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Delayed Nonquadratic $\mathscr{L}_2$-Stabilization of Continuous-Time Nonlinear Takagi-Sugeno Fuzzy Models

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Abstract

This work deals with the design of fuzzy controllers for stabilization of continuous-time nonlinear systems subject to $\mathscr{L}_2$ disturbances, which are represented by nonlinear Takagi-Sugeno fuzzy models, *i.e.*, Takagi-Sugeno fuzzy models with nonlinear consequents. A nonquadratic Lyapunov function is used to derive sufficient design conditions based on linear matrix inequality constraints as well as to reduce the conservativeness when compared to existing control approaches in the literature. Furthermore, the nonquadratic Lyapunov function is defined in terms of an integral membership function, which leads to a delayed nonquadratic $\mathscr{L}_2$-stabilization condition. This condition avoids the well-known difficulties in dealing with time derivatives of membership functions and/or path-independent conditions, found in most of the nonquadratic control approaches for continuous-time Takagi-Sugeno fuzzy models. Two numerical examples are performed to illustrate the reduction in conservativeness provided by the proposed approach.

Keywords: Takagi-Sugeno fuzzy models, nonlinear local models, nonquadratic Lyapunov functions, fuzzy control, robust control.

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1. Introduction

Takagi-Sugeno (TS) fuzzy models have been recognized as an effective way to represent nonlinear systems with an arbitrary precision by means of convex combination of local models, which in general are linear state-space equations [27]. By exploiting the convex structure of TS models, stability analysis and control design conditions can be formulated as convex optimization problems under linear matrix inequality (LMI) constraints [22].

It is well known that exact TS fuzzy models can be obtained to represent nonlinear systems by employing the sector nonlinearity approach [28]. As a consequence, the number of local models (or fuzzy rules) exponentially grows with the amount of the system nonlinearities related to the number of premise variables. Thus, for nonlinear systems with a high number of nonlinearities, the increased number of fuzzy rules leads to a high computational burden to solve stability or stabilization LMI conditions and difficulties for online controller implementation if needed.

An alternative to obtain exact TS models with reduced number of fuzzy rules is to consider TS fuzzy models with nonlinear consequent parts, in short: N-TS fuzzy models. In [16, 29], TS fuzzy models with nonlinear consequents have been used. However, the consequents are polynomial and sum-of-squares conditions were derived for the control design of nonlinear systems. On the other hand, in [8, 10], a class of N-TS fuzzy models was proposed from the introduction of sector-bounded nonlinear functions of states to the local models, and the consequent parts can be viewed as local Lur'e-type systems. Thus, conditions were obtained in terms of LMI constraints. This latter TS fuzzy approach is useful to reduce both the design conservativeness and the computational complexity of the analysis and design conditions compared to the traditional TS fuzzy modeling. In addition, such representation has been used successfully in different applications, such as fault-tolerant control [2] and interconnected systems [1].

For discrete-time N-TS fuzzy models, nonquadratic stabilization approaches

have been developed such as the design of dynamic output feedback fuzzy controllers [9, 13], disturbance attenuation [10], robust fuzzy model predictive control [30], multiple-parameterization approach [5] and constrained output feedback control [21]. On the other hand, for continuous-time N-TS fuzzy models, quadratic Lyapunov functions have been employed to develop conditions for $\mathscr{L}_2$-stabilization [8], distributed control [1], fault-tolerant control [2], and state estimation [24].

A nonquadratic stabilization condition has been proposed in [17] with a particular Lyapunov function, which is composed of a quadratic term and a nonquadratic one constructed from the integral of the nonlinearities. As a result, path-independent assumptions [25] are required, which may introduce extra complexity and conservativeness to the control design approach. Besides, the work in [17] is concerned with obtaining estimates of the domain of attraction and neglects the $\mathscr{L}_2$ control stabilization since the extent of its conditions for this scenario is not trivial. To the best of the authors' knowledge, there is no any other work besides the one in [8] (quadratic stabilization) considering the main topic of this submission, namely to deal with the problem of $\mathscr{L}_2$ control stabilization for N-TS fuzzy models Therefore, there exists a lack of nonquadratic stabilization conditions for continuous-time N-TS systems, especially when $\mathscr{L}_2$-stabilization is concerned. Then, the first motivation for this work is to derive nonquadratic $\mathscr{L}_2$-stabilization conditions for continuous-time N-TS fuzzy systems.

Nevertheless, the major challenge on nonquadratic stabilization of continuous-time fuzzy systems is dealing with the time derivatives of the membership functions [3]. To this end, many works in different scenarios have been done based on the assumption of bounded time-derivatives [3, 4, 18–20] or path-independent conditions [17, 18, 25]. Unlike traditional articles that deal with the time-delay in state variables [14, 23], more recently for TS fuzzy models, the authors in [11, 15] proposed a nonquadratic Lyapunov function depending on the integral of membership functions, such that artificially delayed membership functions are introduced in the design conditions. The idea of delayed membership func-

tions has been exploited to derive less conservative nonquadratic stabilization conditions mainly for discrete-time TS fuzzy models [6, 7, 32] and, recently, discrete-time N-TS fuzzy models [5]. Thus, the second motivation for this work is to employ a nonquadratic Lyapunov function as in [11, 15] to derive a less conservative delayed nonquadratic $\mathscr{L}_2$-stabilization condition than the results in the related literature.

In this work is proposed new nonquadratic stabilization conditions for N-TS fuzzy models subject to $\mathscr{L}_2$ disturbances. The conditions are based on a general nonlinear control law and a delayed nonquadratic Lyapunov function similar to [11]. The main contributions of this paper are summarized as follows.

- The new delayed nonquadratic $\mathscr{L}_2$-stabilization approach avoids dealing with time derivatives of the membership functions, and any specific structures imposed by Lyapunov functions such as path-independent conditions [25].
- We consider a general nonlinear control law whose structure is entirely independent of the used nonquadratic Lyapunov function. This feature contributes to derive an LMI-based condition with significantly reduced conservativeness for continuous-time N-TS fuzzy systems compared to related existing control results.

The paper is organized as follows. The class of N-TS fuzzy systems and the problem formulation are described in Section 2. Section 3 presents the proposed stabilization conditions. In Section 4, two numerical examples are given to illustrate the effectiveness of the proposed control approach. Section 5 concludes this paper.

Notation. For a square matrix X, $X \succ 0$ (respectively $X \prec 0$) means that X is symmetric positive (respectively negative) definite. $\mathrm{diag}(X_1, X_2)$ represents a block-diagonal matrix composed of X_1 and X_2, and "$\star$" represents a symmetric matrix block. Let $\mathcal{I}_p = \{1, \ldots, p\} \subset \mathbb{N}$, with $p \in \mathbb{N}$. The convex hull of the set

$\mathcal{S}$ is denoted by co$\{\mathcal{S}\}$. The following notation is adopted for convex sum:

$$\Upsilon_\theta = \sum_{i=1}^{r} \theta_i(\xi(t))\Upsilon_i$$

for any matrix Υ_i of appropriate dimension. The norm in the $\mathscr{L}_2$ space of continuous square-integrable functions is defined as: $\|\zeta(t)\|_2 = \sqrt{\int_0^\infty \zeta(t)^\top \zeta(t) dt} < \infty$. Arguments are omitted when their meaning is clear.

2. Preliminaries and Problem Formulation

2.1. Takagi-Sugeno Fuzzy Models with Nonlinear Consequents

Consider a continuous-time and input affine nonlinear system given by

$$\begin{aligned} \dot{x}(t) &= f_1(x(t)) + f_2(x(t))w(t) + f_3(x(t))u(t) \\ z(t) &= f_4(x(t)) + f_5(x(t))w(t) + f_6(x(t))u(t) \end{aligned} \tag{1}$$

Following the sector nonlinearity approach [28], it is known that the nonlinear system (1) can be represented by the following class of N-TS fuzzy models [8]:

$$\begin{aligned} \dot{x}(t) &= A_\theta x(t) + B_{1\theta} w(t) + B_{2\theta} u(t) + G_\theta \varphi(x) \\ z(t) &= C_\theta x(t) + D_{1\theta} w(t) + D_{2\theta} u(t) + L_\theta \varphi(x) \end{aligned} \tag{2}$$

where $x(t) \in \mathcal{X} \subset \mathbb{R}^{n_x}$ is the state vector, $\mathcal{X}$ is a convex polytope containing the origin and defines also the validity domain of model (2), $u(t) \in \mathbb{R}^{n_u}$ is the input vector, $w(t) \in \mathbb{R}^{n_w}$ is the $\mathscr{L}_2$ disturbance, $\varphi(x) \in \mathbb{R}^{n_\varphi}$ is the vector of sector-bound nonlinearities, $z(t) \in \mathbb{R}^{n_z}$ is the controlled output. The constant matrices $(A_i, B_{1i}, B_{2i}, G_i, C_i, D_{1i}, D_{2i}, L_i)$ of the ith local model are of appropriate dimensions. The normalized membership functions $\theta_i(\xi(t))$, for $i \in \mathcal{I}_r$, satisfy the following convex sum property:

$$\sum_{i=1}^{r} \theta_i(\xi(t)) = 1, \quad \theta_i(\xi(t)) \geq 0, \quad i \in \mathcal{I}_r,$$

where r the number of fuzzy rules, and $\xi(t) \in \mathbb{R}^{n_v}$ stands for the vector of premise variables, which depends on the state vector. The convex region $\mathcal{X}$ is given as follows:

$$\mathcal{X} = \{x \in \mathbb{R}^{n_x} : h_m^\top x \leq 1;\, h_m \in \mathbb{R}^{n_x}, m \in \mathcal{I}_{n_e}\}, \tag{3}$$

where n_e is the number of hyperplanes. The following assumption is also considered for system (2).

Assumption 1. *The vector $\varphi(x(t))$ satisfies the sector-boundedness condition given by $\varphi_{(i)}(x(t)) \in \text{co}\{0, E_i x(t)\}$, $\forall i \in \mathcal{I}_{n_\varphi}$, which is equivalent to [8]:*

$$\varphi(x(t))^\top \Lambda_\theta^{-1} \left(Ex(t) - \varphi(x(t))\right) \geq 0, \tag{4}$$

where matrix $E = [E_1^\top, \ldots, E_{n_\varphi}^\top]^\top \in \mathbb{R}^{n_\varphi \times n_x}$ is given, and $\Lambda_\theta \in \mathbb{R}^{n_\varphi \times n_\varphi}$ is a positive definite diagonal matrix.

As aforementioned, the classical TS fuzzy modeling may lead to an excessive number of fuzzy rules. Using N-TS fuzzy model (2) can be useful to reduce the number of fuzzy rules, since only some nonlinearities are selected as premise variables, while others belong to the sector satisfying Assumption 1. Hence, an exact N-TS fuzzy model with reduced number of local models is obtained.

2.2. Problem Formulation

Consider the following nonlinear control law:

$$u(t) = K_\theta H_\theta^{-1} x(t) + F_\theta J_\theta^{-1} \varphi(x(t)), \tag{5}$$

where the matrices $K_\theta \in \mathbb{R}^{n_u \times n_x}$, $H_\theta \in \mathbb{R}^{n_x \times n_x}$, $F_\theta \in \mathbb{R}^{n_u \times n_\varphi}$ and $J_\theta \in \mathbb{R}^{n_\varphi \times n_\varphi}$, are to be determined.

Remark 1. *The control law proposed in [8] can be obtained from (5) by choosing $H_\theta = I$ and $J_\theta = I$. Moreover, controller (5) is similar to the one used in [10, Corollary 12] and [5] for the discrete-time case with the particular choice $J_\theta = \Lambda_\theta$. Hence, the new control law (5) is more general than those in previous related works.*

Substituting (5) into (2) leads to the closed-loop system of the form

$$\begin{aligned} \dot{x}(t) &= \mathbf{A}_{cl} x(t) + \mathbf{G}_{cl} \varphi(x(t)) + B_{1\theta} w(t) \\ z(t) &= \mathbf{C}_{cl} x(t) + \mathbf{L}_{cl} \varphi(x(t)) + D_{1\theta} w(t) \end{aligned} \tag{6}$$

where

$$\mathbf{A}_{cl} = A_\theta + B_{2\theta}K_\theta H_\theta^{-1}, \quad \mathbf{G}_{cl} = G_\theta + B_{2\theta}F_\theta J_\theta^{-1}$$
$$\mathbf{C}_{cl} = C_\theta + D_{2\theta}K_\theta H_\theta^{-1}, \quad \mathbf{L}_{cl} = L_\theta + D_{2\theta}F_\theta J_\theta^{-1}.$$

This paper considers the $\mathscr{L}_2$-stabilization control problem for N-TS fuzzy systems (2), *i.e.*, we aim to design a nonlinear control law (5) such that the closed-loop system has the following properties.

(i) If $w(t) = 0$, for $\forall t \geq 0$, the origin of the closed-loop system (6) is asymptotically stable.

(ii) If $w(t) \neq 0$, for $\forall t \geq 0$, the closed-loop system (6) is $\mathscr{L}_2-$stable from $w(t)$ to $z(t)$ with $\mathscr{L}_2-$gain less than or equal to γ under zero initial condition:

$$\|z\|_2 \leq \gamma \|w\|_2, \ \forall t \in [0, \infty). \tag{7}$$

For control design, we consider the following nonquadratic Lyapunov function:

$$V(x) = x^\top \left(\sum_{i=1}^{r} \nu_i(\xi) P_i \right)^{-1} x = x^\top P_\nu^{-1} x, \tag{8}$$

where $P_i \succ 0$, for $i \in \mathcal{I}_r$, and $\nu_i(\xi(t))$ is defined as

$$\nu_i(\xi(t)) = \frac{1}{\alpha} \int_{t-\alpha}^{t} \theta_i(\xi(\tau)) d\tau \geq 0. \tag{9}$$

The given scalar $\alpha > 0$ is an artificial time delay. Note that the convex sum property holds for the integral function $\nu_i(\xi(t))$ since

$$\sum_{i=1}^{r} \nu_i(\xi(t)) = \frac{1}{\alpha} \int_{t-\alpha}^{t} \left(\sum_{i=1}^{r} \theta_i(\xi(\tau)) \right) d\tau = 1. \tag{10}$$

Furthermore, the time derivative of $\nu_i(\xi(t))$ is given by

$$\dot{\nu}_i(\xi(t)) = \frac{1}{\alpha} \left(\theta_i(\xi(t)) - \theta_i(\xi(t-\alpha)) \right), \tag{11}$$

with the initial function given by $x(t) = \phi(t)$, $t \in [-\alpha, 0]$, $\phi \in \mathscr{C}([-\alpha, 0], \mathbb{R})$, where $\mathscr{C}([-\alpha, 0], \mathbb{R})$ is the Banach space of real continuous functions on the interval $[-\alpha, 0]$ with $\max_{t \in [-\alpha, 0]} |\phi(t)|$.

The following technical lemma, adapted from [12, Lemma 2], is useful to derive delayed nonquadratic stabilization conditions for N-TS fuzzy systems.

Lemma 1. *Consider matrices of appropriate dimensions $\mathcal{A}$ and $\mathcal{C}$. If there exists a matrix $\mathcal{P} \succ 0$ such that*

$$\begin{bmatrix} -\lambda(\mathcal{P}) & \star \\ \mathcal{P}\mathcal{A} & -\mathcal{P} - \mathcal{C} \end{bmatrix} \prec 0, \tag{12}$$

where $\lambda(\mathcal{P})$ is an arbitrary matrix expression related to $\mathcal{P}$ or not. Then, there exist matrices $\mathcal{P} \succ 0$ and $\mathcal{Z}$ such that

$$\begin{bmatrix} -\lambda(\mathcal{P}) & \star \\ \mathcal{Z}^\top\mathcal{A} & \mathcal{P} - \mathcal{Z} - \mathcal{Z}^\top - \mathcal{C} \end{bmatrix} \prec 0. \tag{13}$$

3. Delayed Nonquadratic $\mathscr{L}_2$-Stabilization of N-TS Fuzzy Models

The following theorem provides sufficient conditions to design a nonlinear controller (5) for system (2).

Theorem 1. *Given an N-TS fuzzy model (2), positive scalars μ, α and a sufficiently small ϵ. If there exist positive definite matrices P_i, positive definite diagonal matrices Λ_i, symmetric matrices M_{ij}^{kl}, and matrices K_i, F_i, H_i, J_i, of appropriate dimensions, such that*

$$\Upsilon_{ii}^{kl} \preceq M_{ii}^{kl}, \quad i,k,l \in \mathcal{I}_r, \tag{14}$$

$$\Upsilon_{ij}^{kl} + \Upsilon_{ji}^{kl} \preceq M_{ij}^{kl} + {M_{ij}^{kl}}^\top, \quad i,j,k,l \in \mathcal{I}_r \text{ and } i \neq j, \tag{15}$$

$$\begin{bmatrix} M_{11}^{kl} & \cdots & M_{1r}^{kl} \\ \vdots & \ddots & \vdots \\ M_{r1}^{kl} & \cdots & M_{rr}^{kl} \end{bmatrix} \prec 0, \quad k,l \in \mathcal{I}_r, \tag{16}$$

$$\begin{bmatrix} 1 & h_m^\top H_j \\ \star & -P_k + H_j + H_j^\top \end{bmatrix} \succeq 0, \quad j,k \in \mathcal{I}_r, m \in \mathcal{I}_{n_e}, \tag{17}$$

where

$$\Upsilon_{ij}^{kl} = \begin{bmatrix} -\epsilon^{-1}I & \star & \star & \star & \star & \star \\ \Psi_{21} & \Psi_{22} & \star & \star & \star & \star \\ \Psi_{31} & 0 & \Psi_{33} & \star & \star & \star \\ 0 & \Psi_{42} & \epsilon\Psi_{43} & -P_l & \star & \star \\ 0 & \epsilon E H_j & (1-\epsilon)J_j & 0 & -\Lambda_i & \star \\ \epsilon^{-1}D_{1i}^\top & 0 & 0 & B_{1i}^\top & 0 & -\mu\epsilon^{-1}I \end{bmatrix},$$

$$\Psi_{21} = H_j^\top C_i^\top + K_j^\top D_{2i}^\top,$$

$$\Psi_{22} = P_l - \epsilon\alpha^{-1}(P_i - P_k) - H_j - H_j^\top,$$

$$\Psi_{31} = J_j^\top L_i^\top + F_j^\top D_{2i}^\top, \quad \Psi_{33} = \Lambda_i - J_j - J_j^\top,$$

$$\Psi_{42} = H_j + \epsilon(A_i H_j + B_{2i}K_j), \quad \Psi_{43} = G_i J_j + B_{2i}F_j.$$

120 *Then, if $w(t) = 0$, the origin of the closed-loop system (6) is asymptotically stable and the region $\mathscr{L}_V = \{x \in \mathbb{R}^{n_x} : V(x) \leq 1\}$, $\mathscr{L}_V \subset \mathcal{X}$, is included in its region of attraction. Moreover, if $w(t) \neq 0$, then system (6) is $\mathscr{L}_2$-stable from $w(t)$ to $z(t)$ with an $\mathscr{L}_2$-gain less than or equal to $\gamma = \sqrt{\mu}$.*

Proof. By the relaxation result in [31], it follows from (14), (15) and (16) that

$$\sum_{l=1}^{r}\sum_{k=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{r} \nu_l(\xi(t))\theta_k^-(\xi(t))\theta_i(\xi(t))\theta_j(\xi(t))\Upsilon_{ij}^{kl} \prec 0 \tag{18}$$

where we denote $\theta_k^-(\xi(t)) = \theta_k(\xi(t-\alpha))$. Note that the satisfaction of inequality (18) guarantees that $J_\theta + J_\theta^\top \succ 0$, and $H_\theta + H_\theta^\top \succ 0$, for a sufficiently small scalar ϵ. This, in turn, guarantees the existence of J_θ^{-1} and H_θ^{-1}, thus the validity of the control expression (5). Note also that inequality (18) can be represented similar to (13) as

$$\begin{bmatrix} -\lambda(\mathcal{P}) & \star & \star \\ \mathcal{Z}^\top\mathcal{A} & \mathcal{P} - \mathcal{Z} - \mathcal{Z}^\top - \mathcal{C} & \star \\ \Sigma & 0 & -\mu\epsilon^{-1}I \end{bmatrix} \prec 0, \tag{19}$$

with

$$\mathcal{P} = \mathrm{diag}(P_\nu, \Lambda_\theta), \quad \lambda(\mathcal{P}) = \mathrm{diag}(\epsilon^{-1}I, P_\nu, \Lambda_\theta),$$

$$\mathcal{Z} = \mathrm{diag}(H_\theta, J_\theta), \qquad \mathcal{C} = \mathrm{diag}(\epsilon\alpha^{-1}(P_\theta - P_{\theta^-}), 0),$$

$$\mathcal{A} = \begin{bmatrix} \mathbf{C}_{cl}^\top & I + \epsilon\mathbf{A}_{cl}^\top & \epsilon E^\top \\ \mathbf{L}_{cl}^\top & \epsilon\mathbf{G}_{cl}^\top & (1-\epsilon)I \end{bmatrix}, \quad P_{\theta^-} = \sum_{i=1}^{r} \theta_i^-(\xi(t))P_i,$$

$$\Sigma = \begin{bmatrix} \epsilon^{-1}D_{1\theta}^\top & B_{1\theta}^\top & 0 \end{bmatrix}.$$

Applying Lemma 1, inequality (19) is equivalent to

$$\begin{bmatrix} -P_\nu & \star & \star & \star & \star & \star \\ 0 & -\Lambda_\theta & \star & \star & \star & \star \\ \Theta_{31} & \epsilon P_\nu E^\top & \Theta_{33} & \star & \star & \star \\ \epsilon\Theta_{41} & \Theta_{42} & 0 & -\Lambda_\theta & \star & \star \\ 0 & 0 & B_{1h}^\top & 0 & -\mu\epsilon^{-1}I & \star \\ 0 & 0 & \Theta_{63} & \Theta_{64} & \epsilon^{-1}D_{1\theta} & -\epsilon^{-1}I \end{bmatrix} \prec 0, \tag{20}$$

with

$$\Theta_{31} = P_\nu + \epsilon P_\nu \mathbf{A}_{cl}^\top, \quad \Theta_{33} = -P_\nu - \epsilon\alpha^{-1}(P_\theta - P_{\theta^-}),$$

$$\Theta_{41} = \Lambda_\theta \mathbf{G}_{cl}^\top, \qquad \Theta_{42} = (1-\epsilon)\Lambda_\theta,$$

$$\Theta_{63} = \mathbf{C}_{cl}P_\nu, \qquad \Theta_{64} = \mathbf{L}_{cl}\Lambda_\theta.$$

Pre- and postmultiplying inequality (20) with

$$\mathrm{diag}(\sqrt{\epsilon}I, \sqrt{\epsilon}I, (\sqrt{\epsilon}P_\nu)^{-1}, (\sqrt{\epsilon}\Lambda_\theta)^{-1}, \sqrt{\epsilon}I, \sqrt{\epsilon}I),$$

it follows that

$$\begin{bmatrix} -\epsilon\mathscr{P}^{-1} & \star & \star \\ I + \epsilon\mathscr{A}^\top & -\Phi & \star \\ 0 & \Gamma & -\Pi \end{bmatrix} \prec 0, \tag{21}$$

where

$$\mathscr{A} = \begin{bmatrix} \mathbf{A}_{cl} & \mathbf{G}_{cl} \\ E & -I \end{bmatrix}, \qquad \mathscr{P} = \mathrm{diag}(P_\nu^{-1}, \Lambda_\theta^{-1}),$$

$$\Gamma = \begin{bmatrix} B_{1\theta}^\top P_\nu^{-1} & 0 \\ \mathbf{C}_{cl} & \mathbf{L}_{cl} \end{bmatrix}, \quad \Pi = \begin{bmatrix} \mu I & \star \\ -D_{1\theta} & I \end{bmatrix},$$

$$\Phi = \mathrm{diag}\left(\epsilon^{-1}P_\nu^{-1} + \alpha^{-1}P_\nu^{-1}(P_\theta - P_{\theta^-})P_\nu^{-1},\ \epsilon^{-1}\Lambda_\theta^{-1}\right).$$

Note that $\dot{P}_\nu^{-1} = -P_\nu^{-1}\dot{P}_\nu P_\nu^{-1}$ and $\dot{P}_\nu = \alpha^{-1}(P_\theta - P_{\theta^-})$. Then, it follows that

$$\epsilon^{-1}P_\nu^{-1} + \alpha^{-1}P_\nu^{-1}(P_\theta - P_{\theta^-})P_\nu^{-1} = \epsilon^{-1}P_\nu^{-1} - \dot{P}_\nu^{-1}. \tag{22}$$

Hence, inequality (21) can be rewritten in the form

$$\begin{bmatrix} -\epsilon\mathscr{P}^{-1} & \star & \star \\ I + \epsilon\mathscr{A}^\top & -\epsilon^{-1}\mathscr{P} + \mathscr{C} & \star \\ 0 & \Gamma & -\Pi \end{bmatrix} \prec 0, \tag{23}$$

with $\mathscr{C} = \mathrm{diag}(\dot{P}_\nu^{-1}, 0)$. By Schur complement lemma, we can prove that inequality (23) is equivalent to

$$\begin{bmatrix} \mathscr{A}^\top\mathscr{P} + \mathscr{P}\mathscr{A} + \mathscr{C} + \epsilon\mathscr{A}^\top\mathscr{P}\mathscr{A} & \star \\ \Gamma & -\Pi \end{bmatrix} \prec 0. \tag{24}$$

Since $\epsilon\mathscr{A}^\top\mathscr{P}\mathscr{A} \succ 0$, then it follows from inequality (24) that

$$\begin{bmatrix} \Xi & \star & \star & \star \\ \mathbf{G}_{cl}^\top P_\nu^{-1} + \Lambda_\theta^{-1}E & -2\Lambda_\theta^{-1} & \star & \star \\ B_{1\theta}^\top P_\nu^{-1} & 0 & -\mu I & \star \\ \mathbf{C}_{cl} & \mathbf{L}_{cl} & D_{1\theta} & -I \end{bmatrix} \prec 0, \tag{25}$$

with $\Xi = P_\nu^{-1}\mathbf{A}_{cl} + \mathbf{A}_{cl}^\top P_\nu^{-1} + \dot{P}_\nu^{-1}$. Applying Schur complement lemma to (25), then pre- and postmultiplying the result with $\begin{bmatrix} x^\top & \varphi(x)^\top & w^\top \end{bmatrix}$ and its transpose, the following inequality can be obtained:

$$\dot{V}(x) < \mu w^\top w - z^\top z - 2\varphi(x)^\top\Lambda_\theta^{-1}(Ex - \varphi(x)), \tag{26}$$

where $\dot{V}(x)$ is the time derivative of the Lyapunov function (8) along the trajectory of system (6). From (4) and (26), it follows that

$$\dot{V}(x) + z^\top z < \mu w^\top w. \tag{27}$$

We distinguish two following cases.

- If $w(t) = 0$, for $\forall t > 0$, then $\dot{V}(x(t)) < -z(t)^\top z(t) < 0$, for $\forall x(t) \neq 0$. Then, the origin of the closed-loop system (6) is asymptotically stable.

- If $w(t) \neq 0$, for $\forall t > 0$, then integrating both sides of (27) from 0 to ∞ and considering that $V(x(\infty)) > 0$, it follows that

$$\int_0^\infty z(t)^\top z(t) dt \leq \mu \int_0^\infty w(t)^\top w(t) dt + V(x(0)). \tag{28}$$

Under zero initial conditions, it is clear that (28) leads to inequality (7), which means that the closed-loop system (6) is $\mathscr{L}_2$-stable from $w(t)$ to $z(t)$ with an $\mathscr{L}_2$-gain less than or equal to $\gamma = \sqrt{\mu}$.

Finally, it follows from (17) that $\mathscr{L}_V \subset \mathcal{X}$. Thus, since $\dot{V}(x) < 0$, for $w(t) = 0, \forall t$, then it is included in the region of attraction. Hence, if $x(0) \in \mathscr{L}_V$, the state trajectories will never leave $\mathscr{L}_V$. This concludes the proof. □

Remark 2. *One of the major difficulties when deriving the design conditions in Theorem 1 is to deal with the term* $\dot{P}_\nu = \sum_{i=1}^r \dot{\nu}_i(\xi) P_i$ *in* (22). *Exploiting the integral membership functions* $\nu_i(\xi)$, *it follows from* (11) *that* $\dot{P}_\nu = \alpha^{-1}(P_\theta - P_{\theta^-})$, *which is a convex term. This avoids assuming conservative bounds for the time derivative of the membership functions* $\theta_i(\xi)$ *or considering path-independent Lyapunov functions to obtain numerically tractable LMI design conditions.*

Remark 3. *Based on Theorem 1, the* $\mathscr{L}_2$*-gain upper-bound* γ, *with* $\gamma = \sqrt{\mu}$, *can be minimized by solving the following optimization problem:*

$$\begin{aligned} \min_{\psi_i} \quad & \mu \\ \text{subject to } & (14)\text{–}(17), \text{ with } \psi_i = (P_i, \Lambda_i, K_i, F_i, H_i, J_i). \end{aligned} \tag{29}$$

Remark 4. *Note that for a sufficiently small scalar* $0 < \epsilon \ll 1$, *conditions given in* (24) *and* (25) *are* equivalent *since the only difference between them is the term* $\epsilon \mathscr{A}^\top \mathscr{P} \mathscr{A}$. *Then, less conservative results are expected from* (29) *with* $0 < \epsilon \ll 1$, *which can be determined with a simple gridding algorithm.*

4. Numerical Examples

To illustrate the relaxation provided by the proposed approach, comparisons with the control results in [8] are performed hereafter. Note that the method [8]

is the only work currently available in the literature addressing $\mathscr{L}_2$-stabilization for N-TS fuzzy models. Also, the application to a real-world example is employed to discuss the stabilization and the trade-off between the parameter ϵ and the obtained $\mathscr{L}_2$-gain. All optimization problems were done using Yalmip toolbox and Mosek solver.

4.1. Example 1

Consider a nonlinear system whose N-TS fuzzy model (2) is described as follows [17, Example 3]:

$$A_1 = \begin{bmatrix} 2 & -10 \\ 2 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} a & -5 \\ 1 & 2 \end{bmatrix},$$
$$B_{11} = B_{12} = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top, \qquad B_{21} = B_{22} = \begin{bmatrix} b & 2 \end{bmatrix}^\top,$$
$$G_1 = G_2 = \begin{bmatrix} 0.1b & 0 \\ 0 & 0.1a \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

where $x_1, x_2 \in [-2, 2]$. The controlled output of the system is defined by $z(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^\top$. For an artificial time-delay $\alpha = 0.003$ s and $\epsilon = 10^{-3}$, the feasibility regions obtained by solving the optimization problem (29) in Theorem 1 and Corollary 1 of [8], for different values of a and b, are shown in Figure 1. Observe that the feasibility region obtained with the proposed condition is clearly enlarged when compared to the control approach in [8].

Also, by taking $b = 13$, the values of the upper-bound of the $\mathscr{L}_2$-gain γ for different values of a are depicted in Figure 2. It can be noticed that the proposed approach provides smaller upper-bounds for all values of a. In particular, as a is increased, this improvement is even more evident, which clearly illustrates the conservativeness reduction in comparison with [8, Corollary 1].

Finally, the smallest value of γ obtained for all parameters a and b considered in Figure 1 and the related computational complexity are depicted in Table 1. It is worth mentioning that the constraint (17), with $H_j = P_k = P$, is added to the approach in [8] to account the validity domain $\mathcal{X}$ of the N-TS fuzzy model.

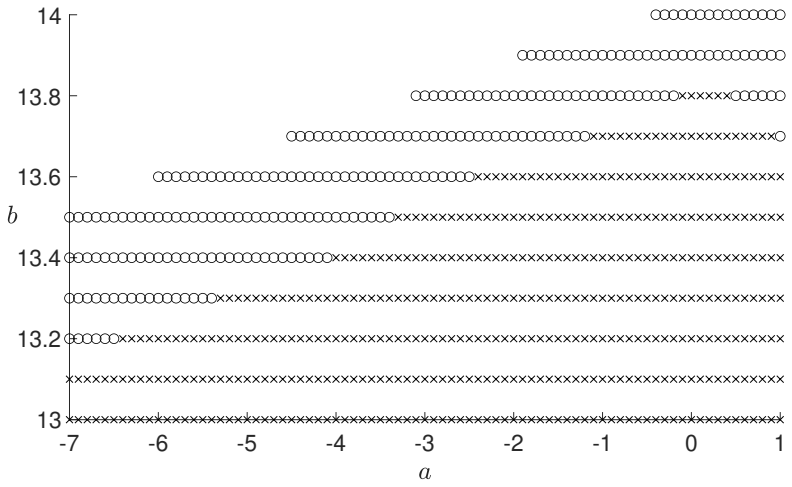

Figure 1: Feasibility regions provided by Corollary 1 [8] ('×') and Theorem 1 ('×' and 'o') – Example 1.

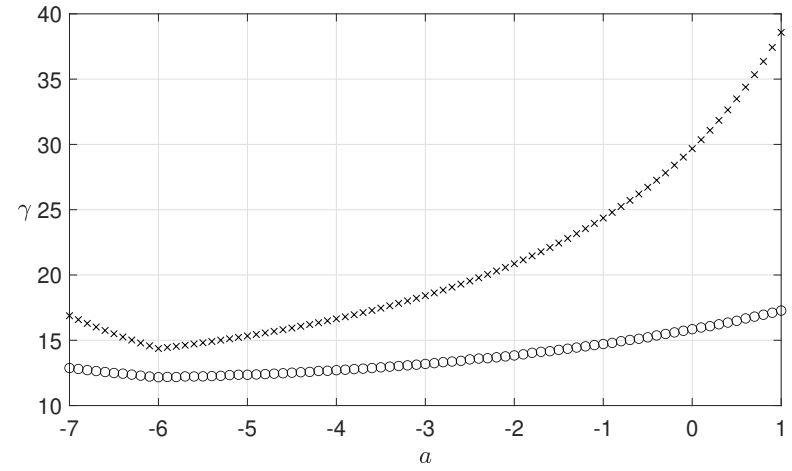

Figure 2: Comparison of upper-bound of the $\mathscr{L}_2-$gain (γ) for Corollary 1 [8] ('×') and Theorem 1 ('×' and 'o'), considering $b = 13$ – Example 1.

The numerical complexity is evaluated in terms of number of decision variables

N_d, and number of LMI rows N_l. For Theorem 1, these values are given by

$$\begin{aligned} N_d &= 1 + (n_\varphi + (n_u + n_x)n_x + (n_u + n_\varphi)n_\varphi)r \\ &\quad + \tfrac{1}{2}\left(n_x(n_x+1) + r^2 N_\Sigma(rN_\Sigma + 1)\right) r, \\ N_l &= \tfrac{1}{2}r^3(r+3)N_\Sigma + (1+n_x)r^2 n_e, \end{aligned}$$

with $N_\Sigma = 2n_x + 2n_\varphi + n_z + n_w$.

Table 1: Computational complexity and comparison of the smallest upper-bound of the $\mathscr{L}_2-$gain γ, considering the variation of parameters in Figure 1.

Design approach	γ (solution of (29))	N_d	N_l	$\log(N_d^3 N_l)$
[8, Corollary 1]	14.3662	119	47	18.1875
Theorem 1	12.1741	1047	268	26.4520

Although the proposed control approach has a slightly higher computational complexity than the design condition in [8], it provides a significant improvement in terms of both the enlargement of the feasibility region and the reduction of the upper-bound of the $\mathscr{L}_2$-gain. This clearly illustrates that the proposed condition to design the control law (5), regarding the nonquadratic Lyapunov function (8), leads to a conservativeness reduction in comparison to [8].

4.2. Example 2

Consider the two-tank system [30, Example 2] illustrated in Figure 3 with the following incremental model:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} -\frac{a_1}{A}\frac{2g}{f(x_1,x_2)} & \frac{a_1}{A}\frac{2g}{f(x_1,x_2)} \\ \frac{a_1}{A}\frac{2g}{f(x_1,x_2)} & -\frac{a_1}{A}\frac{2g}{f(x_1,x_2)} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{k}{A} \\ 0 \end{bmatrix} \Delta v(t) \\ &\quad + \begin{bmatrix} 0 \\ -\frac{2ga_2}{A} \end{bmatrix} \bar{\varphi}(x_2) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} w(t), \end{aligned} \tag{30}$$

in which $x_1(t) = h_1(t) - h_{1e}$, $x_2(t) = h_2(t) - h_{2e}$ with $h_1(t)$ and $h_2(t)$ denoting the water level of each tank, respectively, and h_{1e} and h_{2e} are the given operating points. $\Delta v = v(t) - v_e$ and $v(t)$ denotes the flow rate of the pump (control input).

$w(t)$ represents an exogenous disturbance which can be considered as an extra flow beyond the control input, $A = 100$ cm^2 is the horizontal section, $k = 4$ is a constant, $a_1 = 1$ cm^2 is the section of the connecting the tanks, $a_2 = 0.7$ cm^2 is the section of the outlet valve, $g = 981$ cm/s^2 is the gravitational acceleration constant. The control objective is to keep the water level of the two tanks in the operating point $\begin{bmatrix} h_{1e} & h_{2e} \end{bmatrix}^\top = \begin{bmatrix} 9 & 6 \end{bmatrix}^\top$ cm. Also,

$$f(x_1, x_2) = \sqrt{2g(x_1 - x_2 + h_{1e} - h_{2e})} + \sqrt{2g(h_{1e} - h_{2e})},$$
$$\bar{\varphi}(x_2) = \frac{x_2}{\sqrt{2g(x_2 + h_{2e})} + \sqrt{2gh_{2e}}}.$$

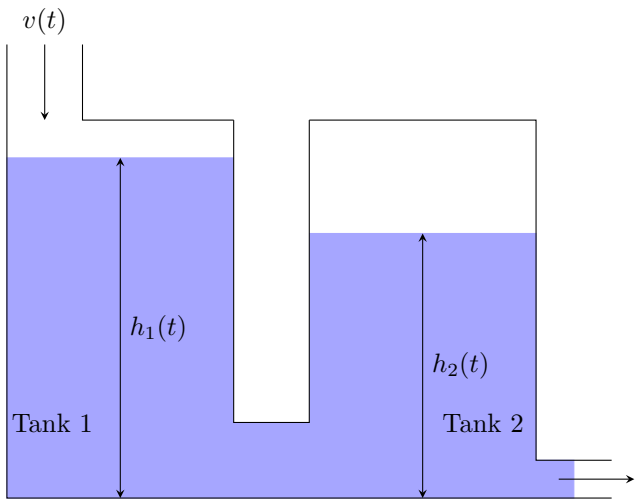

Figure 3: Two-tank system schematic – Example 2.

Assume that $|x_1| \leq 8$, $|x_2| \leq 5$. Then, the premise variable $\xi = \frac{1}{f(x_1,x_2)} \in [0.0054, 0.0130]$ and the sector nonlinearity $\bar{\varphi}(x_2) \in \text{co}\{0.0039x_2(t), 0.0065x_2(t)\}$. After a loop-transformation, it follows that $\varphi(x_2) = \bar{\varphi}(x_2) - 0.0039x_2$. Note that $\varphi(x_2) \in \text{co}\{0, \Omega x_2\}$ with $\Omega = 0.0026$. We consider $\Delta v(t) \equiv u(t)$. Hence, system

(30) can be rewritten in the form

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}-\frac{a_1}{A}\frac{2g}{f(x_1,x_2)} & \frac{a_1}{A}\frac{2g}{f(x_1,x_2)}\\ \frac{a_1}{A}\frac{2g}{f(x_1,x_2)} & -\frac{a_1}{A}\frac{2g}{f(x_1,x_2)} - \frac{0.0078ga_2}{A}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}\frac{k}{A}\\ 0\end{bmatrix}u(t) + \begin{bmatrix}0\\ -\frac{2ga_2}{A}\end{bmatrix}\varphi(x_2) + \begin{bmatrix}0.1\\ 0\end{bmatrix}w(t). \tag{31}$$

Then, using the sector nonlinearity approach [28], the corresponding local state-space matrices of the N-TS fuzzy model (2) for system (31) are given by

$$A_1 = \begin{bmatrix}-0.1059 & 0.1059\\ 0.1059 & -0.1595\end{bmatrix},\ A_2 = \begin{bmatrix}-0.2551 & 0.2551\\ 0.2551 & -0.3086\end{bmatrix},$$
$$B_{1i} = \begin{bmatrix}0.1\\ 0\end{bmatrix},\ B_{2i} = \begin{bmatrix}0.04\\ 0\end{bmatrix},\ G_i = \begin{bmatrix}0\\ -13.734\end{bmatrix},\ \forall i \in \mathcal{I}_2.$$

Let us consider the controlled output as same of the previous example and the artificial time delay $\alpha = 0.001$ s. Fig. 4 depicts the relation between the value of the scalar ϵ and the minimum value of the $\mathscr{L}_2$-gain obtained from the optimization problem (29). Note that the improvement achieved by applying Theorem 1 is not a monotonically increasing function of ϵ. Moreover, to obtain a better result of the upper-bound of the $\mathscr{L}_2$-gain γ, one should solve the optimization problem (29) by iterating over ϵ, see also Remark 2 in [26].

Note that, for $\epsilon \leq 10^{-2}$, the upper bound of the $\mathscr{L}_2$-gain is lower than the one obtained from the approach in [8], which is equal to $\gamma = 1.7205$. The best case corresponds to $\gamma = 1.7102$ obtained when $\epsilon = 3 \times 10^{-3}$. This numerically confirms the statement in Remark 4.

Consider the corresponding controller, Fig. 5 depicts the trajectories of the water levels ($h_1(t)$ and $h_2(t)$) of the two tanks when the following disturbance is applied:

$$w(t) = \begin{cases}1, & 10 \leq t \leq 12,\\ 0, & \text{otherwise.}\end{cases}$$

It is clear that the designed controller effectively attenuates the disturbance effect and guarantees the closed-loop stability.

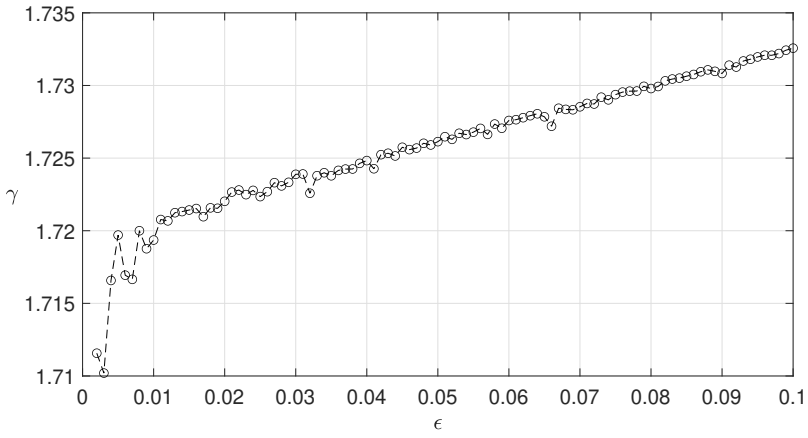

Figure 4: Relation between ϵ and the $\mathscr{L}_2$-gain in Example 2.

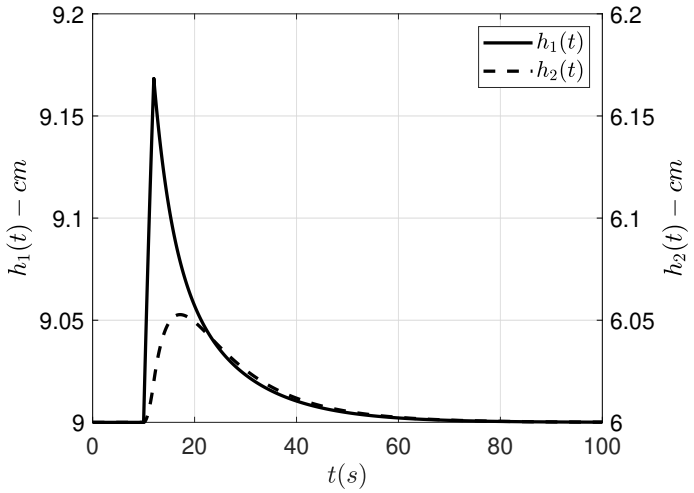

Figure 5: Response of the water levels $h_1(t)$ and $h_2(t)$ in Example 2.

5. Conclusions

185 The stabilization problem for nonlinear Takagi-Sugeno fuzzy systems subject to $\mathscr{L}_2$ disturbances has been investigated in this paper. A nonquadratic Lyapunov function with integral membership functions have been considered and sufficient delayed design conditions have been obtained. The considered non-

quadratic Lyapunov function avoids assuming bounds for the time derivatives of the membership functions or path-independent conditions, which are sources of conservativeness. Moreover, based on appropriate matrix transformations, the control law structure can be entirely independent of the Lyapunov function in a new fashion, contributing to reduce the design conservativeness. Two examples have been provided to illustrate the reduction of conservativeness by the proposed control approach, the application to a real system model, and the trade-off between the $\mathscr{L}_2$-gain and the ϵ parameter introduced in the relaxation of the design constraints. Future directions focus on developing a delayed non-quadratic approach to design observers for the class of nonlinear Takagi-Sugeno fuzzy models [24].

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