -OBTC) was designed and its conditions of existence are given. In addition, to ensure the robust convergence of the estimation and the tracking errors simultaneously, a new sufficient condition was obtained based on the Bounded Real Lemma. Two algorithms are presented to solve the robust stability condition obtained. The first one is based on a two-step procedure. Then a linearisation approach was used to present the robust stability condition of the errors as a convex optimization problem with a Linear Matrix Inequality (LMI) constraint.

The gain matrices of the H_∞ -OBTC can be computed by solving the LMI given, subject to a minimization constraint.

Keywords: H_∞ Observer applications, LMI, Tracking strategy, Minimization of energy consumption, Tramway system.

1. Introduction

In recent years, many researchers have been attracted by the significant increase in the volume of railway traffic and pollutant particle emissions, which is one of the reasons why the issue of speed profiling has become a new problem with specific characteristics that must be resolved. Generally, the goal is to optimize energy consumption and/or running times by

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introducing one or several objectives. The question of speed profiling is thus a specific issue that must be handled in the development phase of eco-aware transportation systems (see [1, 2, 3, 4, 5] and the references therein).

The main goal of the feedback controller design is to stabilize unstable systems or to improve stability in the presence of transient phenomena that do not fade quickly. From a practical point of view, knowledge of all states is not available all the times. This explains the interest of reconstructing the missing states using state observers (see [6, 7, 8, 9, 10] and the references therein). Several studies [11, 12, 13, 14, 15] have dealt with designing different kinds of observers.

In [16], it was noted that taking into account certain factors such as energy efficiency and reducing maximum load lowers overall energy consumption. In [17, 18], the authors modelled train driver behaviour to facilitate the development of Advanced Driver Assistance Systems [19] based on the definition of a trajectory profile to reduce energy consumption. The presence of human/vehicle uncertainties and disturbances makes the conception and design of the control law more complicated [20]. These studies show the motivations and the benefits of this work, which is the continuity of the study initiated in [21] on the tram-driver system eco-driving command.

This work presents a theoretical contribution to avoid the impact of poor driver manoeuvres or behaviour when maintaining the reference tramway system eco-driving speed; These are considered as disturbances with finite energy that affect the system dynamics. To resolve the trajectory-tracking problem, we consider an H_∞ Observer-Based Tracking Controller with two objectives. The first is to ensure the robust estimation of the internal state of the tramway system despite poor driver behaviour. The second is to generate a controller law based on the estimation obtained to track the reference tramway eco-driving speed. The conditions of existence of the proposed observer are formulated and the robust stability of the estimation and the tracking errors is given as an LMI problem subject to a minimization problem.

The paper is organized as follows. In section 2, we provide some definitions and useful lemmas for the computation of the H_∞ -norm of a linear system represented in state space form, or by its transfer function. In section 3, the state space representation of the tramway mathematical model is formulated. In addition, the eco-driving trajectory tracking problem is

formulated and presented as an H_∞ -OBTC design. The main results are presented in section 4, where the problem of the existence of an OBC for the tramway system is formulated, and a necessary and sufficient condition for the existence of such an observer-based controller is given. The second part of section 4 is dedicated to the design of the H_∞ -OBTC, where sufficient conditions for robust stability of the estimation errors are presented as both BMI and LMI formulations. The effectiveness of the proposed method is shown in section 5 through numerical simulation. Moreover, the proposed method is discussed and some prospects for this work are given. Finally, concluding remarks regarding the results are given in section 6.

Notations: Hereinafter, \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively; A^T denotes the transpose of matrix A ; matrix A is symmetric positive definite if and only if $A^T = A$ and $A > 0$. Matrix A^+ denotes the generalized inverse of matrix A which satisfies $AA^+A = A$; $\|\cdot\|_2$ is the L_2 norm; $\|\cdot\|_\infty$ is the H_∞ norm; I and 0 denote the identity matrix and zero matrix of appropriate dimensions, respectively. I_n is the identity matrix of dimensions $n \times n$; $\text{Sym}\{X\}$ is used to denote $X^T + X$. The notation $(*)$ corresponds to the conjugate transpose of the off-diagonal part, and $\text{diag}\{A, B\}$ denotes the diagonal matrix whose diagonal elements are matrices A and B .

2. Fundamentals

In the following, we introduce the state space representation of a disturbed linear system as

$$\dot{x}(t) = A_0x(t) + B_0w(t) \quad (1a)$$

$$z(t) = C_0x(t) + D_0w(t) \quad (1b)$$

where $x \in \mathbb{R}^n$ denotes the state vector, $w \in \mathbb{R}^m$ is the disturbance input, and $z \in \mathbb{R}^p$ is the controlled output. A_0 , B_0 , C_0 and D_0 are known constant matrices of appropriate dimensions.

The corresponding transfer matrix from w to z is defined as

$$Z(s) = T_{zw}(s)W(s) \quad (2)$$

where $W(s)$ and $Z(s)$ are the Laplace transforms of w and z with zero initial conditions i.e. $x(t) = 0, \quad \forall t \leq 0$. Hence, we have

$$T_{zw}(s) = C_0(sI - A_0)^{-1}B_0 + D_0 \quad (3)$$

Definition 1. Let w be the disturbance input. The signal w is of finite energy and belongs to L_2 if the following norm is bounded:

$$\|w\|_2 = \left(\int_0^\infty w(t)^T w(t) dt \right)^{\frac{1}{2}} \quad (4)$$

This norm can be interpreted as the energy of the disturbance input w . \square

Definition 2. System (1) has an H_∞ -norm less than a positive scalar γ , i.e. $\|T_{zw}(s)\|_\infty < \gamma$, if it is stable and the following minimization problem is satisfied for any disturbances w belonging to L_2 and with zero initial conditions:

minimize $\gamma > 0$ subject to

$$J = \int_0^\infty z^T(\tau)z(\tau) - \gamma^2 w^T(\tau)w(\tau) d\tau < 0 \quad (5)$$

\square

The positive scalar γ given in Definition 2 can be interpreted as an upper bound of the L_2 -gain between the disturbance input w and the output z . The solution to the problem (5) can be obtained from the bounded real lemma [22] and is given by the following lemma

Lemma 1. [22] System (1) is asymptotically stable for $w = 0$ and $\|T_{zw}(s)\|_\infty < \gamma$, for $w \neq 0$, if and only if there exists a symmetric positive definite matrix P and a positive scalar γ such that the following LMI condition is satisfied

$$\begin{bmatrix} \text{Sym}\{PA_0\} & PB_0 & C_0^T \\ \star & -\gamma I & D_0^T \\ \star & \star & -\gamma I \end{bmatrix} < 0 \quad (6)$$

Now, we can give the following standard lemma used in robust control.

Lemma 2. [23] Let D , E and F be real matrices of appropriate dimensions and F satisfies $F^T F \leq I$. Then for any scalar $\epsilon > 0$ and vectors $x, y \in \mathbb{R}^n$, we have

$$2x^T DFEy \leq \epsilon^{-1} x^T DD^T x + \epsilon y^T E^T E y \quad (7)$$

3. Problem Formulation & Mathematical model

3.1. Problem Formulation

In the literature, the eco-driving problem is often considered as an optimal control problem where its solution leads to the definition of a speed profile that minimizes energy consumption for a given trip. This is only valid in the case of an autonomous vehicle. In our case, the trams are not autonomous systems, i.e. the presence of a driver is needed. In fact, the driver has to follow a speed trajectory obtained as a solution to the optimal control problem of an autonomous system.

In reality, driver control is not perfect, which leads to a higher consumption in most cases. To overcome this impediment, some researchers have tried to develop new approaches and concepts based on heuristic rules or good practices that are associated with energy-efficient driving [24]. Other existing concepts have also been applied to try to implement eco-driving within a more rigorous framework. Moreover, some of these concepts are predictive, because they are based on the estimation of future external behaviour, while the others are based on information collected in real time, generally extracted from vehicle sensors, and include approaches such as predictive control [25, 26], as well as driver behaviour identification [27, 28]. The disadvantage of these algorithms is that they are executed online, i.e. the controller parameters are computed in real-time, while the convergence of these algorithms is not guaranteed, which represents additional energy consumption during the trip, because these algorithms need a lot of time and energy due to the high number of operations.

The main idea of this work is to address this problem by applying an other existing approach that has demonstrated its effectiveness in several areas [29, 30, 31, 32, 33]: tracking based on an observer who estimates the states of a disturbed system. All the observer and controller parameters are computed offline and it is assumed that the undesirable effect of the driver is bounded. This makes sense since the driver must respect imposed constraints, such as the speed limit and road geometry.

3.2. Mathematical model

The main objective of this section is to present the mathematical model of the tramway system dynamics as well as the state space representation of the system that generates the trajectory to be tracked. The eco-driving trajectory tracking problem is then formulated as an H_∞ -OBTC design.

The tramway system dynamics are modelled to control the tram position and speed at any time along the defined path between two stations. This modelling requires knowledge of some basic parameters such as the traction and brake intensities, adhesion coefficient, and track profile, as well as weather conditions such as wind speed. Usually, the brake and traction curves are given by the constructor.

By applying Newton's fundamental law to the tramway system, we obtain its dynamic equation which states the relation between the forces exerted on the system, its mass m and acceleration $\dot{v} = \ddot{p}$

$$m\dot{v} = F_T(v) - F_R(v) \quad (8)$$

where $F_T(v)$ is the tractive effort that the tram produces in the running phase, and $F_R(v)$ represents the resistance which is the sum of the line $F_{Rl}(v)$, curve $F_{Rc}(v)$ and vehicle $F_{Rm}(v)$ resistances. The latter are given as

$$F_{Rl} = mg \sin(\beta) \quad (9a)$$

$$F_{Rc} = mg \frac{k_e}{r_c} \quad (9b)$$

$$F_{Rm}(v) = a_0 + b_0v + c_0v^2 \quad (9c)$$

From equations (9), we can see that the line resistance $F_{Rl}(v)$ depends on the train mass, the slope angle β , and the gravity constant g . Furthermore, the curve resistance $F_{Rc}(v)$ concerns when a tram passes through some sloping portions of track, and depends on the track gauge coefficient k_e and the curvature radius r_c . In addition, due to the low value of c_0 and the relatively low velocity range $[v_{min}; v_{max}]$, the vehicle resistance $F_{Rm}(v)$ countering the movement of the tram can be approximated by the following $a_1 + b_1v(t)$ using the least squares method.

The tramway system dynamics can now be represented in a state space model as

$$\dot{x}_r(t) = Ax_r(t) + B_u u_r(t) + d_r \quad (10a)$$

$$y_r(t) = C_y x_r(t) \quad (10b)$$

where

$$x_r(t) = \begin{bmatrix} p(t) \\ v(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b_1}{m} \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$C_y = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad d_r = \begin{bmatrix} 0 \\ -\left(\frac{a_1}{m} + g \sin(\beta) + g \frac{k_e}{r_c}\right) \end{bmatrix}$$

In this work, to introduce the poor behaviour when following a reference eco-driving trajectory for the tramway system (10) defined as $x_r(t)$, a new input $w(t)$ can be considered when modelling this kind of system. The new input $w(t)$ is considered as disturbances with finite energy representing poor driver behaviour that affects the the tramway system dynamics.

$$\dot{x}(t) = Ax(t) + B_u u(t) + d_r + B_w w(t) \quad (11a)$$

$$y(t) = C_y x(t) \quad (11b)$$

where the disturbance $w(t)$ models the poor manoeuvres or behaviour of the driver, $x \in \mathbb{R}^n$ is a state vector, $u \in \mathbb{R}^m$ is the input vector, and $y \in \mathbb{R}^q$ is the measurement output vector. A , B_u , B_w and C_y are known matrices of appropriate dimensions.

The main idea of this work is to avoid the impact of the new input $w(t)$, which models the disturbances caused by the poor tram driver manoeuvres or behaviour, by rigorously following the reference trajectory. Therefore, an H_∞ -OBTC is considered and has the following form

$$\dot{\eta}(t) = N\eta(t) + H_u u(t) + J_y y(t) + M d_r \quad (12a)$$

$$\hat{x}(t) = \eta(t) + E_y y(t) \quad (12b)$$

$$u(t) = u_r(t) - K_u (x_r(t) - \hat{x}(t)) \quad (12c)$$

where $\eta \in \mathbb{R}^n$ is the internal state vector of the observer, $\hat{x} \in \mathbb{R}^n$ is the estimate of x , $u \in \mathbb{R}^m$ is the feedback control law, and $u_r \in \mathbb{R}^m$ is the input reference. Matrices N , H_u , J_y , M , E_y , and K_u are unknown and of appropriate dimensions which must be determined.

The tracking problem can be summarized as finding an appropriate control input $u(t)$ such that the estimation error $e_{\hat{x}}(t) = x(t) - \hat{x}(t)$ and the tracking error $e_x(t) = x_r(t) - x(t)$ converge to 0 if the tram driver follows the instructions but otherwise minimizes the effect of the input $w(t)$ on the estimation errors.

4. Main results

4.1. Observer-based Tracking Controller parametrization

To generate a linear feedback control law (12c), the proposed H_∞ observer (12a-12b) must ensure x_r trajectory tracking despite the presence of disturbances in the system (11), where x_r is the state of the reference system (10).

The errors are defined as

$$e(t) = \begin{bmatrix} e_x(t) \\ e_{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} x_r(t) - x(t) \\ x(t) - \hat{x}(t) \end{bmatrix} \quad (13)$$

where e_x and $e_{\hat{x}}$ represent the tracking and the estimation errors, respectively.

The error dynamics are obtained by differentiating each component of (13)

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}_r(t) - \dot{x}(t) \\ &= (A + B_u K_u)e_x + B_u K_u e_{\hat{x}}(t) - B_w w(t) \end{aligned} \quad (14)$$

$$\begin{aligned} \dot{e}_{\hat{x}}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= N e_{\hat{x}}(t) + \underbrace{(RA - NR - J_y C_y)}_{=0} x(t) \\ &\quad + \underbrace{(RB_u - H_u)}_{=0} u(t) + \underbrace{(R - M)}_{=0} d_r + RB_w w(t) \end{aligned} \quad (15)$$

$$\dot{e}_{\hat{x}}(t) = N e_{\hat{x}}(t) + RB_w w(t)$$

where $R = I_n - E_y C_y$.

We obtain

$$\begin{aligned} \dot{e}(t) &= \begin{bmatrix} \dot{e}_x(t) \\ \dot{e}_{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} (A + B_u K_u)e_x + B_u K_u e_{\hat{x}}(t) - B_w w(t) \\ N e_{\hat{x}}(t) + RB_w w(t) \end{bmatrix} \\ &= \begin{bmatrix} (A + B_u K_u) & +B_u K_u \\ 0 & N e_{\hat{x}}(t) \end{bmatrix} \begin{bmatrix} e_x(t) \\ e_{\hat{x}}(t) \end{bmatrix} + \begin{bmatrix} -B_w \\ RB_w \end{bmatrix} w(t) \end{aligned} \quad (16)$$

Proposition 1. *System (12) is an H_∞ -OBTC for system (11) with respect to Definitions 1 and 2 for any initial conditions of x and the estimated \hat{x} if gain matrices N , J , H , E , K_u and $K_{\hat{x}}$ exist such that*

- i) The L_2 -gain of estimation error (16) is bounded by $\gamma > 0$.*

$$ii) NR + J_y C_y - RA = 0$$

$$iii) H_u = RB_u$$

$$iv) M = R$$

where $R = I_n - E_y C_y$.

The design of the H_∞ -OBTC can be summarized as follows: find matrices N , E_y , J_y , M and H such that conditions (i)-(iv) are satisfied.

Remark 1. *Once matrix E_y is computed, the gain matrices H and M can be easily deduced from conditions (iii) and (iv) in Proposition 1.* \square

Firstly, by using the definition of R , equation (ii) is rewritten as

$$N + KC_y + E_y C_y A = A \quad (17)$$

where $K = J_y - NE_y$.

In addition, equation (17) can be written in a compact form as a linear system given as

$$\begin{bmatrix} N & K & E_y \end{bmatrix} \underbrace{\begin{bmatrix} I_n \\ C_y \\ C_y A \end{bmatrix}}_{\mathcal{M}_1} = \underbrace{\begin{bmatrix} A \end{bmatrix}}_{\mathcal{M}_2} \quad (18)$$

which can be solved easily.

The necessary and sufficient condition for the existence of the solution to (18) is given by the following lemma.

Lemma 3. *[34] A solution to (18) exists if and only if*

$$\text{rank} \begin{bmatrix} \mathcal{M}_1 \\ \mathcal{M}_2 \end{bmatrix} = \text{rank} \left[\mathcal{M}_1 \right] . \quad (19)$$

\square

Remark 2. \mathcal{M}_1 is a full column rank matrix, which implies that condition (19) is always satisfied. \square

Then, the general solution to (18) is given by

$$\begin{bmatrix} N & K & E_y \end{bmatrix} = \mathcal{M}_2 \mathcal{M}_1^+ - Z(I - \mathcal{M}_1 \mathcal{M}_1^+) \quad (20)$$

where \mathcal{M}_1^+ is a generalized inverse matrix of \mathcal{M}_1 [34] (i.e $\mathcal{M}_1 = \mathcal{M}_1 \mathcal{M}_1^+ \mathcal{M}_1$) and Z is an arbitrary matrix of appropriate dimensions.

From (20), we obtain

$$N = \mathbb{A}_N - Z\mathbb{B}_N \quad (21a)$$

$$K = \mathbb{A}_K - Z\mathbb{B}_K \quad (21b)$$

$$E_y = \mathbb{A}_{E_y} - Z\mathbb{B}_{E_y} \quad (21c)$$

$$R = \mathbb{A}_R - Z\mathbb{B}_R \quad (21d)$$

where

$$\begin{aligned} \mathbb{A}_N &= (\mathcal{M}_2 \mathcal{M}_1^+) \begin{bmatrix} I & 0 & 0 \end{bmatrix}^T \\ \mathbb{B}_N &= (I - \mathcal{M}_1 \mathcal{M}_1^+) \begin{bmatrix} I & 0 & 0 \end{bmatrix}^T \\ \mathbb{A}_K &= (\mathcal{M}_2 \mathcal{M}_1^+) \begin{bmatrix} 0 & I & 0 \end{bmatrix}^T \\ \mathbb{B}_K &= (I - \mathcal{M}_1 \mathcal{M}_1^+) \begin{bmatrix} 0 & I & 0 \end{bmatrix}^T \\ \mathbb{A}_{E_y} &= (\mathcal{M}_2 \mathcal{M}_1^+) \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T \\ \mathbb{B}_{E_y} &= (I - \mathcal{M}_1 \mathcal{M}_1^+) \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T \\ \mathbb{A}_R &= I_n - \mathbb{A}_{E_y} C_y \\ \mathbb{B}_R &= -\mathbb{B}_{E_y} C_y \end{aligned}$$

Matrices J_y , H and M are deduced directly from the matrix gain E_y as

$$J_y = K + NE_y \quad (22a)$$

$$H = (I_n - E_y C_y) B_u \quad (22b)$$

$$M = (I_n - E_y C_y) \quad (22c)$$

Now, all gain matrices of (12a-12b) can be computed from the gain matrix parameter Z , such that all the constraints (ii), (iii) and (iv) in Proposition 1 are satisfied.

Moreover, using gain matrices (21a) and (21d), the dynamics of estimation error (16) can be rewritten in new compact form as

$$\dot{e}(t) = \tilde{A}e(t) + \tilde{B}w(t) \quad (23)$$

$$\tilde{y}(t) = \tilde{C}e(t) + \tilde{D}w(t) \quad (24)$$

where

$$\tilde{A} = \begin{bmatrix} (A - B_u K_u) & -B_u K_u \\ 0 & (\mathbb{A}_N - Z\mathbb{B}_N) \end{bmatrix} \quad (25)$$

$$\tilde{B} = \begin{bmatrix} -B_w \\ (\mathbb{A}_R - Z\mathbb{B}_R)B_w \end{bmatrix} \quad (26)$$

$$\tilde{C} = I_{2n} \quad (27)$$

$$\tilde{D} = 0 \quad (28)$$

$$e(t) = \begin{bmatrix} e_x(t) \\ e_{\hat{x}}(t) \end{bmatrix} \quad (29)$$

Remark 3. In the next section, the design of H_∞ -OBTC (12) is reduced to find the matrix Z , which satisfies condition (i) of Proposition 1. \square

4.2. Observer-based Tracking Controller design

This section concerns the robust stability analysis problem for the dynamics of estimation error (23) using the bounded real Lemma 1. The impact of the disturbances $w(t)$ on the estimation error $e(t)$ is minimized if the cost function (5) in Definition 2 holds, where L_2 -gain is given by γ .

In this theoretical part, we study the problem in a general way. That is, we propose two algorithms to solve the matrix inequalities obtained. In addition, the observer and controller gain matrices can be deduced from the solutions obtained.

4.2.1. Formulation of the robust stability analysis problem in a BMI form

Firstly, a sufficient condition for the robust stability of the dynamics of the estimation error (23) is given in the following theorem.

Theorem 1. The system described by (12) is an H_∞ -OBTC of system (10) if there are two positive definite matrices P_1 and P_2 , matrices \mathcal{Y}_1 and \mathcal{Y}_2 , and a minimal positive scalar attenuation

γ satisfying the following matrix inequality:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \star & -\gamma I & 0 \\ \star & \star & -\gamma I \end{bmatrix} < 0 \quad (30)$$

$$\Pi_{11} = \text{Sym} \left\{ \begin{bmatrix} (AP_1 - B_u \mathcal{Y}_1) & -B_u K_u \\ 0 & (P_2 \mathbb{A}_N - \mathcal{Y}_2 \mathbb{B}_N) \end{bmatrix} \right\}$$

$$\Pi_{12} = \begin{bmatrix} -B_w \\ (P_2 \mathbb{A}_R - \mathcal{Y}_2 \mathbb{B}_R) B_w \end{bmatrix}$$

$$\Pi_{13} = \text{diag}\{P_1, I_n\}$$

$$\mathcal{Y}_1 = K_u P_1$$

$$\mathcal{Y}_2 = P_2 Z$$

Moreover, the matrix parameter Z and the controller K_u gain are deduced from the expression of \mathcal{Y}_1 and \mathcal{Y}_2 , respectively. In addition, all the gain matrices of system (12) can be computed by equations (21 and (22) . \square

Proof. By letting $P = \text{diag}\{P_1^{-1}, P_2\}$, $A_0 = \tilde{A}$, $B_0 = \tilde{B}$, $C_0 = \tilde{C}$ and $D_0 = \tilde{D}$ where the system matrices \tilde{A} , \tilde{B} , \tilde{C} and \tilde{D} are given in (25), inequality (6) given in Lemma 1 can be rewritten as

$$\begin{bmatrix} \text{Sym} \left\{ \begin{bmatrix} P_1^{-1} & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} (A - B_u K_u) & -B_u K_u \\ 0 & (\mathbb{A}_N - Z \mathbb{B}_N) \end{bmatrix} \right\} \begin{bmatrix} P_1^{-1} & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} -B_w \\ (\mathbb{A}_R - Z \mathbb{B}_R) B_w \\ -\gamma I \\ \star \end{bmatrix} \text{diag}\{I_n, I_n\} \\ \star & 0 \\ \star & -\gamma I \end{bmatrix} < 0 \quad (31)$$

Inequality (31) is equivalent to

$$\begin{bmatrix} \text{Sym} \left\{ \begin{bmatrix} (P_1^{-1} A - P_1^{-1} B_u K_u) & -P_1^{-1} B_u K_u \\ 0 & (P_2 \mathbb{A}_N - P_2 Z \mathbb{B}_N) \end{bmatrix} \right\} \begin{bmatrix} -P_1^{-1} B_w \\ (P_2 \mathbb{A}_R - P_2 Z \mathbb{B}_R) B_w \\ -\gamma I \\ \star \end{bmatrix} \text{diag}\{I_n, I_n\} \\ \star & 0 \\ \star & -\gamma I \end{bmatrix} < 0 \quad (32)$$

Now, we can easily deduce that inequality (30) is obtained by pre- and post-multiplying inequality (32) by suitable matrices given as $\text{diag}\{P_1, I, I, I, I\}$ and $\text{diag}\{P_1, I, I, I, I\}^T$.

$$\begin{bmatrix} \text{Sym} \left\{ \begin{bmatrix} (AP_1 - B_u K_u P_1) & -B_u K_u \\ 0 & (P_2 \mathbb{A}_N - P_2 Z \mathbb{B}_N) \end{bmatrix} \right\} \begin{bmatrix} -B_w \\ (P_2 \mathbb{A}_R - P_2 Z \mathbb{B}_R) B_w \\ -\gamma I \\ \star \end{bmatrix} \text{diag}\{P_1, I_n\} \\ \star & 0 \\ \star & -\gamma I \end{bmatrix} < 0 \quad (33)$$

From the above results, we can deduce a sufficient condition ensuring that the L_2 -gain of system (12) is bounded by γ as follows: if there are two positive definite matrices P_1 , P_2 , two matrices K_u and Z , and a positive scalar γ such that the matrix inequality (30) holds then the L_2 -gain is bounded by γ , which completes the proof. \square

Remark 4. *It is easy to remark that the H_∞ observer-based control synthesis problem given by inequality (30) is a non convex problem. This is due to the presence of the isolated variable K_u and a new decision variable \mathcal{Y}_1 , which is a product between the two decision matrices P_1 and K_u . This leads to a Bi-linear Matrix Inequality (BMI) structure. Then, inequality (30) can not be solved for $(P_1, P_2, K_u, \mathcal{Y}_1, \mathcal{Y}_2)$ at the same time.*

To overcome the problem mentioned in Remark 4, a two-step algorithm procedure can be used (see [35, 10] and references therein). We start by solving the first component of inequality (30). In the second step, after computing matrices P_1 and \mathcal{Y}_1 , which satisfy the first component of inequality (30), we replace them in inequality (30) with the numerical values obtained resulting in an LMI formulation. Finally, the LMI obtained is solved using any free solver.

4.2.2. Formulation of the robust stability analysis problem in LMI form

The easiest algorithm for solving matrix inequalities is to convert them to LMI. A convex objective function can thus solved with convex constraints.

The following Theorem allows us to obtain an optimization problem given in LMI form, by increasing the non-linear term and removing the isolated variable when analysing the robust stability of the estimation error dynamics (23).

Theorem 2. *The system described by (12) is an H_∞ -OBTC of system (10) if there are two positive definite matrices P_1 and P_2 , matrices \mathcal{Y}_1 and \mathcal{Y}_2 , and a minimal positive attenuation scalar γ satisfying the following LMI subject to an optimization problem*

$$\min (\gamma > 0) \quad \text{subject to}$$

$$\begin{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \star & \Omega_{22} \end{bmatrix} & \begin{bmatrix} P_1 \Lambda_1 \\ \Lambda_2 \end{bmatrix}^T \\ \star & \begin{bmatrix} -P_1 & 0 \\ 0 & -P_1 \end{bmatrix} \end{bmatrix} < 0 \quad (34)$$

where

$$\Omega_{11} = \begin{bmatrix} \text{Sym}\{(AP_1 + B_u\mathcal{Y}_1)\} & 0 \\ \star & (P_2\mathbb{A}_N - \mathcal{Y}_2\mathbb{B}_N) \end{bmatrix} \quad (35)$$

$$\Omega_{12} = \begin{bmatrix} -B_w & P_1 & 0 \\ (P_2\mathbb{A}_R - \mathcal{Y}_2\mathbb{B}_R) & 0 & I \end{bmatrix} \quad (36)$$

$$\Omega_{22} = \begin{bmatrix} -\gamma I & 0 & 0 \\ \star & -\gamma I & 0 \\ 0 & 0 & -\gamma I \end{bmatrix} \quad (37)$$

$$\Lambda_1 = \begin{bmatrix} K_u^T B_u^T & 0 & 0 & 0 & 0 \end{bmatrix} \quad (38)$$

$$\Lambda_2 = \begin{bmatrix} 0 & I & 0 & 0 & 0 \end{bmatrix} \quad (39)$$

$$\mathcal{Y}_1 = K_u P_1 \quad (40)$$

$$\mathcal{Y}_2 = P_2 Z \quad (41)$$

Moreover, the matrix parameter Z and the controller K_u gain are given by $Z = P_2^{-1}\mathcal{Y}_2$ and $K_u = \mathcal{Y}_1 P_1^{-1}$, respectively. In addition, all the gain matrices of system (12) can be computed by equations (21) and (22). \square

Proof. We start by replacing matrices A_0 , B_0 , C_0 , and D_0 in inequality (6) by the expressions given in the augmented system (23) \tilde{A} , \tilde{B}_w , \tilde{C} , and \tilde{D} , respectively, and matrix P by $\text{diag}\{P_1^{-1}, P_2\}$. The expression of inequality (6) given in Lemma 1 can be rewritten as

$$\begin{bmatrix} \text{Sym}\left\{ \begin{bmatrix} P_1^{-1} & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} (A-B_u K_u) & -B_u K_u \\ 0 & (\mathbb{A}_N - Z\mathbb{B}_N) \end{bmatrix} \right\} \begin{bmatrix} P_1^{-1} & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} -B_w \\ (\mathbb{A}_R - Z\mathbb{B}_R)B_w \\ -\gamma I \\ \star \end{bmatrix} \begin{matrix} \text{diag}\{I_n, I_n\} \\ 0 \\ -\gamma I \end{matrix} \right\} < 0 \quad (42)$$

Now, pre- and post- multiplication of inequality (42) obtained with matrices $\text{diag}\{P_1, I, I, I, I\}$ and $\text{diag}\{P_1, I, I, I, I\}^T$, respectively, lead to an inequality represented as the sum of a matrix with a new decision variable \mathcal{Y}_1 , which is a product between P_1 and K_u , and a matrix containing the isolated variables K_u given as

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \star & \Omega_{22} \end{bmatrix} + \text{Sym}\{\Lambda_1^T \Lambda_2\} < 0 \quad (43)$$

where

$$\begin{aligned}
\Omega_{11} &= \begin{bmatrix} \text{Sym}\{(AP_1 + B_u\mathcal{Y}_1)\} & 0 \\ \star & (P_2\mathbb{A}_N - \mathcal{Y}_2\mathbb{B}_N) \end{bmatrix} \\
\Omega_{12} &= \begin{bmatrix} -B_w & P_1 & 0 \\ (P_2\mathbb{A}_R - \mathcal{Y}_2\mathbb{B}_R) & 0 & I \end{bmatrix} \\
\Omega_{22} &= \begin{bmatrix} -\gamma I & 0 & 0 \\ \star & -\gamma I & 0 \\ 0 & 0 & -\gamma I \end{bmatrix} \\
\Lambda_1 &= \begin{bmatrix} K_u^T B_u^T & 0 & 0 & 0 & 0 \end{bmatrix} \\
\Lambda_2 &= \begin{bmatrix} 0 & I & 0 & 0 & 0 \end{bmatrix} \\
\mathcal{Y}_1 &= K_u P_1 \\
\mathcal{Y}_2 &= P_2 Z
\end{aligned}$$

Inequality (43) is not an LMI. The main idea to make this inequality linear and overcome this problem is to use Lemma 2 to eliminate the isolated variable K_u with respect to the decision variables P_1 and \mathcal{Y}_1 . Consequently, we obtain

$$\text{Sym}\{\Lambda_1^T \Lambda_2\} \leq - \begin{bmatrix} P_1 \Lambda_1 \\ \Lambda_2 \end{bmatrix}^T \begin{bmatrix} -P_1 & 0 \\ 0 & -P_1 \end{bmatrix}^{-1} \begin{bmatrix} P_1 \Lambda_1 \\ \Lambda_2 \end{bmatrix} \quad (44)$$

Then, from inequalities (43) and (44), and by using the Schur lemma, the following LMI is obtained:

$$\begin{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \star & \Omega_{22} \end{bmatrix} & \begin{bmatrix} P_1 \Lambda_1 \\ \Lambda_2 \end{bmatrix}^T \\ \star & \begin{bmatrix} -P_1 & 0 \\ 0 & -P_1 \end{bmatrix} \end{bmatrix} < 0 \quad (45)$$

which is the same as inequality (45) given in the optimization problem of Theorem 2.

The robust stability of the estimation error and the tracking error given by (16) is satisfied if the optimization problem (45) has a solution. This completes the proof. \square

5. Numerical Simulations

5.1. Solving optimization problems with LMI or BMI constraints

Consider the following state space representation that models the tramway dynamics

$$\dot{x}(t) = Ax(t) + B_u u(t) + d_r + B_w w(t) \quad (46a)$$

$$y(t) = C_y x(t) \quad (46b)$$

where the disturbance $w(t)$ models the poor manoeuvres or behaviour of the driver, the components of the state vector $x \in \mathbb{R}^2$ are position $p(t)$ and speed $v(t)$, $u \in \mathbb{R}$ is the input vector representing the traction force, and $y \in \mathbb{R}$ is the measurement output vector. A , B_u , B_w , d_r and C_y are known matrices of appropriate dimensions.

$$x(t) = \begin{bmatrix} p(t) \\ v(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b_1}{m} \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$C_y = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad d_r = \begin{bmatrix} 0 \\ -\left(\frac{a_1}{m} + g \sin(\beta) + g \frac{k_e}{r_c}\right) \end{bmatrix}$$

By applying Theorem 1, we verify the feasibility of BMI (30) by following the two-step algorithm procedure to overcome the problem mentioned in Remark 4. Inequality (30) was found to be feasible with $\gamma = 2.746$, which is considered as the minimal γ value obtained to satisfy criterion (5). Albeit, the design methodology proposed in Theorem 2 performs successfully by choosing the same design parameters as below. By solving LMI (45), the minimal γ value found, which satisfies the criterion (5) is $\gamma = 0.46079$.

$$P_1 = \begin{bmatrix} 0.19416 & -0.072379 \\ -0.072379 & 0.10574 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 2.5114e^{+08} & -4.804e^{+06} \\ -4.804e^{+06} & 7.7736e^{+07} \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 1409.2 & 2780 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 1.4099e^{+08} & 5.469e^{+07} & -1.4099e^{+08} & -5.469e^{+07} \\ 5.469e^{+07} & 1.9654e^{+08} & -5.469e^{+07} & -1.9654e^{+08} \end{bmatrix}$$

Therefore, we can easily compute the controller gain and the matrix parameter of the observer as follows:

$$K_u = \mathcal{Y}_1 P_1^{-1} = \begin{bmatrix} 22903 & 41967 \end{bmatrix}$$

$$Z = P_2^{-1} \mathcal{Y}_2 = \begin{bmatrix} 0.57556 & 0.26645 & -0.57556 & -0.26645 \\ 0.7391 & 2.5447 & -0.7391 & -2.5447 \end{bmatrix}$$

Finally, the observer-based controller is given by

$$D^{0.5} \eta(t) = \begin{bmatrix} -0.57556 & 0.23355 \\ -0.7391 & -2.5466 \end{bmatrix} \eta(t) + \begin{bmatrix} 0 \\ 2.5312e^{-05} \end{bmatrix} u(t) \begin{bmatrix} 0.72832 \\ -6.303 \end{bmatrix} y(t) + \begin{bmatrix} 0.23355 & 0 \\ -2.5428 & 1 \end{bmatrix} d_r \quad (47a)$$

$$\hat{x}(t) = \eta(t) + \begin{bmatrix} 0.76645 \\ 2.5428 \end{bmatrix} y(t) \quad (47b)$$

$$u(t) = u_r - \begin{bmatrix} 22903 & 41967 \end{bmatrix} (x_r(t) - \hat{x}(t)) \quad (47c)$$

The driver controls the position of the controller, which represents the traction force applied such that the planned trajectory is never followed exactly. The error between the applied and planned commands is represented as a disturbance $w(t)$ plotted in figure 1.

Figures 3 to 6 show the performances of the proposed H_∞ OBC in a closed loop. The robust estimation and reference tracking errors are illustrated in figures 3 and 4.

Due to a scale effect, different time intervals in figure 5 are zoomed in figures 6 to 8 in order to show the influence of the disturbance w on the estimation and tracking errors. Firstly, convergence to zero of the estimation and tracking errors is ensured when the applied and recommended control coincide perfectly, i.e. the disturbance w is equal to zero. Conversely, the disturbance effect on the errors is minimized with respect to the magnitude of this disturbance.

Therefore, figures 1 to 6 show that the proposed H_∞ OBC (12) has robustly tracked the tramway system eco-driving trajectory, despite driver-induced disturbances, with a given disturbance attenuation level γ .

5.2. Discussion

As stated previously, the main idea of the proposed method is to minimize the effect of disturbances. In an open loop, the control law is never robust because it requires precise

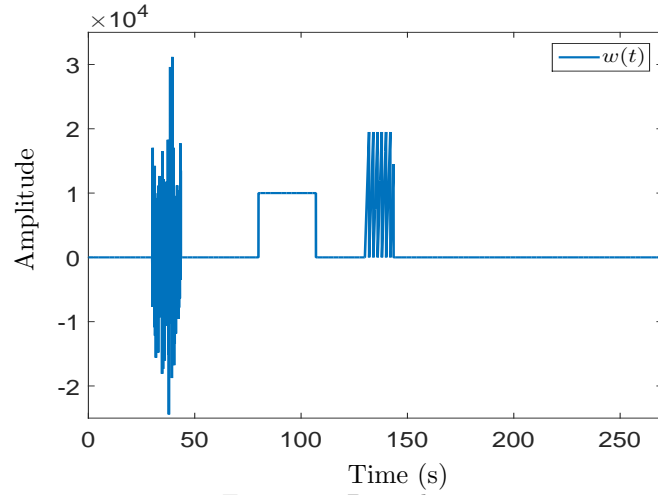


Figure 1: Disturbances

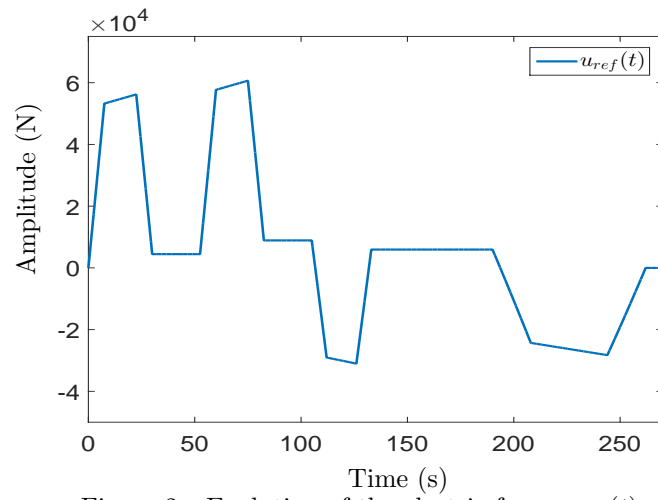


Figure 2: Evolution of the electric force $u_{ref}(t)$

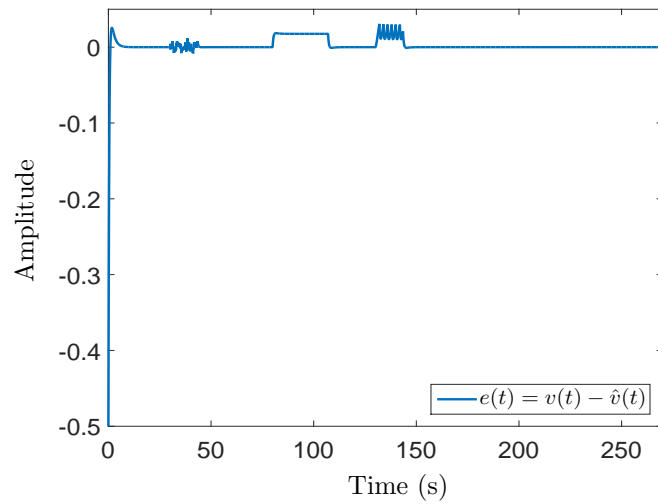


Figure 3: Evolution of the estimation error $e(t) = v(t) - \hat{v}(t)$

knowledge of the state space model, which is complicated or even impossible in the presence of uncertainties or disturbances. The advantage of the proposed methodology is that feedback

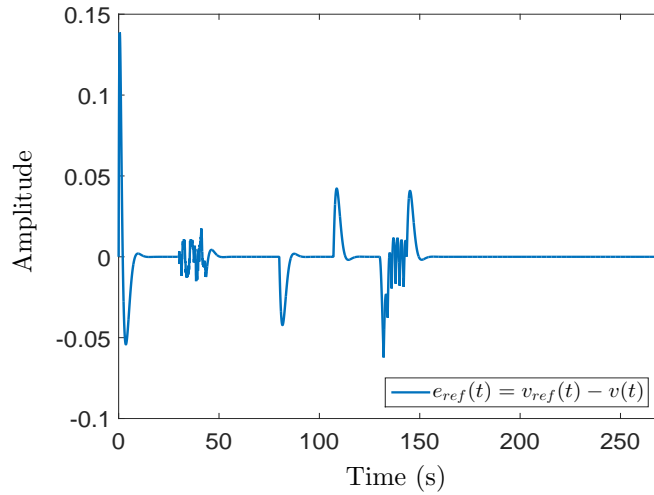


Figure 4: Evolution of the tracking error $e_{ref}(t) = v_{ref}(t) - v(t)$

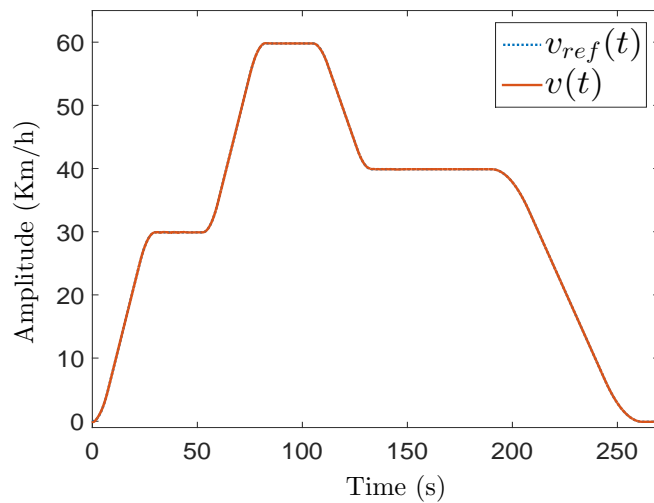


Figure 5: Evolution of the reference velocity $v_{ref}(t)$ and the real controlled velocity $v(t)$

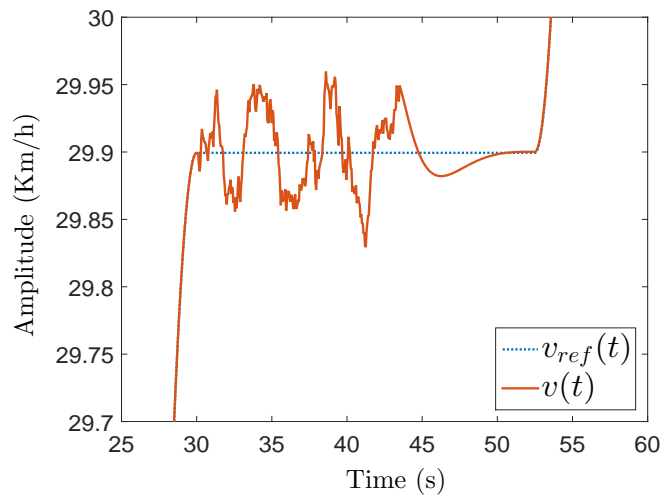


Figure 6: Behaviour of the reference velocity $v_{ref}(t)$ and the real controlled velocity $v(t)$ for $t \in [25, 60]$

based on the estimated state is included in the control. This limits the tracking error of the reference trajectory. In addition, the feedback control law does not allow the imposed speed

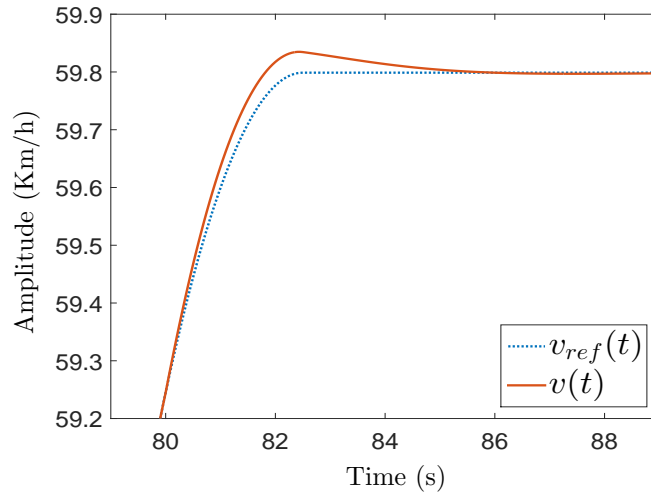


Figure 7: Behaviour of the reference velocity $v_{ref}(t)$ and the real controlled velocity $v(t)$ for $t \in [79, 89]$

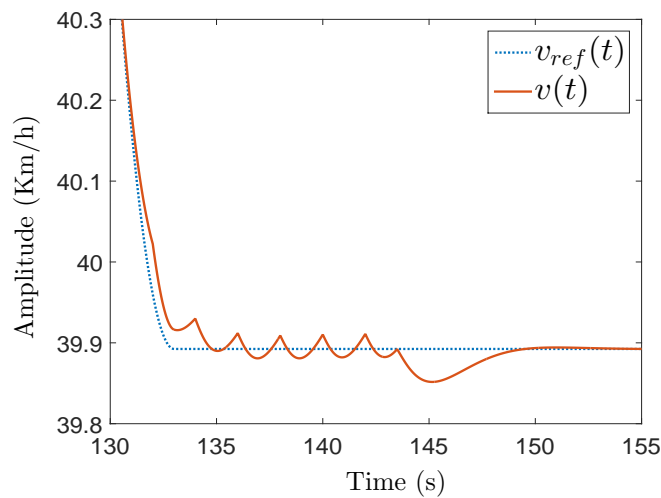


Figure 8: Behaviour of the reference velocity $v_{ref}(t)$ and the real controlled velocity $v(t)$ for $t \in [130, 155]$

limit to be exceeded.

The aim of this work is to simulate the proposed methodology on the PSCHITT platform (French acronym for “Hybrid and Inter-modal Collaborative Simulation Platform in Land Transport”). The PSCHITT platform is a versatile simulator that can be fitted with different cabins (Persons with Reduced Mobility, Rail...) according to the scientific objectives and experimental needs, for example, PSCHITT-Rail¹. For this, a simulation protocol must be prepared because the results obtained using the PSCHITT-Rail simulator will be of significance if professional drivers are involved.

¹<https://www.youtube.com/watch?v=71UBhcCK7hc&t=15s>

6. Conclusion

In this work, an H_∞ Observer-Based Controller was designed to track a reference tramway system eco-driving trajectory. Poor driver behaviour was modelled as disturbances with finite energy, which must be rejected. The conditions of existence of the H_∞ -OBTC were obtained. In addition, observer gain matrices parametrized with a unique gain matrix. Thanks to the Bounded Real Lemma, the problem of the robust stability of the estimation errors and the rejection of the effects of disturbances were formulated as an optimization problem with an LMI constraint to guarantee the H_∞ disturbance attenuation level.

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