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Reduced-Complexity LMI Conditions for Admissibility Analysis and Control Design of Singular Nonlinear Systems

Amine Dehak, Anh-Tu Nguyen*, *Senior Member, IEEE*, Antoine Dequidt, Laurent Vermeiren, Michel Dambrine

Abstract—We present a reduced-complexity control approach for a class of descriptor nonlinear systems with a nonlinear derivative matrix, possibly singular. To this end, a systematic approach is proposed to obtain an *equivalent* polytopic representation of a given nonlinear system within a compact set of the state space. This modeling approach has two particular features compared to the related Takagi-Sugeno (TS) fuzzy model-based framework. First, the model complexity only grows *proportionally*, rather than exponentially, with the number of premise variables. Second, the vertices of the proposed polytopic models can admit an *infinite* number of representations for the same predefined set of premise variables. This *non-uniqueness* feature allows introducing some specific slack variables at the modeling step to reduce the control design conservatism. Based on the proposed polytopic representation and Lyapunov stability theory, we derive reduced-complexity admissibility analysis and design conditions, expressed in terms of linear matrix inequalities, for the considered class of descriptor systems. In particular, a new nonlinear control law is proposed for regular descriptor systems to avoid using the extended redundancy form, which may yield numerically complex and conservative results due to the imposed special control structure. Both numerical and physically motivated examples are given to demonstrate the interests of the new control approach with respect to existing TS fuzzy model-based control results.

Index Terms—Admissibility, Takagi-Sugeno fuzzy descriptor systems, linear matrix inequality (LMI), complexity reduction.

I. INTRODUCTION

Differential-algebraic systems, also known as descriptor systems or generalized state-space systems, provide a natural framework to represent and analyze a large class of engineering applications, including robotics [1], rehabilitation systems [2], chemical processes [3], transportation and power systems [4], etc. However, the analysis of such systems requires more involved techniques when compared to the classical state-space systems since not only stability but also regularity and admissibility have to be addressed [5], [6]. Stability analysis of descriptor systems has been classically studied based on the system index or the coordinates reduction techniques [4]. However, such methods require extensive algebraic manipulations, which can be unsuitable for a large class of engineering

problems [7]. To avoid this drawback, Lyapunov methods directly based on descriptor system formulation have been proposed [8]–[11]. Despite significant advances in numerical analysis and simulation, the problem of stability analysis and control for general descriptor systems remains challenging, especially for systems with a large number of nonlinearities.

Takagi-Sugeno (TS) fuzzy paradigm has become one of the most popular techniques for nonlinear control systems [12]–[15], which is due to several factors. First, TS fuzzy models can be used to approximate any smooth nonlinear system with any degree of accuracy. In particular, an exact TS fuzzy model of a given nonlinear system can be obtained within a state-space compact set via the sector nonlinearity approach [12]. Second, using TS fuzzy modeling and Lyapunov stability theory, stability analysis and control design conditions can be derived for nonlinear systems in the form of linear matrix inequalities (LMIs), which can be efficiently solved using convex optimization techniques [16]. Third, TS fuzzy model-based technique has been successfully applied to various engineering applications [17]–[19]. From a theoretical viewpoint, it is possible to derive asymptotically necessary and sufficient stability conditions for TS fuzzy systems [20]. Nevertheless, in practice these stability conditions are conceptual rather than implementable since the computational burden swiftly increases such that most numerical solvers crash [15]. This leads to another challenge of TS fuzzy approaches in deriving less conservative sufficient conditions for stability analysis of nonlinear systems with a reasonable numerical burden. To reduce the design conservatism, various fuzzy Lyapunov functions have been effectively exploited for TS fuzzy systems [9], [21]–[24]. In contrast to quadratic stability, exploiting the information on the membership functions and their time-derivatives plays a key role for fuzzy Lyapunov-based stability of TS fuzzy systems [9], [24]. However, the information on the time-derivative of premise variables is generally not available for control design, which implies much more numerical and theoretical challenges when using fuzzy Lyapunov functions for TS fuzzy stability analysis and control design. Note that within local TS fuzzy control framework, it is still not possible to *theoretically* demonstrate that nonquadratic results include those derived from quadratic Lyapunov functions [15].

Concerning descriptor nonlinear systems, most of TS fuzzy based approaches have been devoted to the case with regular derivative matrices, see for instance [25]–[28] and related references, for which the regularity and admissibility analysis is not required. However, such a class of descriptor systems

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cannot take into account the physical differential-algebraic equations into the model description. TS fuzzy model-based admissibility analysis conditions, in the form of LMI constraints, have been proposed in [29]–[37]. However, these results can only be applied to singular TS fuzzy systems with a constant derivative matrix, which may be restrictive for many engineering problems. To avoid this drawback, LMI-based design conditions have been developed for singular TS fuzzy systems with a nonlinear derivative matrix, which is more challenging [9], [11], [38]. For existing TS fuzzy approaches, the numerical complexity of stability analysis, observation and control design conditions *exponentially* grows with respect to the number of premise variables [12]. This limits the applicability of these results to systems with only few nonlinearities. To overcome this major limitation, several approaches have been proposed to reduce the numerical complexity of TS fuzzy systems. A singular value decomposition was proposed in [39] to reduce the fuzzy rules, leading to approximate TS fuzzy models. The authors in [40] proposed to transform some nonlinearities into system uncertainties to reduce the number of vertices of TS fuzzy models. Based on the linear dependencies between the TS fuzzy local sub-models obtained with the sector nonlinearity approach, a reduced-complexity model can be directly derived from the initial TS fuzzy model in [41]. However, the reduction methods in [40], [41] can lead to over-conservative design results. Data-based methods have been proposed for dimensionality reduction of the premise variables, *e.g.*, principal component analysis [42], deep neural network [43]. However, these modeling methods suffer some major drawbacks. First, we can only deal with nonlinear systems in the sense of approximation, which could be unsuitable for fast dynamical systems. Second, these methods fundamentally rely on experimental data obtained from typical trajectories of the premise variables, which require not only additional optimization steps but also extensive simulations to collect data. Third, due to the “data-based” feature, the stability analysis and control performance obtained with the resulting TS fuzzy models highly depend on the collected data.

Motivated by the above technical and practical issues, this paper aims at finding an effective solution to reduce the numerical complexity of LMI-based stability analysis and design conditions with respect to classical TS fuzzy approaches. A special focus is paid on a class of descriptor systems with a nonlinear derivative matrix. We propose a new approach to derive an equivalent polytopic representation for a given nonlinear system within a compact set. Although all powerful tools of TS fuzzy framework can be directly applied to the proposed approach, the model complexity only grows *proportionally* with the number of premise variables, rather than *exponentially* when compared to the conventional TS fuzzy modeling. Moreover, for the same predefined set of premise variables, the vertices of the proposed polytopic models can admit an *infinite* number of representations. This non-uniqueness feature allows introducing specific slack variables at the modeling step, which are useful to reduce the control design conservatism. Using the proposed modeling and the descriptor redundancy approach [25], [27], reduced-complexity admissibility analysis and control design condi-

tions for singular nonlinear systems are derived in terms of LMIs. Then, we propose a new nonlinear control law for the nonsingular case to avoid using the extended redundancy form, which is unnecessarily complex from a numerical viewpoint and may yield conservative results due to the imposed special control structure as similarly discussed in [26], [28] for fuzzy descriptor observer design. The main contributions can be summarized as follows.

- A reduced-complexity polytopic representation of nonlinear systems, which offers a possibility to introduce specific slack variables for control design relaxation.
- LMI-based admissibility analysis and control design conditions for descriptor nonlinear systems with less complex and less conservative results compared to related TS fuzzy model-based results.

This paper is organized as follows. Section II formalizes the control problem of singular nonlinear systems. Section III presents the LMI-based control design of descriptor nonlinear systems for both singular and regular cases. Numerical and physically motivated examples are given in Section IV to demonstrated the proposed control approach. Section V concludes the paper and discusses future works.

Notations. \mathcal{I}_n denotes the set of natural numbers $\{1, 2, \dots, n\}$. For a vector z , we denote by z_i its i th entry. $\mathbb{R}^{m \times n}$ denotes the set of $m \times n$ matrices with real elements. I is an identity matrix of appropriate dimensions. For a matrix X , X^\top is the transpose of X , X^{-1} is its inverse, $\det(X)$ is its determinant, and $\text{He}[X] = X + X^\top$. The expression $X \succ Y$ (respectively $X \prec Y$) means that $X - Y$ (respectively $Y - X$) is a symmetric positive (respectively negative) definite matrix. The symbol \star stands for matrix blocks that can be deduced by symmetry. Arguments are omitted when their meaning is straightforward.

II. PROBLEM FORMULATION

This section first recalls preliminary results on admissibility analysis and TS fuzzy modeling of singular nonlinear systems. Then, we propose a new polytopic representation to reduce the numerical complexity compared to the conventional TS fuzzy technique. Finally, the control problem is formalized.

A. Preliminaries

We consider the nonlinear differential-algebraic system

$$\begin{aligned} E_1(z)\dot{x}_d(t) &= A_{11}(z)x_d(t) + A_{12}(z)x_a(t) + B_1(z)u(t), (1a) \\ 0 &= A_{21}(z)x_d(t) + A_{22}(z)x_a(t) + B_2(z)u(t), (1b) \end{aligned}$$

where $x_d(t) \in \mathbb{R}^q$ is the differential state vector, $x_a(t) \in \mathbb{R}^s$ is the vector of algebraic variables, and $u(t) \in \mathbb{R}^m$ is the control input. The vector of premise variables $z(t) \in \mathbb{R}^r$ continuously depends on the system state and

$$z(t) \in \mathcal{D}_z = \{z \in \mathbb{R}^r : z_{i \min} \leq z_i \leq z_{i \max}, \forall i \in \mathcal{I}_r\}, (2)$$

where $z_{i \min}$ and $z_{i \max}$ are given bounds. The system matrices $E_1(z) \in \mathbb{R}^{q \times q}$, $A_{11}(z) \in \mathbb{R}^{q \times q}$, $A_{12}(z) \in \mathbb{R}^{q \times s}$, $A_{21}(z) \in \mathbb{R}^{s \times q}$, $A_{22}(z) \in \mathbb{R}^{s \times s}$, $B_1(z) \in \mathbb{R}^{q \times m}$, and $B_2(z) \in \mathbb{R}^{s \times m}$ *affinely* depend on $z(t)$. Moreover, we consider the case where

$E_1(z)$ is regular for all $z \in \mathcal{D}_z$. For compactness, system (1) can be rewritten in the singular form

$$E(z)\dot{x}(t) = A(z)x(t) + B(z)u(t), \quad (3)$$

where $x = [x_d^\top \ x_a^\top]^\top \in \mathbb{R}^n$, with $n = q + s$, $E(z) = \text{diag}(E_1(z), 0)$ and

$$A(z) = \begin{bmatrix} A_{11}(z) & A_{12}(z) \\ A_{21}(z) & A_{22}(z) \end{bmatrix}, \quad B(z) = \begin{bmatrix} B_1(z) \\ B_2(z) \end{bmatrix}.$$

Remark 1. Various engineering systems can be represented in the form (1), e.g., constrained robot systems [44], chemical processes [3], etc. Moreover, if $x = x_d$ and $x_a \in \emptyset$, then system (1a) becomes regular, which has been widely studied in the literature [1], [12], [25], [40].

The following definition is important to analyze the regularity and impulse-free proprieties of system (3).

Definition 1 ([45]). The unforced singular system (3), i.e., $u = 0$, is of index-one if there exists a unique solution $x_a = f(x_d)$ in some neighborhood of the equilibrium $x(0) = 0$ satisfying $f(0) = 0$ and

$$A_{21}(z)x_d(t) + A_{22}(z)f(x_d(t)) = 0. \quad (4)$$

We consider the following assumption for system (1).

Assumption 1. The premise variables only depend on x_d , i.e., $z(x) = g(x_d)$, where $g: \mathbb{R}^q \rightarrow \mathbb{R}^r$ is a differentiable function with respect to x_d .

Under Assumption 1, the unforced system (3) is of index-one if $\det(A_{22}(z)) \neq 0$, $\forall z \in \mathcal{D}_z$. Moreover, in this case the unique solution of equation (4) is given by

$$x_a = f(x_d) = -A_{22}(z)^{-1}A_{21}(z)x_d. \quad (5)$$

Note from (5) that if $x_d \rightarrow 0$, then $x_a \rightarrow 0$. The following definition is useful for the admissibility analysis of system (1).

Definition 2. The unforced singular system (3), i.e., $u = 0$, is admissible if it is of index-one and its solution $x = 0$ is asymptotically stable.

The nonlinear singular system (3) can be described by the following TS fuzzy model with IF-THEN fuzzy rules:

$$\begin{aligned} \text{RULE } R_i: & \text{ IF } z_1(t) \text{ is } \mathcal{M}_1^i \text{ and } \dots \text{ and } z_r(t) \text{ is } \mathcal{M}_r^i, \\ & \text{ THEN } \tilde{E}_i \dot{x}(t) = \tilde{A}_i x(t) + \tilde{B}_i u(t), \end{aligned} \quad (6)$$

where $(\tilde{E}_i, \tilde{A}_i, \tilde{B}_i)$ are known constant matrices with appropriate dimensions, R_i denotes the i th fuzzy inference rule, and \mathcal{M}_j^i is the fuzzy set, for $i \in \mathcal{I}_{2r}$ and $j \in \mathcal{I}_r$. The fuzzy membership functions are defined as $h_i(z) = \frac{\prod_{j=1}^r \mu_j^i(z_j)}{\sum_{i=1}^{2r} \prod_{j=1}^r \mu_j^i(z_j)}$, $\forall i \in \mathcal{I}_{2r}$, where $\mu_j^i(z_j)$ represents the membership grade of z_j in the respective fuzzy set \mathcal{M}_j^i . Using the fuzzy method in [12], system (6) can be inferred as

$$\sum_{i=1}^{2r} h_i(z) \tilde{E}_i \dot{x}(t) = \sum_{i=1}^{2r} h_i(z) (\tilde{A}_i x(t) + \tilde{B}_i u(t)). \quad (7)$$

The membership functions $h_i(z)$, for $i \in \mathcal{I}_{2r}$, satisfy the property $\sum_{i=1}^{2r} h_i(z) = 1$ and $0 \leq h_i(z) \leq 1$, $\forall i \in \mathcal{I}_{2r}$.

TS fuzzy model-based technique has been shown as one of the most promising approaches for LMI-based admissibility analysis and control design of the singular system (3), see e.g., [11], [32], [34], [37], [38], [45]. However, the numerical complexity of model (7), specifically the number of vertices 2^r and by conjunction the number of related LMI conditions, exponentially grows with the number of premise variables r . This may prevent the use of the TS fuzzy model (7) to deal with complex engineering systems, i.e., $r \gg 1$. To avoid this major drawback, we propose hereafter a reduced-complexity polytopic representation of the nonlinear singular system (3), where the number of vertices only grows proportionally with r rather than exponentially as for TS fuzzy model (7).

B. Reduced-Complexity Polytopic Representation

Since the matrices $E(z)$, $A(z)$ and $B(z)$ of the nonlinear system (3) affinely depend on z , then they can be represented in the form

$$X(z) = \hat{X}_0 + \sum_{p=1}^r z_p \hat{X}_p, \quad (8)$$

where $X \in \{E, A, B\}$ and \hat{X}_i , for $i \in \mathcal{I}_r \cup \{0\}$, are constant matrices of appropriate dimensions, which can be directly obtained from the affine structure of $X(z)$. Moreover, for $z \in \mathcal{D}_z$, the term z_p can be convexly rewritten as

$$z_p = \omega_p^0(z) z_{p \min} + \omega_p^1(z) z_{p \max}, \quad (9)$$

with $\omega_p^0(z) = \frac{z_{p \max} - z_p}{z_{p \max} - z_{p \min}}$ and $\omega_p^1(z) = 1 - \omega_p^0(z)$, $\forall p \in \mathcal{I}_r$. Then, it follows from (9) that expression (8) can be equivalently rewritten as

$$\begin{aligned} X(z) &= \hat{X}_0 + \sum_{p=1}^r (\omega_p^1(z) z_{p \max} + \omega_p^0(z) z_{p \min}) \hat{X}_p \\ &= \sum_{p=1}^r (w_{2p-1}(z) X_{2p-1} + w_{2p}(z) X_{2p}) = \sum_{i=1}^{2r} w_i(z) X_i, \end{aligned} \quad (10)$$

with

$$\begin{aligned} w_{2p-1}(z) &= \frac{1}{r} \omega_p^1(z), \quad X_{2p-1} = \hat{X}_0 + r z_{p \max} \hat{X}_p, \\ w_{2p}(z) &= \frac{1}{r} \omega_p^0(z), \quad X_{2p} = \hat{X}_0 + r z_{p \min} \hat{X}_p. \end{aligned} \quad (11)$$

Using the matrix decomposition (10), the nonlinear singular system (3) can be rewritten in the polytopic form

$$\sum_{i=1}^{2r} w_i(z) E_i \dot{x}(t) = \sum_{i=1}^{2r} w_i(z) (A_i x(t) + B_i u(t)), \quad (12)$$

where the nonlinear functions $w_i(z)$, $\forall i \in \mathcal{I}_{2r}$, satisfy the following properties:

$$\begin{aligned} \sum_{i=1}^{2r} w_i(z) &= 1, \quad 0 \leq w_i(z) \leq 1, \\ w_{2p-1}(z) + w_{2p}(z) &= \frac{1}{r}, \quad \forall p \in \mathcal{I}_r. \end{aligned} \quad (13)$$

Let Ω be the set of nonlinear functions $w_i(z)$, $\forall i \in \mathcal{I}_{2r}$, satisfying (13), i.e., $w(z) = [w_1(z), w_2(z), \dots, w_{2r}(z)] \in \Omega$.

The vertices X_i can be directly obtained from $X(z)$, for $X \in \{E, A, B\}$ and $i \in \mathcal{I}_{2r}$. To this end, we define the vectors ζ^1, \dots, ζ^r forming the canonical basis of \mathbb{R}^r as

$$\zeta^i = [0, \dots, 0, \overbrace{1}^{\text{ith}}, 0, \dots, 0]^\top, \quad i \in \mathcal{I}_r.$$

Then, it follows from the matrix decomposition (10)–(11) that

$$X_i = \begin{cases} X(r \cdot z_{p \max} \cdot \zeta^p) & \text{if } i = 2p - 1, \\ X(r \cdot z_{p \min} \cdot \zeta^p) & \text{if } i = 2p, \end{cases} \quad \text{for } i \in \mathcal{I}_{2r}. \quad (14)$$

It is important to note that the polytopic model (12) is an *algebraic* rewriting of the nonlinear model (3), *i.e.*, there is no approximation error between these two models. For model (12), the nonlinear functions $w_i(z)$, $\forall i \in \mathcal{I}_{2r}$, *precisely* defined in (11), capture the nonlinearities in model (3).

Remark 2. Note that the nonlinear functions $w_j(z)$, $j \in \mathcal{I}_{2r}$, in (11), of the new polytopic model (12) only depend on the j th component z_j of z . Indeed, their construction is fundamentally different compared to that of the membership functions $h_i(z)$, for $i \in \mathcal{I}_{2r}$, of the TS fuzzy model (7), for which a combination product is required [12], *i.e.*,

$$h_i(z) = \prod_{p=1}^r \omega_p^{i_p}(z_p), \quad i \in \mathcal{I}_{2r}, \quad i_p \in \{0, 1\},$$

with $\omega_p^{i_p}(z_p)$ defined in (9). Hence, model (12) only has $2r$ vertices, which can significantly reduce the numerical complexity of LMI-based design conditions of system (3) compared to the conventional TS fuzzy model (7) with 2^r vertices. This feature is particularly interesting when dealing with nonlinear complex systems with a large number r of premise variables. However, as shown in (14), the bounds $z_{p \min}$ and $z_{p \max}$, $p \in \mathcal{I}_r$, are multiplied by r when constructing the vertices of model (12). This may lead to a modeling overbounding [42], thus the design conservatism.

Remark 3. Let $\{T_p\}_{p \in \mathcal{I}_{2r}}$ be a family of matrices with appropriate dimensions such that $T_{2p} = T_{2p-1}$, for $p \in \mathcal{I}_r$, and $T = \frac{1}{r} \sum_{p=1}^r T_{2p}$. We define

$$X_i^* = X_i + T_i - T, \quad X \in \{E, A, B\}, \quad i \in \mathcal{I}_{2r}. \quad (15)$$

Then, for $X \in \{E, A, B\}$, it follows from (13) and (15) that

$$\begin{aligned} \sum_{i=1}^{2r} w_i(z) X_i^* &= \sum_{i=1}^{2r} w_i(z) (X_i + T_i - T) \\ &= \sum_{i=1}^{2r} w_i(z) X_i + \sum_{i=1}^{2r} w_i(z) T_i - T \\ &= X(z) + \sum_{p=1}^r (w_{2p-1}(z) T_{2p-1} + w_{2p}(z) T_{2p}) - T \\ &= X(z) + \frac{1}{r} \sum_{p=1}^r T_{2p} - T = X(z). \end{aligned} \quad (16)$$

Note from (16) that $X(z)$ can admit an *infinite* number of representations of the form (10) with the same predefined set of nonlinear functions $w(z) \in \Omega$. This non-uniqueness feature of the vertices of model (12) offers a flexibility to introduce r

slack variables, *i.e.*, similar to matrices T_p in (15), into LMI-based design conditions to further reduce the conservatism that may be induced by the overbounding discussed in Remark 2. This will be numerically illustrated in Section IV.

This paper investigates the following control problem.

Problem 1. Consider the singular nonlinear system (3) under Assumption 1. Determine a control law $u(t)$ such that the closed-loop system (3) is admissible and the corresponding trajectory $x(t)$ exponentially converges towards the origin with a decay rate less than a predefined positive scalar α . Moreover, for any admissible initial state $x_d(0)$, the corresponding closed-loop trajectory $x_d(t)$ of system (3) is required to remain inside the validity domain $\mathcal{D}_{x_d} \subseteq \mathbb{R}^q$, described by

$$\mathcal{D}_{x_d} = \{x_d \in \mathbb{R}^q : H_{(l)} x_d \leq 1, \quad l \in \mathcal{I}_{n_s}\}, \quad (17)$$

where $H_{(l)}$ denotes the l th row of the matrix $H \in \mathbb{R}^{n_s \times q}$, which characterizes the domain \mathcal{D}_{x_d} .

Remark 4. The system states are generally bounded in engineering applications due to physical and/or safety reasons [12], [46]. Then, the characterizing matrix H can be directly derived from the ultimate bounds $x_{di \min}$ and $x_{di \max}$ of the state x_d , *i.e.*, $x_{di \min} \leq x_{di} \leq x_{di \max}$, for $i \in \mathcal{I}_q$, as illustrated in Example 3 in Section IV. Note that due to Assumption 1 and the admissibility property, any limitation bounds on the algebraic variables x_a of system (3) can be also transformed into those on the differential state x_d . Moreover, since $z(x) = g(x_d)$, from (17) we can directly obtain the ultimate bounds $z_{i \min}$ and $z_{i \max}$, for $i \in \mathcal{I}_r$, defining the admissible set \mathcal{D}_z in (2) of the premise variables. Hence, if $x_d(t) \in \mathcal{D}_{x_d}$, then the bounds of the premise variables in (2) are always valid for the proposed polytopic representation.

The following lemma is useful for the control design.

Lemma 1 ([47]). Let Υ_{ij} be symmetric matrices of appropriate dimensions where $i, j \in \mathcal{I}_r$. Then, the inequality

$$\sum_{i=1}^r \sum_{j=1}^r w_i(z) w_j(z) \Upsilon_{ij} \prec 0, \quad (18)$$

holds if

$$\begin{aligned} \Upsilon_{ii} &\prec 0, \quad i \in \mathcal{I}_r \\ \frac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} &\prec 0, \quad i, j \in \mathcal{I}_r, \quad i \neq j. \end{aligned}$$

Note that other relaxation results to convert (18) into a finite set of LMI constraints with different degrees of complexity and/or conservatism can be found in [12], [20].

III. LMI-BASED CONTROL DESIGN FOR NONLINEAR DESCRIPTOR SYSTEMS

This section first presents the control design for system (1) using the proposed model (12). Then, we pay a special attention to the control design of regular systems, *i.e.*, there is no algebraic equation (1b), which has been widely treated in TS fuzzy control framework.

A. Descriptor-Redundancy Based Control Design

We consider the nonlinear singular system (3), which can be rewritten in the following extended form:

$$\bar{E}\dot{x}_e(t) = \bar{A}(z)x_e(t) + \bar{B}(z)u(t), \quad (19)$$

with

$$x_e = \begin{bmatrix} x_d \\ \dot{x}_d \\ x_a \end{bmatrix}, \quad \bar{A}(z) = \begin{bmatrix} 0 & I & 0 \\ A_{11}(z) & -E_1(z) & A_{12}(z) \\ A_{21}(z) & 0 & A_{22}(z) \end{bmatrix},$$

$$\bar{B}(z) = \begin{bmatrix} 0 \\ B_1(z) \\ B_2(z) \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Applying the reduced-complexity modeling in (12), system (19) can be equivalently represented as

$$\bar{E}\dot{x}_e(t) = \sum_{i=1}^{2r} w_i(z)(\bar{A}_i x_e(t) + \bar{B}_i u(t)). \quad (20)$$

For the admissibilization of system (20), we consider the following control law:

$$u(t) = \sum_{i=1}^{2r} w_i(z)\bar{K}_i x_e(t) \doteq \bar{K}(z)x_e(t), \quad (21)$$

where the feedback gains $\bar{K}_i = [K_i \ 0]$, with $K_i \in \mathbb{R}^{m \times q}$, $i \in \mathcal{I}_{2r}$, are to be determined such that the closed-loop system (3) is admissible. The following theorem provides LMI conditions to design controller (21).

Theorem 1. Consider system (3) with Assumption 1. If there exist a positive definite matrix $V \in \mathbb{R}^{q \times q}$, symmetric matrices $T_i, S_j \in \mathbb{R}^{2(n+q) \times 2(n+q)}$, matrices $L_j, R_j \in \mathbb{R}^{(n+q) \times (n+q)}$, $F_j \in \mathbb{R}^{m \times q}$, $W_{ij} \in \mathbb{R}^{n \times q}$, $Z_{ij} \in \mathbb{R}^{n \times n}$, for $i, j \in \mathcal{I}_{2r}$, and a positive scalar α , such that

$$\Xi_{ii} \prec 0, \quad i \in \mathcal{I}_{2r}, \quad (22)$$

$$\frac{2}{2r-1}\Xi_{ii} + \Xi_{ij} + \Xi_{ji} \prec 0, \quad i, j \in \mathcal{I}_{2r}, \quad i \neq j, \quad (23)$$

$$\begin{bmatrix} V & \star \\ H_{(l)}V & 1 \end{bmatrix} \succeq 0, \quad l \in \mathcal{I}_{n_s}, \quad (24)$$

with $T_{2p} = T_{2p-1}$, $S_{2p} = S_{2p-1}$, $\forall p \in \mathcal{I}_r$, and

$$\Xi_{ij} = \Psi_{ij} + T_i + S_j - \frac{1}{r}(T + S),$$

$$S = \sum_{p=1}^r S_{2p}, \quad T = \sum_{p=1}^r T_{2p},$$

$$\Psi_{ij} = \text{He} \begin{bmatrix} (\bar{A}_i + \alpha \bar{E})L_j + \bar{B}_i \bar{F}_j & \bar{A}_i R_j \\ \mathcal{P}_{ij} - L_j & -R_j \end{bmatrix},$$

$$\bar{F}_j = [F_j \ 0], \quad \mathcal{P}_{ij} = \begin{bmatrix} V & 0 \\ W_{ij} & Z_{ij} \end{bmatrix}.$$

Then, the closed-loop system (20) is admissible with a decay rate less than α , and for any $x_d(0)$ belonging to the set \mathcal{E}_V , specified in (38), the corresponding trajectory $x_d(t)$ remains inside \mathcal{D}_{x_d} . Moreover, the feedback gains of the control law (21) are given by

$$K_i = F_i V^{-1}, \quad i \in \mathcal{I}_{2r}. \quad (25)$$

Proof. For brevity, we denote

$$\Theta(z) = \sum_{j=1}^{2r} w_j(z)\Theta_j, \quad \Theta \in \{L, R, F\},$$

$$\Lambda(z) = \sum_{i=1}^{2r} \sum_{j=1}^{2r} w_i(z)w_j(z)\Lambda_{ij}, \quad \Lambda \in \{\Xi, \Psi\},$$

$$\mathcal{P}(z) = \begin{bmatrix} V & 0 \\ W(z) & Z(z) \end{bmatrix} = \sum_{i=1}^{2r} \sum_{j=1}^{2r} w_i(z)w_j(z)\mathcal{P}_{ij}.$$

It follows from the definition of Ψ_{ij} that

$$\Psi(z) = \text{He} \begin{bmatrix} (\bar{A}(z) + \alpha \bar{E})L(z) + \bar{B}(z)\bar{F}(z) & \bar{A}(z)R(z) \\ \mathcal{P}(z) - L(z) & -R(z) \end{bmatrix},$$

with $\bar{F}(z) = [F(z) \ 0]$. Moreover, it follows from (13) that

$$\sum_{i=1}^{2r} \sum_{j=1}^{2r} w_i(z)w_j(z)(T_i + S_j) = \sum_{i=1}^{2r} w_i(z)T_i + \sum_{j=1}^{2r} w_j(z)S_j$$

$$= \sum_{k=1}^r (w_{2k-1}(z)T_{2k} + w_{2k}(z)T_{2k})$$

$$+ \sum_{\ell=1}^r (w_{2\ell-1}(z)S_{2\ell} + w_{2\ell}(z)S_{2\ell}) \quad (26)$$

$$= \frac{1}{r} \sum_{p=1}^r (T_{2p} + S_{2p}) = \frac{1}{r}(T + S).$$

Then, it follows from the definition of $\Psi(z)$ and relation (26) that $\Xi(z) = \Psi(z)$. Applying Lemma 1, LMI conditions (22) and (23) imply that $\Xi(z) \prec 0$, thus

$$\text{He} \begin{bmatrix} (\bar{A}(z) + \alpha \bar{E})L(z) + \bar{B}(z)\bar{F}(z) & \bar{A}(z)R(z) \\ \mathcal{P}(z) - L(z) & -R(z) \end{bmatrix} \prec 0. \quad (27)$$

Pre- and post-multiplying (27) with $[I \ (\bar{A}(z) + \alpha \bar{E})^\top]$ and its transpose, it follows that

$$\Delta(z) = \text{He} [(\bar{A}(z) + \alpha \bar{E})\mathcal{P}(z) + \bar{B}(z)\bar{F}(z)] \prec 0. \quad (28)$$

Note that

$$\Delta(z) = \begin{bmatrix} \bullet & \bullet \\ \bullet & -\text{He}[\bar{A}_{22}(z)Z(z)] \end{bmatrix}, \quad (29)$$

where “ \bullet ” denotes the matrix terms irrelevant to the theoretical developments, and

$$\bar{A}_{22}(z) = \begin{bmatrix} -E_1(z) & A_{12}(z) \\ 0 & A_{22}(z) \end{bmatrix} \in \mathbb{R}^{n \times n}.$$

It follows from (28) and (29) that

$$\bar{A}_{22}(z)Z(z) + Z(z)^\top \bar{A}_{22}(z)^\top \succ 0,$$

which ensures that $\det(\bar{A}_{22}(z)Z(z)) \neq 0$, thus $\det(Z(z)) \neq 0$, $\forall z \in \mathcal{D}_z$. Combining with the fact that $V \succ 0$, it follows that $\mathcal{P}(z)$ is invertible on \mathcal{D}_z with

$$\mathcal{P}(z)^{-1} = \begin{bmatrix} V^{-1} & 0 \\ -Z^{-1}(z)W(z)V^{-1} & Z^{-1}(z) \end{bmatrix}. \quad (30)$$

Note from (30) that

$$\bar{E}^\top \mathcal{P}^{-1}(z) = \mathcal{P}^{-1}(z)\bar{E} = \text{diag}(V^{-1}, 0) \succeq 0. \quad (31)$$

From (25) and (30), it follows that

$$\bar{K}(z) = \bar{F}(z)\mathcal{P}(z)^{-1}. \quad (32)$$

Pre- and post-multiplying (28) with $\mathcal{P}(z)^{-\top}$ and $\mathcal{P}(z)^{-1}$ while considering expression (32), it follows that

$$\text{He} [\mathcal{P}(z)^{-\top}(\bar{A}(z) + \bar{B}(z)\bar{K}(z)) + 2\alpha\bar{E}^{\top}\mathcal{P}(z)^{-1}] \prec 0. \quad (33)$$

We consider the Lyapunov function candidate

$$\mathcal{V}(z, x_e) = x_e^{\top} \bar{E}^{\top} \mathcal{P}(z)^{-1} x_e. \quad (34)$$

It follows from (31) and (34) that $\mathcal{V}(z, x_e) = x_d^{\top} V^{-1} x_d > 0$, $\forall x_e \neq 0$. The time derivative of $\mathcal{V}(z, x_e)$ in (34) along the solution of system (19) is given by

$$\dot{\mathcal{V}}(z, x_e) = \text{He} [x_e^{\top} \mathcal{P}(z)^{-\top} (\bar{A}(z) + \bar{B}(z)\bar{K}(z)) x_e]. \quad (35)$$

It is clear from (33) and (35) that $\dot{\mathcal{V}}(z, x_e) + 2\alpha\mathcal{V}(z, x_e) < 0$, $\forall x_e \neq 0$. Hence, the solution $x_e = 0$ of system (19) is asymptotically stable with a decay rate less than α . Moreover, since $E(z)$ is regular, the expression of $\dot{\mathcal{V}}(z, x_e)$ can be also computed using (1) as

$$\begin{aligned} \dot{\mathcal{V}}(z, x_e) &= \text{He} [\dot{x}_d^{\top} V^{-1} x_d] \\ &= \text{He} [\dot{x}_d^{\top} E_1(z)^{\top} E_1(z)^{-\top} V^{-1} x_d] \\ &= \text{He} [(A_{c11}(z)x_d + A_{12}(z)x_a)^{\top} Y(z)^{-1} x_d], \end{aligned} \quad (36)$$

with $A_{c11}(z) = A_{11}(z) + B_1(z)K(z)$ and $Y(z) = VE_1(z)^{\top}$. Note that

$$A_{c21}(z)x_d + A_{22}(z)x_a = 0,$$

with $A_{c21}(z) = A_{21}(z) + B_2(z)K(z)$. Then, $\dot{\mathcal{V}}(z, x_e)$ in (36) can be rewritten as follows:

$$\begin{aligned} \dot{\mathcal{V}}(z, x_e) &= \text{He} [(A_{c11}(z)x_d + A_{12}(z)x_a)^{\top} Y(z)^{-1} x_d] \\ &\quad + \text{He} [(A_{c21}(z)x_d + A_{22}(z)x_a)^{\top} Y(z)^{-1} x_a] \\ &= \begin{bmatrix} x_d \\ x_a \end{bmatrix}^{\top} \begin{bmatrix} \bullet & \bullet \\ \bullet & \text{He} [A_{22}(z)^{\top} Y(z)^{-1}] \end{bmatrix} \begin{bmatrix} x_d \\ x_a \end{bmatrix}. \end{aligned} \quad (37)$$

Since $\dot{\mathcal{V}}(z, x_e) < 0$, $\forall x \neq 0$, it follows from (37) that $\text{He} [A_{22}(z)^{\top} Y(z)^{-1}] \prec 0$, or $\det(A_{22}(z)Y(z)^{-1}) \neq 0$. This latter ensures that $\det(A_{22}(z)) \neq 0$. Hence, the nonlinear singular system (3) under Assumption 1 is of index-one.

Following the same line as in [16], [46], we can prove that condition (24) guarantees the inclusion $\mathcal{E}_V \subseteq \mathcal{D}_{x_d}$, with

$$\mathcal{E}_V = \{x_d \in \mathbb{R}^q : x_d^{\top} V^{-1} x_d \leq 1\}. \quad (38)$$

Since $\dot{\mathcal{V}}(z, x_e) < 0$, $\forall x \neq 0$, then by the set invariance property [48], we deduce that $\forall x_d(0) \in \mathcal{E}_V$, the corresponding trajectory $x_d(t)$ converges to the origin while remaining inside this set, thus $x_d(t) \in \mathcal{D}_{x_d}$. This concludes the proof. \square

The following corollary provides LMI-based design conditions without using slack variables T_i and S_i , $\forall i \in \mathcal{I}_{2r}$, specifically offered by the proposed polytopic model (12).

Corollary 1. Consider system (3) with Assumption 1. If there exist a positive definite matrix $V \in \mathbb{R}^{q \times q}$, matrices $L_j, R_j \in \mathbb{R}^{(n+q) \times (n+q)}$, $F_j \in \mathbb{R}^{m \times q}$, $W_{ij} \in \mathbb{R}^{(n+q) \times q}$, $Z_{ij} \in \mathbb{R}^{(n+q) \times (n+q)}$, for $i, j \in \mathcal{I}_{2r}$, and a positive scalar α ,

such that condition (24) is verified, and

$$\Phi_{ii} \prec 0, \quad i \in \mathcal{I}_{2r}, \quad (39)$$

$$\frac{2}{2r-1} \Phi_{ii} + \Phi_{ij} + \Phi_{ji} \prec 0, \quad i, j \in \mathcal{I}_{2r}, \quad i \neq j, \quad (40)$$

with

$$\Phi_{ij} = \text{He} \begin{bmatrix} (\bar{A}_i + \alpha\bar{E})L_j + \bar{B}_i\bar{F}_j & \bar{A}_i R_j \\ \mathcal{P}_{ij} - L_j & -\bar{R}_j \end{bmatrix}.$$

Then, the closed-loop system (20) is admissible with a decay rate less than α , and for any $x_d(0) \in \mathcal{E}_V$, the corresponding trajectory $x_d(t)$ remains inside the set \mathcal{D}_{x_d} . Moreover, the feedback gains of the control law (21) are given in (25).

Proof. In Theorem 1, setting $T_i = S_j = 0$, $\forall i, j \in \mathcal{I}_{2r}$, it follows that $\Phi_{ij} = \Xi_{ij}$. Then, conditions (22) and (23) become (39) and (40), respectively. The proof is concluded following the result of Theorem 1. \square

Remark 5. Following the same lines of fuzzy Lyapunov-based control approaches for TS fuzzy systems, see [9], [21]–[24] and related references, the Lyapunov function candidate

$$\mathcal{V}(z, x_e) = x_e^{\top} \bar{E}^{\top} \mathcal{P}(z)^{-1} x_e, \quad (41)$$

with

$$\mathcal{P}(z) = \begin{bmatrix} V(z) & 0 \\ W(z) & Z(z) \end{bmatrix} = \sum_{i=1}^{2r} \sum_{j=1}^{2r} w_i(z) w_j(z) \begin{bmatrix} V_{ij} & 0 \\ W_{ij} & Z_{ij} \end{bmatrix},$$

can be directly applied to reduce the design conservatism. However, such control results require a fundamental assumption that

$$x \in \mathcal{R} = \{x \in \mathbb{R}^n : |\dot{w}_i(z)| \leq \phi_i, \quad \forall i \in \mathcal{I}_{2r}\},$$

for some *predefined* positive scalars ϕ_i . Note that such an assumption can be only verified *a posteriori* by extensive simulations with the designed controllers. Hence, as for conventional TS fuzzy control approaches [15], it is still not possible to *theoretically* demonstrate that control results based on $V(z, x_e)$ in (41) include those derived from the quadratic Lyapunov function $\mathcal{V}(z, x_e)$ in (34). By numerical experiments in Section IV, we show that in many cases, the proposed quadratic control approach with specific slack variables in Theorem 1, *i.e.*, S_i and T_i , for $i \in \mathcal{I}_{2r}$, offered by the new representation (12) can provide a better performance, in terms of conservatism reduction, compared to many existing fuzzy Lyapunov based control results in TS fuzzy control framework.

B. Control Design without Descriptor-Redundancy Approach

In the following, a special attention is paid to the control design of regular systems, for which the use of the well-known descriptor-redundancy approach [27], *i.e.*, the extended form (19), is still possible but can be unnecessarily complex and conservative from a numerical viewpoint. To this end, let us consider the following descriptor nonlinear system:

$$E(z)\dot{x}(t) = A(z)x(t) + B(z)u(t), \quad (42)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, the matrix $E(z)$ is non-singular, *i.e.*, $\text{rank}(E(z)) = n$, and

$$[E(z) \quad A(z) \quad B(z)] = \sum_{i=1}^{2r} w_i(z) [E_i \quad A_i \quad B_i].$$

In this case, as in (17) we consider the validity domain

$$\mathcal{D}_x = \{x \in \mathbb{R}^n : H_{(l)}x \leq 1, l \in \mathcal{I}_{n_s}\}, \quad (43)$$

where the matrix $H \in \mathbb{R}^{n_s \times n}$ characterizes the domain \mathcal{D}_x . Note that system (42) is admissible as long as it admits an asymptotically stable solution $x = 0$. We consider the nonlinear control law

$$u(t) = K(z)x(t). \quad (44)$$

The following theorem provides LMI conditions to determine a feedback gain $K(z) \in \mathbb{R}^{m \times n}$ such that the origin of system (42) is asymptotically stable, and its trajectory $x(t)$ remains inside \mathcal{D}_x defined in (43) under an admissible initial condition.

Theorem 2. If there exist a positive definite matrix $P \in \mathbb{R}^{n \times n}$, symmetric matrices $T_i, S_j \in \mathbb{R}^{2n \times 2n}$, matrices $F_i \in \mathbb{R}^{m \times n}$, $L_i, R_j \in \mathbb{R}^{n \times n}$, for $i, j \in \mathcal{I}_{2r}$, and a positive scalar α , such that

$$\Gamma_{ii} \prec 0, \quad i \in \mathcal{I}_{2r}, \quad (45)$$

$$\frac{2}{2r-1} \Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} \prec 0, \quad i, j \in \mathcal{I}_{2r}, \quad i \neq j, \quad (46)$$

$$\begin{bmatrix} P & \star \\ H_{(l)}P & 1 \end{bmatrix} \succeq 0, \quad l \in \mathcal{I}_n, \quad (47)$$

with $T_{2p} = T_{2p-1}$, $S_{2p} = S_{2p-1}$, $\forall p \in \mathcal{I}_r$, and

$$\begin{aligned} \Gamma_{ij} &= \Sigma_{ij} + T_i + S_j - \frac{1}{r}(T + S), \\ S &= \sum_{p=1}^r S_{2p}, \quad T = \sum_{p=1}^r T_{2p}, \\ \Sigma_{ij} &= \text{He} \begin{bmatrix} (A_i + \alpha E_i)L_j + B_i F_j & A_i R_j \\ P E_i^\top - L_j & -R_j \end{bmatrix}. \end{aligned}$$

Then, the solution $x = 0$ of the non-singular system (42) is asymptotically stable with a decay rate less than α , and $\forall x(0)$ belonging to the set \mathcal{E}_P , specified in (51), the corresponding trajectory $x(t)$ remains inside \mathcal{D}_x . Moreover, the feedback gain of the control law (44) is given by

$$K(z) = F(z)E(z)^{-\top}P^{-1}, \quad F(z) = \sum_{j=1}^{2r} w_j(z)F_j. \quad (48)$$

Proof. Following the same steps in the proof of Theorem 1, we can prove that conditions (45)–(46) guarantee that

$$\text{He}[(A(z) + \alpha E(z))PE(z)^\top + B(z)F(z)] \prec 0. \quad (49)$$

Since $P \succ 0$ and $E(z)$ is non-singular, pre- and post-multiplying (49) with $P^{-1}E(z)^{-1}$ and $E(z)^{-\top}P^{-1}$, it follows that

$$\Pi = \text{He}[P^{-1}E(z)^{-1}(A(z) + \alpha E(z) + B(z)K(z))] \prec 0 \quad (50)$$

Condition (50) ensures that $\dot{V}(z, x) + 2\alpha V(z, x) = x^\top \Pi x < 0$, $\forall x \neq 0$, where $\dot{V}(z, x)$ is the time derivative of the

Lyapunov function candidate $V(z, x) = x^\top P^{-1}x$ along the solution of system (42).

As in the proof of Theorem 1, we can prove that condition (47) guarantees the inclusion $\mathcal{E}_P \subseteq \mathcal{D}_x$, with

$$\mathcal{E}_P = \{x \in \mathbb{R}^n : x^\top P^{-1}x \leq 1\}. \quad (51)$$

Since $\dot{V}(z, x) < 0$, $\forall x \neq 0$, using again the set invariance property [48], we deduce that $\forall x(0) \in \mathcal{E}_P$, the corresponding trajectory $x(t)$ converges to the origin while remaining inside this set, thus $x(t) \in \mathcal{D}_x$. This concludes the proof. \square

Remark 6. The new nonlinear control law (44) with the feedback gain (48) exploits the invertibility of $E(z)$ to avoid using the extended form as in (19). Then, no specific matrix structure is required for the feedback gain as $\bar{K}(z)$ in (21) or the Lyapunov matrix as $\bar{E}^\top P^{-1}(z)$ in (31). Compared to the conventional TS fuzzy descriptor approaches, see for instance [9], [11], [12], [25], these features can further reduce not only the numerical complexity but also the conservatism of the control results for complex non-singular nonlinear systems as illustrated in Example 3.

Remark 7. The computational complexity of LMI-based optimization problems can be evaluated with the number of scalar decision variables \mathcal{N}_{var} and the number of LMI constraints \mathcal{N}_{row} . To illustrate the complexity reduction of the proposed approach, Table I shows these characteristics numbers corresponding to the LMI constraints of different control results for non-singular descriptor systems with $n = q$ states, m control inputs and $r = r_e + r_a$ premise variables, where r_e and r_a respectively represent the number of the premise variables in the matrix $E(z)$ and in both matrices $A(z)$ and $B(z)$. For illustrations, the evolution of \mathcal{N}_{var} and \mathcal{N}_{row} with respect to r for the case of stability analysis with $n = 5$ and $m = 0$ is depicted in Fig. 1. Note that for a relatively low number of premise variables, *i.e.*, $r \leq 6$, all considered methods have a similar level of numerical complexity. We can also observe an exponential growth of \mathcal{N}_{var} and \mathcal{N}_{row} with respect to r for TS fuzzy model-based results, which is not the case of the proposed approach. Further numerical studies are performed in the next section to demonstrate the interests of the new method in reducing both computational complexity and design conservatism compared to existing TS fuzzy model-based results.

Remark 8. The decay rate α is related to the closed-loop time performance, *i.e.*, a larger value of α leads to a faster convergence time [12].

Remark 9. When there are no required state constraints, *i.e.*, $\mathcal{D}_{x_d} \equiv \mathbb{R}^q$ for system (3) or $\mathcal{D}_x \equiv \mathbb{R}^n$ for system (42), the control design for these systems can be performed by simply removing condition (24) in Theorem 1 and Corollary 1, or condition (47) in Theorem 2, respectively. Moreover, it is possible to maximize the size of the invariant sets \mathcal{E}_V and \mathcal{E}_P using convex optimization techniques. This issue, omitted here, has been well addressed in [16] and related references.

TABLE I: Complexity Characteristics Numbers of Different Control Results.

Control Design	Theorem 1	Theorem 2	[11, Theorem 3.3]	[25, Theorem 2]
Number of vertices	$2r$	$2r$	2^r	2^r
\mathcal{N}_{row}	$n(16r^2 + 1)$	$n(8r^2 + 1)$	$n(2^{r+r_a+2} + 2^r + 2^{r+1})$	$n(2^{r+r_a+1} + 2^r + 2^{r+1}) + m2^{r+r_a+1}$
\mathcal{N}_{var}	$n^2(8r^2 + 32r + \frac{1}{2}) + n(\frac{1}{2} + 4r) + 2rmn$	$n^2(12r + \frac{1}{2}) + \frac{n}{2} + 2rmn$	$n^2(\frac{3}{2}2^r + 2^{r_e-1} + 2^{r_a-1} + 6) + n(2^{r-1} + 2^{r_e-1} + 2^{r_a-1}) + nm2^r$	$n^2(1 + 2^{2r_a+1} + 2^{r-1}) + n(1 + 2^{r-1} + 3m2^r + m2^{2r_a}) + m^22^r$

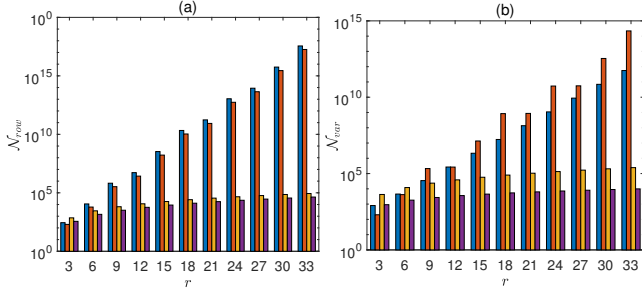


Fig. 1: Characteristics numbers of numerical complexity with respect to the number of premise variables r for the case $n = 5$ and $m = 0$: Theorem 1 (■); Theorem 2 (■); [11, Theorem 3.3] (■); [25, Theorem 2] (■).

IV. NUMERICAL EXAMPLES

This section presents three examples with different degrees of numerical complexity to illustrate the effectiveness of the proposed approach. All the involved LMI-based conditions are solved using YALMIP toolbox with SeDuMi solver [49]. Note that without an explicit statement, all the design conditions are solved with the decay rate $\alpha = 0$ for fair comparisons with related existing results. Moreover, for Examples 1 and 2 where $\mathcal{D}_{x_d} \equiv \mathbb{R}^q$, the LMI condition (24) is not considered in the admissibility analysis and control design, see Remark 9.

Example 1. This example is used for two purposes: i) to illustrate the proposed polytopic modeling procedure, ii) to put in evidence the interest of the specific slack variables offered by the proposed modeling method. To this end, we consider a simple *unforced* singular nonlinear system of the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & z_1(t) + 5z_2(t) \\ az_2(t) & b \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad (52)$$

where $a \in [-10, 10]$ and $b \in [-10, 10]$ are the parameters. The two premise variables are given by $z_1 = \cos(x_1)$ and $z_2 = \sin(x_1)$, i.e., $z = [z_1 \ z_2]^\top$, which both depend on the differential state x_1 . We consider the validity domain $\mathcal{D}_{x_d} \equiv \mathbb{R}$. The matrices of system (52) are given by

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A(z) = \begin{bmatrix} -1 & z_1 + 5z_2 \\ az_2 & b \end{bmatrix}. \quad (53)$$

Applying the proposed modeling procedure in (10), (11) and (14) to matrix $A(z)$ with $-1 \leq z_1 \leq 1$ and $-1 \leq z_2 \leq 1$, $\forall x_1 \in \mathbb{R}$, we obtain $2 \times r = 4$ following vertices:

$$\begin{aligned} A_1 &= \begin{bmatrix} -1 & -2 \\ 0 & b \end{bmatrix}, & A_2 &= \begin{bmatrix} -1 & 2 \\ 0 & b \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -1 & -10 \\ -2a & b \end{bmatrix}, & A_4 &= \begin{bmatrix} -1 & 10 \\ 2a & b \end{bmatrix}. \end{aligned} \quad (54)$$

The nonlinear functions $w_i(z)$, for $i \in \mathcal{I}_4$, corresponding to the four above vertices are given by

$$\begin{aligned} w_1(z) &= \frac{1 - \cos(x_1)}{4}, & w_2(z) &= \frac{\cos(x_1) + 1}{4}, \\ w_3(z) &= \frac{1 - \sin(x_1)}{4}, & w_4(z) &= \frac{\sin(x_1) + 1}{4}. \end{aligned} \quad (55)$$

Let us define $x = [x_1 \ x_2]^\top$. From (53), (54) and (55), the singular nonlinear system (52) can be equivalently rewritten in the polytopic form $E\dot{x} = \sum_{i=1}^4 w_i(z)A_i x$. We examine the conservatism of the admissibility analysis for system (52) using the following approaches:

- Theorem 1 with $u(t) = 0$, i.e, $F_i = 0$, for $i \in \mathcal{I}_4$,
- Corollary 1 with $u(t) = 0$, i.e, $F_i = 0$, for $i \in \mathcal{I}_4$,
- Affine TS fuzzy-model-based result in [41, Theorem 2] adapted to singular systems,
- TS fuzzy-model-based result in [11, Theorem 3.3] with $u(t) = 0$, i.e, $N_{jk} = 0$, for $j, k \in \mathcal{I}_4$.

Fig. 2 shows the feasibility regions obtained with the four above control results. Observe that using the reduced-complexity affine representation for TS fuzzy systems, the result in [41] leads to a conservative admissibility analysis compared to the proposed modeling method in this paper. Theorem 1 provides a larger feasibility region compared to that obtained with Corollary 1. This means that using the slack variables, specifically offered by the proposed modeling, can contribute to reduce the conservatism induced by overbounding, see Remarks 2 and 3. For this simple singular system (52), the proposed approach and the recent result in [11] lead to the same feasibility region.

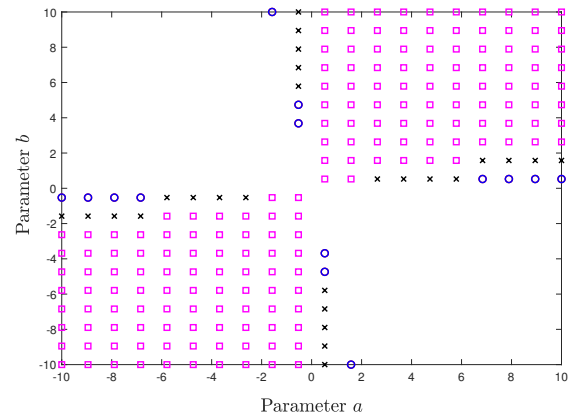


Fig. 2: Feasibility regions obtained with [41, Theorem 2] (□); Corollary 1 (×, □); Theorem 1 and [11, Theorem 3.3] (○, ×, □).

Example 2. This example aims at studying the design conservatism of the proposed approach with respect to recent related TS fuzzy control results. For this purpose, we consider the singular system (3), whose matrices are taken from [11] as

$$E(z) = \begin{bmatrix} 1 & 0 & 0 \\ z_1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B(z) = \begin{bmatrix} 0 \\ 0.16z_3 + a \\ 0.12z_3 + b \end{bmatrix},$$

$$A(z) = A_0 z_3 + \begin{bmatrix} -z_2 - 5 & 1 & z_2 \\ z_2 & bz_2 - b & 1 \\ (a+2)z_2 - 2 & 0 & 1 \end{bmatrix}, \quad (56)$$

$$A_0 = \begin{bmatrix} 0.05 & -0.05 & 0.15 \\ 0.04 & -0.04 & 0.12 \\ 0.06 & -0.06 & 0.18 \end{bmatrix},$$

where $a \in [-13, -8]$ and $b \in [-27, -20]$ are system parameters. The three premise variables are given by

$$z_1 = \frac{e^{-2x_1}}{1 + e^{-2x_1}}, \quad z_2 = \frac{1 + \sin^2(x_2)}{2}, \quad z_3 = \sin(0.1x_1).$$

For this example, we have $x_d = [x_1 \ x_2]^\top$, $x_a = x_3$ and $\mathcal{D}_{x_d} \equiv \mathbb{R}^2$. Note that $0 \leq z_1 \leq 1$, $0 \leq z_2 \leq 1$ and $-1 \leq z_3 \leq 1$, $\forall x_d \in \mathbb{R}^2$. Applying the proposed modeling approach, the considered nonlinear system can be exactly represented in the polytopic form (12) with $2 \times 3 = 6$ vertices, whose six nonlinear functions $w_i(z)$, $\forall i \in \mathcal{I}_6$, can be derived using (9) and (11) as

$$w_1(z) = \frac{1}{3 + 3e^{-x_1}}, \quad w_2(z) = \frac{e^{-x_1}}{3 + 3e^{-x_1}},$$

$$w_3(z) = \frac{1 - \sin^2(x_2)}{6}, \quad w_4(z) = \frac{1 + \sin^2(x_2)}{6},$$

$$w_5(z) = \frac{1 - \sin(0.1x_1)}{6}, \quad w_6(z) = \frac{1 + \sin(0.1x_1)}{6}.$$

The details on the corresponding system matrices of the six vertices are omitted here for brevity. We examine the design conservatism between the following control results: i) LMI conditions in Corollary 1; ii) LMI conditions in Theorem 1; iii) TS fuzzy-model-based results in [11, Theorem 3.3] and [9, Theorem 3]; iv) TS fuzzy-model-based result in [25, Theorem 2]; v) affine TS fuzzy-model-based result in [41, Theorem 2]. Fig. 3 depicts the feasibility regions obtained with these control results. Remark that Corollary 1 cannot provide any feasible control solution for the considered parameter space, which confirms the negative overbounding effect of the proposed modeling approach on the control design conservatism. However, taking into account the slack variables specifically introduced by this modeling, Theorem 1 outperforms other recent TS fuzzy control results. The characteristics numbers \mathcal{N}_{row} and \mathcal{N}_{var} , representing the numerical burden of LMI-based optimization problems, for the considered control results are given in Table II. Remark that as a price for conservatism reduction, the numerical complexity of design conditions in Theorem 1 is slightly higher than that of other control results.

For illustrations, we consider system (56) with $a = -11.37$ and $b = -20$ to evaluate the control performance. Note that only Theorem 1; [11, Theorem 3.3] and [9, Theorem 3] can provide a feasible solution for this system. Table III shows the comparison of the decay rate performance obtained with these

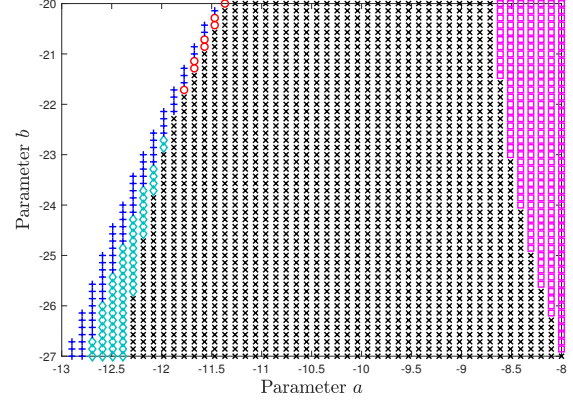


Fig. 3: Feasibility regions obtained with [41, Theorem 2] (\square); [11, Theorem 3.3] and [9, Theorem 3] (\circ, \times, \square); [25, Theorem 2] ($\diamond, \times, \square$); and Theorem 1 ($+$, \diamond , \circ, \times, \square).

control results. Observe that the proposed control approach provides a larger value of the maximal decay rate α_{\max} . Fig. 4 illustrates the closed-loop behaviors obtained with the corresponding controllers. As expected, we can see that compared to the recent related results, the proposed control approach yields a faster closed-loop time response under the same initial condition, see Remark 8.

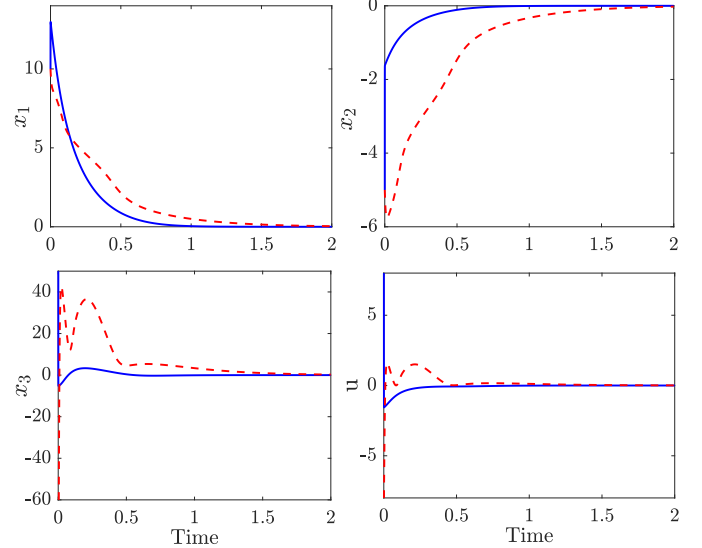


Fig. 4: Closed-loop responses of system (56) with $a = -11.37$ and $b = -20$ obtained under the same initial condition from Theorem 1 with $\alpha_{\max} = 2.25$ (—); [11, Theorem 3.3] and [9, Theorem 3] with $\alpha_{\max} = 0.2$ (- - -).

Example 3. To demonstrate the interests of the proposed control approach for complex engineering systems with a large number of nonlinearities, we consider a three-degree-of-freedom (3DoF) serial robot depicted in Fig. 5, whose dynamics can be described as [41]

$$\mathbf{M}(\theta)\ddot{\theta}(t) + \mathbf{N}(\theta, \dot{\theta})\dot{\theta}(t) + \mathbf{G}(\theta) = u(t), \quad (57)$$

where $\theta(t) \in \mathbb{R}^3$ is the vector of generalized coordinates in

TABLE II: Numerical Complexity of Different Control Results.

Control Design	Theorem 1	Corollary 1	[41, Theorem 2]	[25, Theorem 2]	[11, Theorem 3.3]
Number of vertices	6	6	4	8	8
\mathcal{N}_{row}	362	362	146	224	408
\mathcal{N}_{var}	1185	855	119	334	306

TABLE III: Comparison of Decay Rate for Example 2.

Control Design	Theorem 1	[11, Theorem 3.3]	[9, Theorem 3]
α_{max}	2.25	0.20	0.20

joint space, $u(t) \in \mathbb{R}^3$ is the vector of generalized control forces, $\mathbf{M}(\theta) \in \mathbb{R}^{3 \times 3}$ is the inertia matrix, $\mathbf{N}(\theta, \dot{\theta}) \in \mathbb{R}^{3 \times 3}$ is the Coriolis/centripetal matrix plus the viscous friction coefficients of the joints, and $\mathbf{G}(\theta) \in \mathbb{R}^n$ represents the gravity matrix. Since the vector-valued function $\mathbf{G}(\theta)$ is smooth and $\mathbf{G}(0) = 0$, we can then parameterize $\mathbf{G}(\theta) = \mathbf{H}(\theta)\theta$. System (57) can be rewritten in the nonlinear descriptor form (3) with $\theta(t) = [\theta_1(t) \ \theta_2(t) \ \theta_3(t)]^\top$ and

$$\begin{aligned} x(t) &= \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix} \in \mathbb{R}^6, \quad B(z) = \begin{bmatrix} 0 \\ I \end{bmatrix}, \\ E(z) &= \begin{bmatrix} I & 0 \\ 0 & \mathbf{M}(z) \end{bmatrix}, \quad A(z) = \begin{bmatrix} 0 & I \\ -\mathbf{H}(z) & -\mathbf{N}(z) \end{bmatrix}. \end{aligned} \quad (58)$$

The matrices $\mathbf{M}(z)$, $\mathbf{N}(z)$ and $\mathbf{H}(z)$ in (58) are given by

$$\begin{aligned} \mathbf{N}(z) &= \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}, \\ \mathbf{M}(z) &= \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ \star & 2c_6z_2 + c_8 & c_6z_2 + c_9 \\ \star & \star & c_9 \end{bmatrix}, \\ \mathbf{H}(z) &= \begin{bmatrix} c_1z_{10} + c_2z_{11} + c_3z_{12} & c_2z_{11} + c_3z_{12} & c_3z_{12} \\ \star & c_2z_{11} + c_3z_{12} & c_3z_{12} \\ \star & \star & c_3z_{12} \end{bmatrix}, \end{aligned}$$

with

$$\begin{aligned} m_{11} &= 2(c_4z_1 + c_6z_2 + c_5z_3 + c_7), \\ m_{12} &= c_4z_1 + 2c_6z_2 + c_5z_3 + c_8, \\ m_{13} &= c_6z_2 + c_5z_3 + c_9, \\ n_{11} &= c_5(z_6 - z_9) + c_6(z_7 - z_8) + f_v, \\ n_{12} &= -c_4z_4 + c_6(z_7 - z_8) - c_5z_9, \\ n_{13} &= -c_6z_8 - c_5z_9, \\ n_{21} &= c_4z_5 + c_5z_6 + c_6(z_7 - z_8), \\ n_{22} &= c_6(z_7 - z_8) + 2f_v, \quad n_{23} = -c_6z_8, \\ n_{31} &= c_5z_6 + c_6z_7, \quad n_{32} = c_6z_7, \quad n_{33} = 2f_v. \end{aligned}$$

The expressions of the premise variables z_i , for $i \in \mathcal{I}_{12}$, are given in Table IV. Note that $\{z_1, z_2, z_3\}$ are the nonlinear terms involved in the inertia matrix $\mathbf{M}(\theta)$; $\{z_4, z_5, \dots, z_9\}$ are the nonlinear terms involved in the Coriolis matrix $\mathbf{N}(\theta, \dot{\theta})$; and $\{z_{10}, z_{11}, z_{12}\}$ are the nonlinear terms involved in the gravity-related matrix $\mathbf{H}(\theta)$. For the robot model (57), we consider the joint mechanical limits and the maximum motor velocities as $|\theta_i| \leq \theta_{max}$ [rad], and $|\dot{\theta}_i| \leq 50$ [rad/s], for

$i \in \mathcal{I}_3$. The parameter $\theta_{max} \in [0, \pi]$ is used in the following to study the control design conservatism. As a result, the validity domain \mathcal{D}_x of the robot model (57) can be directly defined as

$$\mathcal{D}_x = \{x \in \mathbb{R}^6 : H_{(l)}x \leq 1, l \in \mathcal{I}_{12}\}, \quad (59)$$

where the matrix H , characterizing the physical limitations of the robot states, is given by

$$H = \begin{bmatrix} \frac{1}{\theta_{max}} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{\theta_{max}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\theta_{max}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\theta_{max}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\theta_{max}} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\theta_{max}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{50} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{50} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{50} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{50} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{50} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{50} \end{bmatrix}.$$

Given the above bounds of θ_i and $\dot{\theta}_i$, for $i \in \mathcal{I}_3$, we can straightforwardly derive the bounds of the twelve premise variables from their mathematical expressions shown in Table IV, *i.e.*, $z_p \min \leq z_p \leq z_p \max$, for $p \in \mathcal{I}_{12}$, which are used to construct the TS fuzzy model (7) and the polytopic model (12) of the 3DoF robot model (57). We consider the case without the viscous friction coefficient, *i.e.*, $f_v = 0$, and the values of the constant parameters c_i , for $i \in \mathcal{I}_9$, are given in Table V.

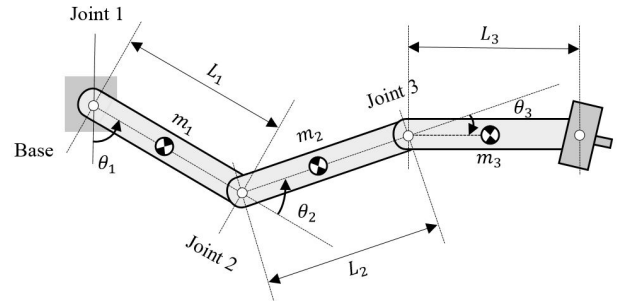


Fig. 5: Schematic of the 3DoF robot in the vertical plane.

With $r = 12$ premise variables z_i , for $i \in \mathcal{I}_{12}$, the TS fuzzy modeling (7), using the classical sector nonlinearity approach [12], requires $2^{12} = 4096$ vertices to exactly represent system (57)–(58), which is computationally excessive for control design. Moreover, the corresponding TS fuzzy controller is *impractical* for real-time control purposes. However, the proposed modeling (12) leads to only $2 \times 12 = 24$ vertices, allowing for a practically feasible control solution.

TABLE IV: Premise Variables of the 3DoF Robot.

Premise variables	Expression
z_1	$\cos(\theta_2)$
z_2	$\cos(\theta_3)$
z_3	$\cos(\theta_2 + \theta_3)$
z_4	$(2\dot{\theta}_1 + \dot{\theta}_2) \sin(\theta_2)$
z_5	$\dot{\theta}_1 \sin(\theta_2)$
z_6	$\dot{\theta}_1 \sin(\theta_2 + \theta_3)$
z_7	$(\dot{\theta}_1 + \dot{\theta}_2) \sin(\theta_3)$
z_8	$(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \sin(\theta_3)$
z_9	$(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \sin(\theta_2 + \theta_3)$
z_{10}	$\text{sinc}(\theta_1)$
z_{11}	$\text{sinc}(\theta_1 + \theta_2)$
z_{12}	$\text{sinc}(\theta_1 + \theta_2 + \theta_3)$

TABLE V: Mechanical Parameters of the 3DoF Robot.

Symbol	Expression	Value
c_1	$g(L_1 m_2 + L_1 m_3 + m_1 r_1)$	121.64
c_2	$g(L_2 m_3 + m_2 r_2)$	67.69
c_3	$g m_3 r_3$	23.54
c_4	$L_1(L_2 m_3 + m_2 r_2)$	6.9
c_5	$L_1 m_3 r_3$	2.4
c_6	$L_2 m_3 r_3$	2.16
c_7	$(m_2 + m_3)L_1^2 + L_2^2 m_3 + \sum_{i=1}^3 I_i$	19.75
c_8	$L_2^2 m_3 + I_2 + I_3$	7.85
c_9	I_3	1.9

For comparison purposes, we examine the following *quadratic* control results: i) LMI conditions in Theorems 1 and 2 with $\alpha = 0$, $L_j = L$, $R_j = R$, $W_{ij} = W$ and $Z_{ij} = Z$, $\forall i, j \in \mathcal{I}_{24}$; ii) TS fuzzy descriptor results in [12, Chapter 10] and [25, adapted Theorem 2]; iii) affine TS fuzzy-model-based result in [41, Theorem 2]. Note that the existing fuzzy Lyapunov-based control approaches cannot be applied to this example due to the numerical limitations of LMI solvers. To evaluate the design conservatism, we check the existence of a stabilizing controller for system (57)–(58) with respect to the *physical* variations of two parameters: the mass of the third arm $m_3 \in [1, 15]$ [kg], and the upper bound $\theta_{\max} \in [0, \pi]$ [rad] of the robot positions. Fig. 6 shows the feasibility regions obtained with the five considered control results. Remark that without including the acceleration information $\ddot{\theta}(t)$ (which is generally unavailable in practice) in the controller structure, the control result in [41, Theorem 2] cannot provide any feasible solution. The existing TS fuzzy control results in [12], [25] and Theorem 1 lead to the same feasibility region with a very restrictive joint range, *i.e.*, $\theta_{\max} \leq \frac{\pi}{5}$ [rad]. We can observe that without using the descriptor-redundancy approach, Theorem 2 allows finding feasible control solutions for the whole robot workspace. This clearly confirms the interests of the new control approach for complex nonlinear systems in reducing the numerical complexity and design conservatism. The complexity characteristics numbers \mathcal{N}_{row} and \mathcal{N}_{var} of the considered control approaches are given in Table VI. Remark that despite the introduction of slack variables, the computational complexity of the proposed control results are much lower compared to the existing TS fuzzy control results. The reduced-complexity affine model-based approach in [41, Theorem 2] leads to numerically simple design conditions at the price of over-conservativeness.

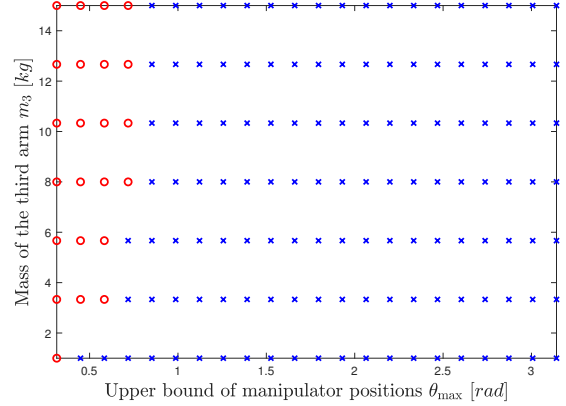


Fig. 6: Feasibility regions obtained with TS fuzzy approaches [12, Chapter 10] (\circ); [25, adapted Theorem 2] (\circ); Theorem 1 (\circ); and Theorem 2 (\circ, \times).

For illustrations, we consider system (57) with $m_3 = 1$ [kg] and $\theta_{\max} = \frac{\pi}{10}$ [rad]. Fig. 7 depicts the closed-loop response of the corresponding robot system (57) obtained from the nonlinear controller (21), designed with Theorem 1 and $\alpha = 3$. Note that the initial condition $x(0) = 0.7 [\theta_{\max} \quad -\theta_{\max} \quad -\theta_{\max} \quad 50 \quad -50 \quad -50]^T$ is selected such that $x(0) \in \mathcal{E}_V$. We observe that the robot states converge to the origin while respecting their physical limitations defined by the validity domain \mathcal{D}_x in (59). Hence, the computed bounds of the premise variables given in Table IV are always valid for the proposed polytopic representation during the simulation as discussed in Remark 4. However, without taking into account condition (24) in the control design to ensure the invariance property, see Remark 9, the closed-loop trajectories can go outside the validity domain \mathcal{D}_x under the same simulation conditions as shown in Fig 8. Note that in this case, the bounds of the premise variables are not respected anymore for the proposed polytopic representation as well as the TS fuzzy modeling.

To evaluate numerically the control performance, Table VII compares the decay rates obtained with different related control results for the robot system (57) with $m_3 = 1$ [kg] and $\theta_{\max} = \frac{\pi}{10}$ [rad]. We note that the proposed conditions in Theorems 1 and 2 yield much larger values of α_{\max} compared to the control approach in [25]. In particular, the largest value of α_{\max} obtained with Theorem 2 confirms the great potential of this control result, in terms of design conservatism and control performance, for non-singular nonlinear systems.

V. CONCLUDING REMARKS

A new LMI-based control approach has been proposed for a class of descriptor nonlinear systems. Compared to the classical TS fuzzy-model-based control approaches, the numerical complexity of the new approach grows proportionally, rather than exponentially, with respect to the number of premise variables. This is particularly interesting when dealing with complex descriptor systems with a large number of nonlinearities. Moreover, the system vertices obtained from

TABLE VI: Numerical Complexity of Different Control Results.

Control Design	Theorem 1	Theorem 2	[41, Theorem 2]	[12, Chapter 10]	[25, Theorem 2]
Number of vertices	24	24	13	4096	4096
\mathcal{N}_{row}	582	294	162	49158	61446
\mathcal{N}_{var}	813	525	255	73821	73821

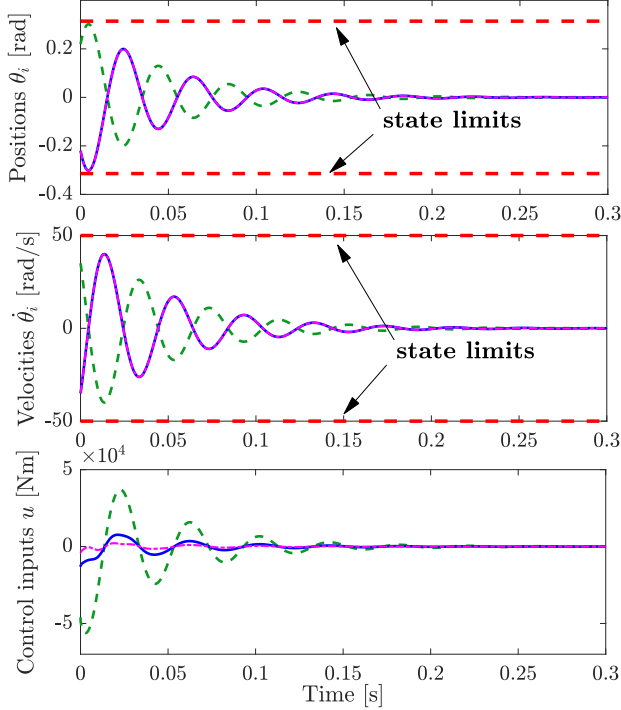


Fig. 7: Closed-loop response of the robot system (57) with $m_3 = 1$ [kg] and $\theta_{\max} = \frac{\pi}{10}$ [rad] obtained with controller (21) designed from Theorem 1, $\alpha = 3$ and $x(0) \in \mathcal{E}_V$.

TABLE VII: Comparison of Decay Rate for Example 3.

Control Design	Theorem 1	Theorem 2	[25, adapted Theorem 2]
α_{\max}	10.6×10^4	25×10^4	750

the proposed polytopic representation are not unique even with the same predefined set of premise variables. This non-uniqueness representation feature allows introducing specific slack variables into the control design to reduce the conservatism of the proposed approach. Strict LMI conditions are derived via Lyapunov stability theory for control design and admissibility analysis. A special attention is paid to the control design of non-singular nonlinear systems, for which the descriptor-redundancy approach is not required to further reduce the design complexity and conservatism. The interests of the new control results are clearly demonstrated with both numerical and physically motivated examples. From numerical experiments, it is observed that for nonlinear systems with a low number of premise variables the proposed approach leads to admissibility analysis and control design results with a similar level of conservatism and numerical complexity as related existing TS fuzzy model-based approaches. However, when the number of premise variables becomes sufficiently

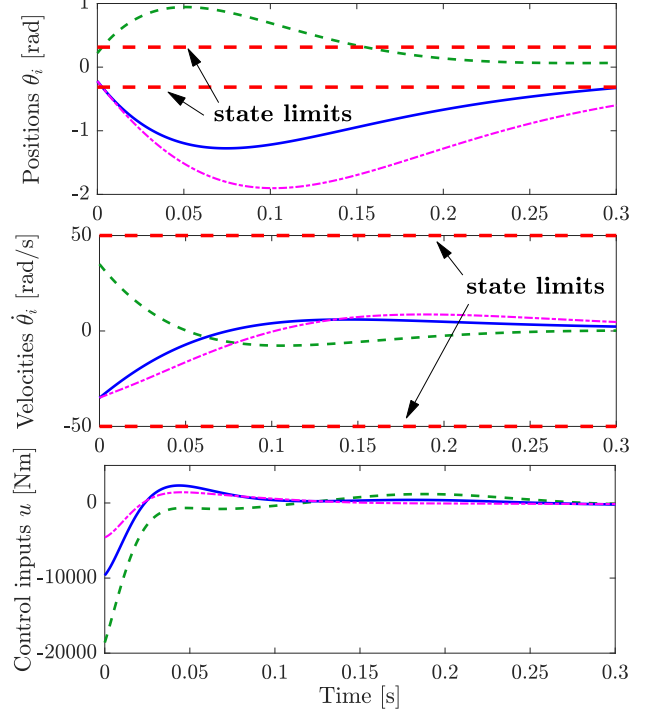


Fig. 8: Closed-loop response of the robot system (57) with $m_3 = 1$ [kg] and $\theta_{\max} = \frac{\pi}{10}$ [rad] obtained with controller (21) designed from Theorem 1 without condition (24), $\alpha = 3$ under the same initial condition as in Fig. 7.

large, as illustrated in Example 3, the strong interest of the new method related to numerical complexity and design conservatism reduction over TS fuzzy model-based approaches is put in evidence. Future works focus on extending the proposed approach to output feedback tracking control of singular nonlinear systems. Considering the proposed polytopic modeling method to deal with the fault detection issue of singular nonlinear systems with unmeasured premise variables and delayed/quantized output signals [50] is another promising research direction.

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