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Adaptive gain-scheduling control for continuous-time systems with polytopic uncertainties: An LMI-based approach

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Abstract

We develop a new adaptive gain-scheduling control scheme for continuous-time linear systems with polytopic uncertainties. The gain-scheduled control law is proposed as a convex sum of a fixed set of controller gains, exploiting the polytopic representation of the system uncertainty, which is not possible with classical robust control results in the literature. To realize this scheme, an adaptation law is proposed to *adaptively* provide the tuning parameter for the gain-scheduling implementation. The admissible domain of the stabilizing control feedback gains, defined by the fixed set of controller gains, can be determined *offline* by solving a set of linear matrix inequality constraints over a scalar line search. Using Lyapunov-based arguments, the proposed design conditions and the adaptation law ensure that all closed-loop signals are bounded. In particular, if the uncertain parameters are not time-varying, then the system states asymptotically converge to the origin. Theoretical arguments and appropriate numerical illustrations are provided to demonstrate the effectiveness of the proposed control scheme.

Keywords: Robust control, adaptive control, linear systems, polytopic uncertainty, parameter estimation, linear matrix inequality.

1. Introduction

Dealing with system uncertainty is a central issue in control theory and applications [1]. Within this context, robust control of uncertain polytopic systems [2] and gain-scheduling control of linear parameter-varying (LPV) systems [3] have received considerable attention. An appealing feature of these approaches is that using Lyapunov-based arguments, a large number of robust analysis and control design problems can be reformulated as convex optimization procedures under linear matrix inequalities (LMIs), which are effectively solved with numerical solvers [2]. Significant advances have been achieved with various classes of parameter-dependent Lyapunov functions and/or control laws, see *e.g.* [4–9] and related references.

A key assumption for the gain-scheduling technique is that the time-varying parameters must be available online (measured or estimated) to design parameter-dependent control laws for LPV systems [3, 10–12]. The design of LPV controllers with *inexact* scheduling parameters affected by bounded uncertainties has been discussed in [13–16]. However, for uncertain

polytopic systems, the uncertain parameters are essentially *unknown* for control design. Hence, the main goal has been to propose less conservative methods to design a linear parameter-independent (LPI) control law [17–19]. The so-called *S-variable* approach has played a key role to accomplish this goal [20]. However, an LPI controller may not exist or may not be found with numerical routines [21]. Moreover, using a single LPI controller may lead to poor closed-loop performance when the uncertainty domain is large.

Adaptive control is known as another effective methodology to deal with system uncertainties [22]. Generally speaking, two parameter estimation schemes can be distinguished in adaptive control [23], *i.e.*, direct [24, 25] and indirect [22, 26–28] schemes. In a direct adaptive scheme, the controller parameters are directly updated online without requiring any estimation of the plant dynamics. However, the plant parameters must be first estimated for control design/implementation in an indirect adaptive scheme. Hence, parameter estimation is a core component in the second scheme. Up to now, several parameter estimation algorithms have been proposed in the literature for adaptive control, see *e.g.* [26–31] and a recent survey in [32].

Motivated by the advantages of gain-scheduling control and indirect adaptive control, we developed in the preliminary version [33] a new robust control method for linear time-invariant (LTI) systems with polytopic uncertainties. This method can be viewed as an *adaptive gain-scheduling* scheme for uncertain polytopic systems, whose fixed set of controller gains, which represent the possible values the stabilizing control feedback gains may assume, is predetermined *offline* by solving a set of LMIs over a scalar line search. Using Lyapunov-based arguments, we prove that the proposed adaptive gain-scheduling

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controller guarantees the asymptotic convergence of the system state to the origin and the stability of the parameter adaptation scheme.

One of the attractive characteristics of the adaptive control law proposed in [33] is that the control gains and the terms defining the adaptation law are found *offline* by solving an optimization problem subject to matrix inequalities. In that same manner, [34] presents a set of conditions for a system to be strictly positive real (SPR) in closed loop, which allows the use of a simple adaptive control law, [35] presents conditions which ensure that, given an almost strictly positive real system, a simple adaptive control law will guarantee an upper bound for the \mathcal{H}_∞ norm, and [36] provides a set of conditions that ensure that an adaptive diagonal output feedback control law guarantees an \mathcal{L}_2 performance in closed loop.

This paper generalizes the result in our conference version [33]¹ using a parameter-dependent Lyapunov function and allowing the uncertain parameters to be time-varying. Such a generalization is not trivial since both the parameter adaptation law and the control law in [33] depend on the inverse of the Lyapunov matrix. To avoid a nonconvex control formulation, we judiciously introduce *S-variables* in the design procedure through specific algebraic equations and appropriate matrix properties. As a result, the Lyapunov matrix is no longer involved in the parameter adaptation law and the control gain expression, leading to an LMI-based control formulation. In particular, it is possible to prove theoretically that the proposed formulation is less conservative than some existing robust control results, which cannot be demonstrated in [33]. Illustrative examples numerically confirm the interests of the new adaptive gain-scheduling scheme.

Notation. $\mathbb{R}^{m \times n}$ is the set of m -by- n real matrices. For a vector \mathbf{x} , x_i denotes its i th entry and the Euclidean norm of \mathbf{x} is defined by $\|\mathbf{x}\| = \sqrt{\mathbf{x}^T \mathbf{x}}$. Given a matrix X , X^T indicates its transpose, X^{-1} indicates its inverse, and $\text{He}(X) = X + X^T$. For symmetric matrices P and X , $P \succ X$ (respectively $X \prec Y$) if and only if $P - X$ is positive (respectively negative) definite. $\text{diag}(X, Y)$ denotes a block-diagonal matrix composed of matrices X and Y , $X \otimes Y$ denotes the Kronecker product of X and Y . \star denotes a component that can be inferred by symmetry in a symmetric matrix. I denotes the identity matrix of appropriate dimensions. $\mathcal{K}_r = \{i \in \mathbb{N} : 1 \leq i \leq r\}$ denotes the set of positive integer indices less than or equal to r . Arguments are omitted whenever their meaning is clear.

2. Problem Statement

Consider the following uncertain system:

$$\dot{\mathbf{x}}(t) = A(\boldsymbol{\alpha})\mathbf{x}(t) + B(\boldsymbol{\alpha})\mathbf{u}(t), \quad (1)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ is the control input, and $\boldsymbol{\alpha} = [\alpha_1 \ \dots \ \alpha_r]^T$ is the uncertain, possibly time-varying, parameter vector. Matrices $A(\boldsymbol{\alpha})$ and $B(\boldsymbol{\alpha})$ are not

precisely known, but belong to a polytopic uncertainty domain given by

$$[A(\boldsymbol{\alpha}) \ B(\boldsymbol{\alpha})] = \sum_{i=1}^r \alpha_i [A_i \ B_i], \quad \boldsymbol{\alpha} \in \Xi_r,$$

where the unit simplex Ξ_r is defined as

$$\Xi_r = \left\{ \boldsymbol{\alpha} \in \mathbb{R}^r : 0 \leq \alpha_i \leq 1, \sum_{i=1}^r \alpha_i = 1 \right\}.$$

Note that r is the number of vertices of the polytope, and the known matrices (A_i, B_i) , for $i \in \mathcal{K}_r$, are the vertices.

Since $\boldsymbol{\alpha}$ is *unknown*, a parameter-dependent control law of the form $\mathbf{u}(t) = \sum_{i=1}^r \alpha_i K_i \mathbf{x}(t)$ cannot be implemented for practical uses. Hence, a single LPI control law $\mathbf{u}(t) = K \mathbf{x}(t)$, *i.e.*, $K_i = K$, for $\forall i \in \mathcal{K}_r$, has been largely used for control design of uncertain polytopic systems in most of existing results, see *e.g.* [2, 17–20] and related references.

Note that a parameter-independent control law cannot fully exploit the polytopic structure of system (1). Hence, the results obtained with an LPI control law may be conservative, especially when the domain of the uncertainty is large. To overcome this major drawback, we consider a parameter-dependent controller of the form

$$\mathbf{u}(t) = \sum_{i=1}^r \hat{\alpha}_i(t) K_i \mathbf{x}(t) = K(\hat{\boldsymbol{\alpha}}) \mathbf{x}(t), \quad (2)$$

where the control gains $K_i \in \mathbb{R}^{m \times n}$, for $i \in \mathcal{K}_r$, and the time-varying scheduling parameter $\hat{\boldsymbol{\alpha}}(t) = [\hat{\alpha}_1(t) \ \dots \ \hat{\alpha}_r(t)]^T$ are adapted in closed-loop. Throughout this paper, we consider adaptation laws which ensure that $\sum_{i=1}^r \hat{\alpha}_j(t) = 1$, so that from (1) and (2), the closed-loop system is expressed by

$$\dot{\mathbf{x}} = \sum_{i=1}^r \sum_{j=1}^r \alpha_i \hat{\alpha}_j (A_i + B_i K_j) \mathbf{x}. \quad (3)$$

We are now in the position to formulate the control problem.

Problem 1. Given a polytopic uncertain system (1). Determine the control gains K_i , for $\forall i \in \mathcal{K}_r$, and the time-varying scheduling parameter $\hat{\boldsymbol{\alpha}}(t)$ such that the closed-loop system (3) is asymptotically stable for $\forall \boldsymbol{\alpha} \in \Xi_r$.

3. Adaptive Gain-Scheduling Control Design

Based on Lyapunov arguments and an adaptive parameter tuning, two theorems are presented in this next section. The first one presents a novel set of LMI-based conditions to deal with the control problem when the uncertain parameters are constant, *i.e.* $\dot{\boldsymbol{\alpha}} = 0$. Whereas the second one modifies the first theorem conditions and adaptive control law to deal with the case of time-varying uncertain parameters with a bounded variation rate, $|\dot{\alpha}_j| \leq \phi_j$, for $j \in \mathcal{K}_r$.

¹Note that a quadratic parameter-independent Lyapunov function was used in [33] to derive the design conditions.

Theorem 1. Consider an uncertain system (1) with $\alpha \in \Xi_r$ constant and a parameter-dependent controller (2). If there exist symmetric positive definite matrices $P_i \in \mathbb{R}^{n \times n}$, symmetric matrices $J_{ii} \in \mathbb{R}^{n \times n}$, $M \in \mathbb{R}^{rn \times rn}$, $H \in \mathbb{R}^{m \times m}$, $V \in \mathbb{R}^{rn \times rn}$, $\Omega_i \in \mathbb{R}^{n \times n}$, $\Lambda_i \in \mathbb{R}^{n \times n}$, and matrices $J_{ij} \in \mathbb{R}^{n \times n}$ ($i \neq j$), $J_{ji} = J_{ij}^T$, $L_{ij} \in \mathbb{R}^{n \times n}$, $\bar{L}_i \in \mathbb{R}^{n \times n}$, $N \in \mathbb{R}^{n \times n}$, $E \in \mathbb{R}^{m \times n}$, $X_i \in \mathbb{R}^{m \times n}$, with $i, j \in \mathcal{K}_r$, and a positive scalar μ , such that

$$\mathcal{Q}_{ii} \prec 0, \quad \mathcal{Q}_{ij} + \mathcal{Q}_{ji} \prec 0, \quad i, j \in \mathcal{K}_r, i < j \quad (4)$$

$$\frac{1}{2}(\mathcal{S} + \mathcal{S}^T - M) - V \succ 0 \quad (5)$$

$$\begin{bmatrix} V & \star \\ \mathcal{X} & H \end{bmatrix} \succ 0 \quad (6)$$

$$\begin{bmatrix} \mathcal{J} & \star \\ \mathcal{L}^T & 2M \end{bmatrix} \succ 0 \quad (7)$$

where $\mathcal{X} = [X_1 - E \quad \dots \quad X_r - E]$ and

$$\mathcal{Q}_{ij} = \text{He} \begin{bmatrix} A_i N + B_i X_j + \frac{1}{2} J_{ij} & -N \\ P_i + \mu(A_i N + B_i X_j) & \frac{\mu^2}{2} B_i H B_i^T - \mu N \end{bmatrix} \quad (8)$$

$$\mathcal{J} = \begin{bmatrix} J_{11} & \dots & J_{1r} \\ \vdots & \ddots & \vdots \\ J_{r1} & \dots & J_{rr} \end{bmatrix} \quad (9)$$

$$\mathcal{L} = \begin{bmatrix} L_{11} + \bar{L}_1 & \dots & L_{1r} + \bar{L}_1 \\ \vdots & \ddots & \vdots \\ L_{r1} + \bar{L}_r & \dots & L_{rr} + \bar{L}_r \end{bmatrix} \quad (10)$$

$$\mathcal{S} = \begin{bmatrix} \text{He}(B_1 X_1) + \Psi_{11} & \dots & \text{He}(B_1 X_r) + \Psi_{r1} \\ \vdots & \ddots & \vdots \\ \text{He}(B_r X_1) + \Psi_{1r} & \dots & \text{He}(B_r X_r) + \Psi_{rr} \end{bmatrix} \quad (11)$$

with $\Psi_{ij} = \Omega_i + \Lambda_j + L_{ji}$, for $i, j \in \mathcal{K}_r$. Then, the controller (2) together with its parameter adaptation law

$$\dot{\hat{\alpha}}_j(t) = g_j(t) - \frac{1}{r} \sum_{k=1}^r g_k(t), \quad \sum_{j=1}^r \hat{\alpha}_j(0) = 1,$$

where

$$g_j(t) = -\gamma \sum_{k=1}^r \hat{\alpha}_k \mathbf{x}(t)^T R_{kj} \mathbf{x}(t), \quad \gamma > 0, \quad (12)$$

$$R_{kj} = N^{-1} L_{kj} N^{-1} + N^{-1} B_k K_j + K_j^T B_k^T N^{-1},$$

guarantee that the states of the closed-loop system (3) asymptotically converge to the origin and that all closed-loop signals are bounded. Furthermore, the control gains are given by

$$K_i = X_i N^{-1}, \quad i \in \mathcal{K}_r. \quad (13)$$

Proof. Consider a Lyapunov candidate function of the form

$$\mathcal{V}(\mathbf{x}, \Delta\alpha) = \mathbf{x}^T \mathcal{P}(\alpha) \mathbf{x} + \frac{1}{2\gamma} \sum_{j=1}^r \Delta\alpha_j^2,$$

where $\Delta\alpha_j = \hat{\alpha}_j - \alpha_j$ and

$$\mathcal{P}(\alpha) = \sum_{i=1}^r \alpha_i \mathcal{P}_i, \quad \mathcal{P}_i = N^{-T} P_i N^{-1}, \quad i \in \mathcal{K}_r. \quad (14)$$

Note that condition (4) ensures the existence of N^{-1} . Hence, by definition (14), matrix $\mathcal{P}(\alpha)$ is well defined and positive definite since $P_i \succ 0, \forall i \in \mathcal{K}_r$. As the unknown parameters α_j are constant, then the time derivative of $\mathcal{V}(\mathbf{x}, \Delta\alpha)$ is given by

$$\dot{\mathcal{V}}(\mathbf{x}, \Delta\alpha) = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}^T \begin{bmatrix} 0 & \mathcal{P}(\alpha) \\ \mathcal{P}(\alpha) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} + \frac{1}{\gamma} \sum_{j=1}^r \Delta\alpha_j \left(g_j - \frac{1}{r} \sum_{k=1}^r g_k \right). \quad (15)$$

The adaptation law includes the projection term $-\frac{1}{r} \sum_{k=1}^r g_k$, which ensures that, if $\sum_{j=1}^r \hat{\alpha}_j(0) = 1$, then $\sum_{j=1}^r \hat{\alpha}_j(t) = 1, \forall t$. This, in turn, implies that $\sum_{j=1}^r \Delta\alpha_j = 0$ and

$$\sum_{j=1}^r \Delta\alpha_j \left(\frac{1}{r} \sum_{k=1}^r g_k \right) = 0. \quad (16)$$

Using the *S-variable* approach [20], and considering the closed-loop dynamics in (3), we have that

$$\begin{aligned} & \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}^T \begin{bmatrix} 0 & \mathcal{P}(\alpha) \\ \mathcal{P}(\alpha) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} = \\ & \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}^T \left(\begin{bmatrix} 0 & \mathcal{P}(\alpha) \\ \mathcal{P}(\alpha) & 0 \end{bmatrix} \right. \\ & \left. + \text{He} \left(\begin{bmatrix} N^{-T} \\ \mu N^{-T} \end{bmatrix} \begin{bmatrix} A^T(\alpha) + K^T(\hat{\alpha}) B^T(\alpha) \\ -I \end{bmatrix} \right)^T \right) \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} \end{aligned} \quad (17)$$

which allows decoupling the Lyapunov parameter matrix from the system's dynamics as well as introducing the slack variable N . From (16) and (17), it follows that (15) can be rewritten as

$$\begin{aligned} \dot{\mathcal{V}}(\mathbf{x}, \Delta\alpha) &= \sum_{i=1}^r \sum_{j=1}^r \alpha_i \hat{\alpha}_j \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}^T \begin{bmatrix} Q_{ij}^{(11)} & \star \\ Q_{ij}^{(21)} & Q_{ij}^{(22)} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} \\ &+ \frac{1}{\gamma} \sum_{j=1}^r \Delta\alpha_j g_j, \end{aligned} \quad (18)$$

with

$$\begin{aligned} Q_{ij}^{(11)} &= N^{-T} A_i + A_i^T N^{-1} + N^{-T} B_i K_j + K_j^T B_i^T N^{-1} \\ Q_{ij}^{(21)} &= N^{-T} P_i N^{-1} - N^{-1} + \mu (N^{-T} A_i + N^{-T} B_i K_j) \\ Q_{ij}^{(22)} &= -\mu (N^{-1} + N^{-T}). \end{aligned}$$

Note that $\hat{\alpha}_j = \alpha_j + \Delta\alpha_j$. Then, substituting g_j from (12) to (18), it follows that

$$\begin{aligned} \dot{V}(\mathbf{x}, \Delta\alpha) &= \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}^T \begin{bmatrix} \hat{Q}_{ij}^{(11)} & \star \\ \hat{Q}_{ij}^{(21)} & \hat{Q}_{ij}^{(22)} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} \\ &+ 2\mu \sum_{i=1}^r \sum_{j=1}^r \alpha_i \Delta\alpha_j \mathbf{x}^T K_j^T B_i^T N^{-1} \dot{\mathbf{x}} \\ &+ \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \alpha_i \mathbf{x}^T \text{He} (K_j^T B_i^T N^{-1}) \mathbf{x} \\ &- \sum_{j=1}^r \sum_{k=1}^r \Delta\alpha_j \hat{\alpha}_k \mathbf{x}^T N^{-1} L_{kj} N^{-1} \mathbf{x} \\ &- \sum_{j=1}^r \sum_{k=1}^r \Delta\alpha_j \hat{\alpha}_k \mathbf{x}^T \text{He} (K_j^T B_i^T N^{-1}) \mathbf{x}. \end{aligned} \quad (19)$$

Note that

$$\begin{aligned} &2\mu \sum_{i=1}^r \sum_{j=1}^r \alpha_i \Delta\alpha_j \mathbf{x}^T K_j^T B_i^T N^{-1} \dot{\mathbf{x}} \\ &\leq \sum_{i=1}^r \sum_{j=1}^r \Delta\alpha_i \Delta\alpha_j \mathbf{x}^T K_i^T H^{-1} K_j \mathbf{x} \\ &+ \mu^2 \sum_{i=1}^r \alpha_i \dot{\mathbf{x}}^T N^{-T} B_i H B_i^T N^{-1} \dot{\mathbf{x}}, \end{aligned} \quad (20)$$

for any matrix of appropriate dimension $H \succeq 0$. Then, considering the variable transformation $\mathbf{x} = N\mathbf{z}$, it follows from (19) and (20) that

$$\begin{aligned} \dot{V}(\mathbf{x}, \Delta\alpha) &\leq \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix}^T \begin{bmatrix} \hat{Q}_{ij}^{(11)} & \star \\ \hat{Q}_{ij}^{(21)} & \hat{Q}_{ij}^{(22)} \end{bmatrix} \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix} \\ &- \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T \Upsilon_{ij} \mathbf{z} \\ &- \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \hat{\alpha}_i \mathbf{z}^T L_{ij} \mathbf{z}, \end{aligned} \quad (21)$$

with $\Upsilon_{ij} = B_i X_j + X_j^T B_i^T - X_i^T H^{-1} X_j$, and

$$\begin{aligned} \hat{Q}_{ij}^{(11)} &= A_i N + N^T A_i^T + B_i X_j + X_j^T B_i^T \\ \hat{Q}_{ij}^{(21)} &= P_i - N^T + \mu (A_i N + B_i X_j) \\ \hat{Q}_{ij}^{(22)} &= \mu^2 B_i H B_i^T - \mu (N + N^T). \end{aligned}$$

Using again the relation $\hat{\alpha}_i = \alpha_i + \Delta\alpha_i$, it follows from condition (21) that

$$\begin{aligned} \dot{V}(\mathbf{x}, \Delta\alpha) &\leq \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix}^T \begin{bmatrix} \hat{Q}_{ij}^{(11)} & \star \\ \hat{Q}_{ij}^{(21)} & \hat{Q}_{ij}^{(22)} \end{bmatrix} \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix} \\ &- \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T \Upsilon_{ij} \mathbf{z} \\ &- \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T L_{ij} \mathbf{z} - \Pi(\mathbf{z}, \alpha), \end{aligned} \quad (22)$$

with $\Pi(\mathbf{z}, \alpha) = \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \alpha_i \mathbf{z}^T L_{ij} \mathbf{z}$. Since $\sum_{j=1}^r \Delta\alpha_j = 0$, we can introduce slack variables \bar{L}_i in $\Pi(\mathbf{z}, \alpha)$, leading to

$$\begin{aligned} \Pi(\mathbf{z}, \alpha) &= \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \alpha_i \mathbf{z}^T (L_{ij} + \bar{L}_i) \mathbf{z} \\ &= \mathbf{z}^T (\alpha^T \otimes I) \mathcal{L} (\Delta\alpha \otimes I) \mathbf{z}, \end{aligned} \quad (23)$$

with \mathcal{L} in (10) and $\Delta\alpha = [\Delta\alpha_1 \ \Delta\alpha_2 \ \dots \ \Delta\alpha_r]^T$. Since $-2(\alpha^T \otimes I) \mathcal{L} (\Delta\alpha \otimes I) \preceq (\Delta\alpha^T \otimes I) M (\Delta\alpha \otimes I) + (\alpha^T \otimes I) \mathcal{L} M^{-1} \mathcal{L}^T (\alpha \otimes I)$,

and inequality (7) implies that $\frac{1}{2} \mathcal{L} M^{-1} \mathcal{L}^T \preceq \mathcal{J}$, with \mathcal{J} defined in (9), it follows that

$$\begin{aligned} & - (\alpha^T \otimes I) \mathcal{L} (\Delta\alpha \otimes I) \\ & \preceq \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j J_{ij} + \frac{1}{2} (\Delta\alpha^T \otimes I) M (\Delta\alpha \otimes I). \end{aligned} \quad (24)$$

From (22), (23) and (24), it follows that

$$\begin{aligned} \dot{V}(\mathbf{x}, \Delta\alpha) &\leq \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix}^T \mathcal{Q}_{ij} \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix} \\ &- \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T (B_i X_j + X_j^T B_i^T + L_{ij}) \mathbf{z} \\ &+ \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T (X_i^T H^{-1} X_j) \mathbf{z} \\ &+ \frac{1}{2} \Delta\alpha^T (\mathbf{z}^T \otimes I) M (\mathbf{z} \otimes I) \Delta\alpha, \end{aligned} \quad (25)$$

with \mathcal{Q}_{ij} defined in (8). Once again, we make use of the fact that $\sum_{i=1}^r \Delta\alpha_i = 0$ to introduce slack variables, leading to

$$\begin{aligned} & \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T (B_i X_j + X_j^T B_i^T + L_{ij}) \mathbf{z} \\ &= \sum_{i=1}^r \sum_{j=1}^r \Delta\alpha_i \Delta\alpha_j \mathbf{z}^T (B_i X_j + X_j^T B_i^T + L_{ij} + \Lambda_i + \Omega_j) \mathbf{z}, \end{aligned} \quad (26)$$

for any symmetric matrices Λ_i and Ω_j of appropriate dimensions, with $i, j \in \mathcal{K}_r$. A suitable slack matrix E can also be introduced, leading to

$$\begin{aligned} & \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T (X_i^T H^{-1} X_j) \mathbf{z} \\ &= \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T (X_i^T - E^T) H^{-1} (X_j - E) \mathbf{z}. \end{aligned} \quad (27)$$

To obtain LMI-based design conditions, an upper bound for this term is assured from inequality (6), which implies that

$$\begin{aligned} & \sum_{j=1}^r \sum_{i=1}^r \Delta\alpha_j \Delta\alpha_i \mathbf{z}^T (X_i^T - E^T) H^{-1} (X_j - E) \mathbf{z} \\ & \leq \Delta\alpha^T (\mathbf{z}^T \otimes I) V (\mathbf{z} \otimes I) \Delta\alpha. \end{aligned} \quad (28)$$

Taking into account (26), (27) and (28), it follows from inequality (25) that

$$\begin{aligned} \dot{\mathcal{V}}(\mathbf{x}, \Delta\alpha) &\leq \sum_{i=1}^r \sum_{j=1}^r \alpha_i \alpha_j \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix}^T \mathcal{Q}_{ij} \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix} \\ &\quad - \Delta\alpha^T (\mathbf{z}^T \otimes I) \Sigma (\mathbf{z} \otimes I) \Delta\alpha, \end{aligned} \quad (29)$$

where $\Sigma = \frac{1}{2} (\mathcal{S} + \mathcal{S}^T - M) - V$ and \mathcal{S} defined in (11). It is clear now that conditions (4) and (5) guarantee $\dot{\mathcal{V}}(\mathbf{x}, \Delta\alpha) \leq 0$. Furthermore, note from condition (29) that $\dot{\mathcal{V}}(\mathbf{x}, \Delta\alpha) = 0$ only if $\mathbf{x} = 0$. By LaSalle's invariance principle [37], we conclude that the conditions in Theorem 1 guarantee that the state of the closed-loop uncertain system (3) asymptotically converges to the origin, *i.e.*, $\lim_{t \rightarrow \infty} \mathbf{x} = 0$. \square

Remark 1. From the design conditions in Theorem 1, by selecting $X_i = X = E$ and $L_{ij} = L_i = -\bar{L}_i$, for $\forall i, j \in \mathcal{K}_r$, we can make the matrices $J_{ij} \preceq \epsilon I$, $H \preceq \epsilon I$ and $M \preceq \epsilon I$, with an arbitrarily small scalar $\epsilon > 0$. Then, the conditions

$$\text{He} \begin{bmatrix} A_i N^T + B_i X & P_i \\ \mu (A_i N^T + B_i X) - N & -\mu N \end{bmatrix} \prec 0, \quad i \in \mathcal{K}_r.$$

can be directly recovered. This means that given the same Lyapunov candidate function, the design results obtained from Theorem 1 include those given by [33, Lemma 2], which is a common set of conditions obtained from either the S -variable approach or Finsler's lemma [20]. This inclusion cannot be proved with the results proposed in our preliminary work [33]. In particular, following similar arguments with appropriate choices of the decision variables $N = P = P_i$, $X_i = E$, for $\forall i \in \mathcal{K}_r$, $M = \epsilon I$, $H = \epsilon I$, with an arbitrarily small scalar $\epsilon > 0$, we can also precisely recover the control results given in [33, Theorem 1] from the proposed design conditions.

Theorem 2. Consider an uncertain system (1) where the *time-varying* parameter $\alpha(t) \in \Xi_r$ is with a bounded variation rate, *i.e.* $|\dot{\alpha}_j| < \phi_j$, for $j \in \mathcal{K}_r$. If the design conditions in Theorem 1 are satisfied with a fixed Lyapunov matrix, *i.e.* $P_i = P$, for $\forall i \in \mathcal{K}_r$, and \mathcal{Q}_{ij} in (4) given by

$$\mathcal{Q}_{ij} = \text{He} \begin{bmatrix} A_i N + B_i X_j + \frac{1}{2} J_{ij} + \sigma P & -N \\ P + \mu (A_i N + B_i X_j) & \frac{\mu^2}{2} B_i H B_i^T - \mu N \end{bmatrix},$$

with $\sigma > 0$, instead of expression (8). Then, the adaptive controller (2) guarantees that the states of the closed-loop system (3) are ultimately bounded by

$$\|\mathbf{x}\| \leq \sqrt{\frac{\sum_{j=1}^r (\phi_j + \frac{\sigma}{4})}{\gamma \sigma \lambda_{\min}}}, \quad (30)$$

with λ_{\min} the smallest eigenvalue of $N^T P N^{-1}$, and $\gamma > 0$ a learning parameter of the adaptation law. The control gains are given in (13) while the parameter adaptation law is defined as

$$\begin{aligned} \dot{\hat{\alpha}}_j(t) &= \mathcal{P}_{\Xi_r} \left(g_j(t) - \frac{1}{r} \sum_{k=1}^r g_k(t) \right), \quad \hat{\alpha}(0) \in \Xi_r, \\ g_j(t) &= -\gamma \left(\sum_{k=1}^r \hat{\alpha}_k \mathbf{x}(t)^T R_{kj} \mathbf{x}(t) \right) - \frac{\sigma}{2} \left(\hat{\alpha}_j - \frac{1}{2} \right), \end{aligned}$$

with $\gamma > 0$, R_{kj} given by (12) and \mathcal{P}_{Ξ_r} a projection that guarantees that $\hat{\alpha}(t) \in \Xi_r$, for $\forall t > 0$.

Proof. As presented in adaptive control theory, the term introduced in the Lyapunov stability analysis by a projection onto a convex set can only make the Lyapunov function's time derivative more negative, and can be disregarded [22, Chapter 4.4]. Hence, following the proof from Theorem 1, we can prove that

$$\dot{\mathcal{V}}(\mathbf{x}, \Delta\alpha) \leq -\sigma \mathbf{x}^T P \mathbf{x} + \frac{1}{\gamma} \sum_{j=1}^r \Delta\alpha_j \left(-\dot{\alpha}_j - \frac{\sigma}{2} \left(\hat{\alpha}_j - \frac{1}{2} \right) \right).$$

Using the fact that $\hat{\alpha}_j = \alpha_j + \Delta\alpha_j$ and due to the projection, $|\Delta\alpha_j| < 1$, it follows that

$$\begin{aligned} \dot{\mathcal{V}}(\mathbf{x}, \Delta\alpha) &\leq -\sigma \mathcal{V}(\mathbf{x}, \Delta\alpha) + \frac{1}{\gamma} \sum_{j=1}^r \left(|\dot{\alpha}_j| + \frac{\sigma}{4} \right) \\ &\leq -\sigma \mathcal{V}(\mathbf{x}, \Delta\alpha) + \frac{1}{\gamma} \sum_{j=1}^r \left(\phi_j + \frac{\sigma}{4} \right). \end{aligned} \quad (31)$$

Integrating both sides of (31) from 0 to t , it follows that

$$\mathcal{V}(\mathbf{x}, \Delta\alpha) \leq \mathcal{V}(0) + (1 - e^{-\sigma t}) \left(\frac{\sum_{j=1}^r (\phi_j + \frac{\sigma}{4})}{\gamma \sigma} \right).$$

Finally, the ultimate bound given in (30) is derived using the fact that $\mathbf{x}^T N^T P N^{-1} \mathbf{x} \leq \mathcal{V}(\mathbf{x}, \Delta\alpha)$. \square

Remark 2. Note that since the term $-\frac{1}{r} \sum_{k=1}^r g_k(t)$ already guarantees that $\sum_{k=1}^r \hat{\alpha}_k(t) = 1$, the projection \mathcal{P}_{Ξ_r} only needs to guarantee that $0 \leq \hat{\alpha}(t) \leq 1$ without breaking the constraint already imposed. To do so, it checks if $\hat{\alpha}_j = 0$ and $\dot{\hat{\alpha}}_j < 0$, or if $\hat{\alpha}_j = 1$ and $\dot{\hat{\alpha}}_j > 0$, for $\forall j \in \mathcal{K}_r$. If any of those affirmations is true, then it makes $\dot{\hat{\alpha}}_j = 0$, for $\forall j \in \mathcal{K}_r$. Otherwise, it does not make any change.

4. Illustrative Examples

This section presents two illustrative examples to demonstrate the effectiveness of the proposed control method. All numerical optimizations are performed with Matlab software using YALMIP toolbox [38] and Mosek solver.

Example 1 (Comparative study). Consider the following uncertain polytopic system taken from Example 1 in [33]:

$$\dot{\mathbf{x}} = \sum_{i=1}^4 \alpha_i (A_i \mathbf{x} + B_i \mathbf{u}), \quad \alpha \in \Xi_4, \quad (32)$$

with

$$\begin{aligned} A_1 &= \begin{bmatrix} -8.1818 & 0 \\ 0.0909 & 0 \end{bmatrix}, & B_1 &= \begin{bmatrix} -18.1818 \\ 0.0909 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1.6364 & 0 \\ 0.0909 & 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} -3.6364 \\ 0.0909 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} \frac{10(k-1)}{k+1} & 0 \\ \frac{k}{k+1} & 0 \end{bmatrix}, & B_3 &= \begin{bmatrix} \frac{-20}{k+1} \\ \frac{k}{k+1} \end{bmatrix}, \\ A_4 &= \begin{bmatrix} \frac{2(k-1)}{k+1} & 0 \\ \frac{k}{k+1} & 0 \end{bmatrix}, & B_4 &= \begin{bmatrix} \frac{-4}{k+1} \\ \frac{k}{k+1} \end{bmatrix}, \end{aligned}$$

and k is a positive parameter. The goal here is to determine the largest feasible value of $k > 0$, denoted by k^* , for which a control solution can be found for system (32). Table 1 shows the results obtained with the design conditions in Theorem 1 and related existing control approaches. Specifically, the proposed control conditions are compared to the adaptive conditions in [33], and fixed-gain conditions (*i.e.*, linear control laws) in [2, 17–19] (without taking into account the disturbances).

Table 1: Maximum feasible value of $k > 0$ in Example 1.

Design conditions	k^*
Theorem 1, $\mu = 10^{-11}$	30.32
[33, Theorem 1]	11.80
[19, adapted Theorem 1], $\xi = -0.999$, $\varepsilon = 0.11$	9.77
[18, adapted Theorem 1], $\xi = 10^{-11}$	9.77
[17, adapted Lemma 3.1], $\epsilon = 0.99 \times 10^{-5}$	9.74
[2, equation (7.12)]	9.76

Observe from Table 1 that the new control approach provides a significant improvement compared to the previous adaptive result in [33] which, for this example, already outperforms the existing linear robust control approaches.

Example 2 (Physically motivated system). Consider a brushless DC (BLDC) motor system whose dynamics is given by

$$\dot{\mathbf{x}} = \sum_{i=1}^6 \alpha_i(\theta)(A_i \mathbf{x} + B \mathbf{u}), \quad \alpha(\theta) \in \Xi_6, \quad (33)$$

with $\mathbf{x} = [i_a \ i_b \ i_c \ \omega]^T$, $\mathbf{u} = [v_a \ v_b \ v_c]^T$. The states i_a, i_b and i_c represent the phase currents and ω is the angular velocity of the motor. The control inputs v_a, v_b and v_c respectively represent the voltage of each phase. The uncertain parameters $\alpha_i(\theta)$, for $i \in \mathcal{K}_6$, are functions of the *unmeasurable* angle θ of the motor shaft. Using the parameters in the datasheet of the BG75 \times 75 SI motor from Dunkermotoren company, the state-space matrices of system (33) are expressed as follows:

$$A_i = \begin{bmatrix} -111.11 & 0 & 0 & \beta\phi_{i1} \\ 0 & -111.11 & 0 & \beta\phi_{i2} \\ 0 & 0 & -111.11 & \beta\phi_{i3} \\ \delta\phi_{i1} & \delta\phi_{i2} & \delta\phi_{i3} & -4.65 \end{bmatrix}$$

$$B = \begin{bmatrix} \tau & 0 & 0 \\ 0 & \tau & 0 \\ 0 & 0 & \tau \\ 0 & 0 & 0 \end{bmatrix}, \quad \phi_i = [\phi_{i1} \ \phi_{i2} \ \phi_{i3}],$$

with $\beta = 128.25$, $\delta = -1825.2$, $\tau = 1587.3$, and

$$\begin{aligned} \phi_1 &= [-1 \ 1 \ 0], & \phi_2 &= [-1 \ 0 \ 1], \\ \phi_3 &= [0 \ -1 \ 1], & \phi_4 &= [1 \ -1 \ 0], \\ \phi_5 &= [1 \ 0 \ -1], & \phi_6 &= [0 \ 1 \ -1]. \end{aligned}$$

Note that the parameter $\alpha(\theta)$ of system (33) is unknown and time-varying. In this case, conditions from [2, 17–19] are *infeasible*, whereas conditions from Theorem 2 are feasible with $\mu \in [10^{-10}, 10^{-4}]$.

For illustrations, Figure 1 depicts the closed-loop state induced by the adaptive gain-scheduling controller obtained with $\mu = 5 \times 10^{-5}$, $\sigma = 0.2$ and $\gamma = 10^{-3}$. The uncertain time-varying parameters $\alpha_i(\theta(t))$, the respective estimated parameters and control input are shown in Figure 2. Note that the obtained closed-loop behavior physically corresponds to that of the BLDC motor during a braking phase.

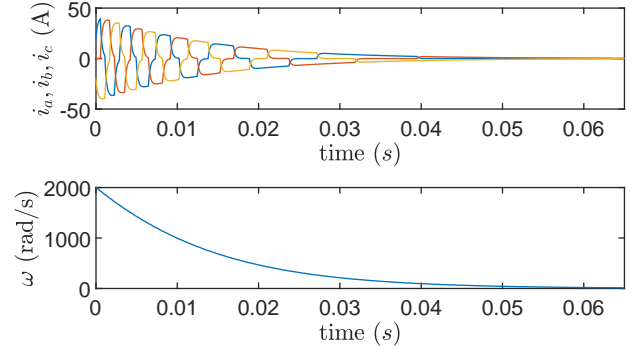


Figure 1: States from the BLDC motor in Example 2 in closed loop with the adaptive controller found using the conditions from Theorem 2.

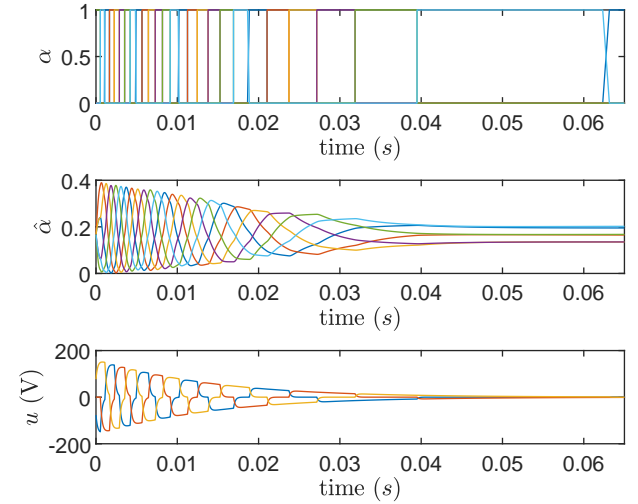


Figure 2: Time-varying uncertain parameters, estimated parameters and control input from the BLDC motor in Example 2 with the adaptation law presented in Theorem 2.

5. Concluding Remarks

A new method to design an adaptive gain-scheduling controller for uncertain polytopic systems has been proposed. This enables exploiting the system polytopic structure for the control design, which contributes to overcome the conservatism issue of the standard robust control approaches. The possible feedback gains, employed in a gain-scheduled manner with the estimated convex weights, as well as the adaptation law gains

are determined *offline* by solving a set of LMI design conditions. Using Lyapunov theory, the asymptotic stability of the closed-loop system and the stability of the parameter adaptation scheme are guaranteed. Numerical examples are given to point out the interests of the proposed method over the state-of-the-art control results in robust control.

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