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Abstract

In this article we ask whether a privately-owned natural renewable resource can be conserved and managed efficiently when households have a joy-of-giving resource bequest motive. We model an overlapping generations economy in which firms have access to a CES production technology combining the natural resource, physical capital and labor. Our results shed light on the interplay between the resource bequest motive and the substitutability/complementarity relationship between capital and the natural resource in the determination of

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the equilibrium propensity to use the resource. The mere existence of the bequest motive does not guarantee that the resource will be conserved in the long run. When the resource is highly substitutable with capital, the equilibrium actually never exhausts the resource stock whatever the intensity of the bequest motive. When the resource is a poor substitute for capital, the equilibrium preserves the resource only if the taste for bequeathing is strong enough. Be the economy in over-accumulation or in under-accumulation of the natural resource, it always increases aggregate consumption to run the stock of capital at a level lower than the efficiency level.

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1 Introduction

It is hard to pretend that individuals are purely selfish but, on the other hand, it may be equally unrealistic to pretend that individuals have a perfectly universal concern for the entire posterity. Such a discussion holds in particular for environmental and natural resource assets, because these assets should be shared among generations to guarantee sustainability. Thus, it seems reasonable to assume that individuals have some degree of altruism for future generations. In this paper, we shall consider

that individuals enjoy the idea to accomplish their duty regarding future generations because they experience a *warm glow*, or a *joy-of-giving* from fulfilling their duty, whatever it may be. This motive comes from Andreoni (1989) who used this idea to model the so-called joy-of-giving bequest motive and applied it to charities giving and transfers inside the family. In this article we ask whether a privately-owned renewable productive resource can be conserved and managed efficiently when households have a joy-of-giving resource bequest motive. In particular, we scrutinize the conditions for resource extinction not to happen.

In this paper the natural resource stock is privately owned by individuals and it is not traded.[1] The motive behind this modeling choice is to get rid of all motives that may drive non-optimal forest exploitation or depletion. Because the resource is privately owned, the family bears all costs and benefits of resource exploitation. This allows us to focus on the motive for intergenerational transmission of the resource, which is the scope of the paper, and its interplay with other private decisions, in particular the usual trade-off in an overlapping generations models between savings and consumption over the agent's life-cycle. Extracted resource is used for production. So the last key ingredient will be the interplay between individual's degree of altruism and the characteristics of the production process, notably the degree of substitutability among production factors.

Family ownership for resource is more the rule than the exception, in particular for forests, and this has little to do with the countries' stage of development. An enquiry on private forest ownership conducted in 2006/2007 by the Timber Section

[1]It is the extracted resource, rather than the in situ resource, that is traded.

of the United Nations Economic Commission for Europe and the FAO [2] confirmed the significance of private forestry across Europe. This study is reported by Hirsch *et al.* (2007) and covers 23 European countries. It turned out that 49.6 percent of forest and other wooded land is privately owned. The ownership structure varies among countries. In Austria, France, Norway and Slovenia, privately owned forests account for more than 75 percent of the total forest area, whereas, for historical reasons, in Bulgaria, the Czech Republic, Romania and Poland they represent less than onequarter. Several countries are characterized by a relatively balanced forest ownership structure, like the Netherlands (52 percent), Belgium (60 percent) or Germany (44 percent). Furthermore, private ownership is mostly by individuals: 82 percent of private forest in Europe is held by individuals or families, followed by private institutions (13 percent) and forest industries (5 percent). Because they own half of Europes forested area, private forest owners constitute an important contribution to promoting the sustainable management of the regions forests and sustaining their productivity. Understanding the rational behind privately owned natural resources and its implication on economic growth and resource preservation is thus of a major importance.

We address this issue in an overlapping generations model which generalizes other contributions in the following respects. First, papers in the literature on forestry generally study the bequest of timber between generations without modeling a final good production sector, *e.g.* Amacher *et al.* (1999), Ollikainen (1998). We model the pro-

[2]This study has been conducted in cooperation with the Ministerial Conference on the Protection of Forests in Europe (MCPFE) and the Confederation of European Private Forest Owners (CEPF). The paper is available on the web site of FAO.

duction process of a consumption-investment good like in the Diamond model (1965). Second the papers which do take into account the production process sometimes do not include the aggregate stock of physical capital beside labor and extracted resource, *e.g.* Olson and Knapp (1997), Koskela *et al.* (2000) and (2002). At odd with these papers, but like Mourmouras (1991) and Farmer (2000), we assume a three-factor production function. This allows us to study the substitutability between capital and resource as production factors within a CES production function. Third, in an overlapping generations (OLG) setting there needs to be a mechanism of transmission of the resource from one generation to the next one. This can take two forms, either by selling the unextracted resource stock or by bequeathing it. Usually, when the resource is sold, households are assumed to be selfish (Koskela *et al.*, 2002; Mourmouras, 1991) whereas, in the other type of transmission, households are assumed to have a resource bequest motive. In many papers the assumed bequest motive is based on the altruism *à la* Barro (1974). In such a setting, parents care about their offspring's utility (Amacher *et al.*, 1999). As Becker (1993) admits, this form of intergenerational concern requires human foresight capacities that are beyond the capacities of the most prescient. Alternatively, parents may be assumed to care about their offspring's adult income, motivated by family altruism (see Bréchet and Lambrecht (2009) and, without natural resource, Lambrecht *et al.* (2005) and Lambrecht *et al.*, 2006) or to have a joy-of-giving resource bequest motive (Ollikainen, 1998). In this paper we will consider a natural resource joy-of-giving bequest motive.

We model an OLG economy in which individuals are privately-endowed with a renewable resource. This resource can be extracted at no cost by the young households and provided to production as a source of revenue. However, the joy-of-giving bequest

motive motivates the transfer of the unexploited resource to the heirs so as to let them the opportunity to raise their own revenues from the resource. The extracted resource is combined with physical capital and labor to produce a consumption-investment good. The issue of substitution between natural resource and capital is addressed, as well as the issue of selfishness versus altruism and their implications on the opportunities set left to future generations.

Our main results are the following. The equilibrium propensity to use the extracted resource as an input to production is determined both by preferences and technology. The mere existence of a taste for bequeathing the resource is not a sufficient condition to avoid the extinction of the aggregate natural resource stock. The taste for bequeathing may also be too strong, leading the economy to over-accumulation of the natural resource stock. We show that there exists a degree of the joy-of-giving bequest motive compatible with a maximized stationary aggregate consumption. When the natural extracted resource is not essential to production (high substitutability with capital), it is technically feasible to maintain positive consumption without natural resource. But we also show that, whatever the degree of the bequest motive, resource extinction will never occur in equilibrium. Conversely, when factors are poor substitutes, the resource is technologically seen as essential to maintain a positive consumption level. Then, in this case equilibrium does not guarantee that the aggregate resource stock will be preserved, except if the degree of the bequest motive is high enough. Lastly, we explore the maximization of aggregate consumption in the case where the resource stock is not used efficiently, this being understood as 'not delivering the highest sustainable outcome'. We show that, be the resource stock excessively high or excessively low with respect to its efficient sta-

tionary level, it always increases aggregate consumption to run the stationary capital stock at a lower level than the efficient one. Put differently, whatever an economy is resource-conservationist or resource-wasting, it must not compensate with more capital accumulation, and the capital level should always be below the optimal one.

The following of the paper is organized as follows. In section 2 we describe an OLG economy with physical capital and a renewable resource in which households have a resource bequest motive. We pay a special attention to the resource own's dynamics and we characterize the intertemporal equilibrium. Section 3 analyzes how the joy-of-giving bequest motive and technological constraints interplay to challenge sustainability. Section 4 shows why the resource may be misused, the implications on the whole dynamics of the economy and the interplay with capital accumulation. Section 5 summarizes our main conclusions.

2 An OLG economy with a natural resource bequest motive

The economy is of the Diamond's (1965) type with a constant population, but with the two extensions of an extracted renewable resource and a joy-of-giving bequest motive. The N young households at time $t = 0$ hold equal shares z_{-1} of the global stock of resource Z_{-1}. This section presents the natural resource dynamics, the agents' and the firms' behavior and characterizes the equilibrium.

2.1 The natural resource dynamics

Let us first describe the resource own dynamics, *i.e.* without human exploitation (the harvest decision will be studied in subsection 2.2). The equation which governs the evolution of each individual endowment in the renewable resource, with no harvest, is given by

$$z_t = H\left(z_{t-1}\right) z_{t-1}, \qquad \text{where} \qquad H(z_{t-1}) = 1 + Nh(z_{t-1}), \tag{1}$$

where the variable z_{t-1} is the individual stock inherited from time $t-1$ by each of the N time t young individuals and the expression $Nh\left(h_{t-1}\right)$ is the individual resource natural return. The presence of the generation size N indicates that the individual resource return linearly depends on all other individual resource returns. This function h satisfies the following properties:

1. $h''\left(z\right) < 0, \forall z$,
2. $\exists! \hat{z} > 0 : h\left(\hat{z}\right) = -1/N$ or, equivalently, $H(\hat{z}) = 0$,
3. $lim_{z \to 0} h'(z) = \lambda$, with $\lambda > 1$.

These hypotheses imply that the function $H(z_{t-1})z_{t-1}$ reaches a maximum at some $\bar{z}$, defined by the $H'(\bar{z}_{t-1})\bar{z} - t - 1 + H(\bar{z}_{t-1}) = 0$. The phase line of the natural resource dynamics in the plane $z_{t-1} - z_t$ thus display the typical bell-shape.[3]

[3]See Dasgupta and Heal (1979), page 115 and followings for an illustration with a quadratic specification.

2.2 Households' behavior

2.2.1 Timing of decisions and budget constraints

Each individual lives for two periods: youth and old age. The individual is endowed with one unit of labor which she supplies inelastically during her first period of life for a real wage w_t. She is also endowed with the total available individual resource stock $H(z_{t-1})z_{t-1}$ composed of her parents' bequest z_{t-1} augmented by the resource natural return $Nh(z_{t-1})z_{t-1}$. She decides how much to extract of this inherited stock. Harvesting is costless. She provides the extracted amount e_t to the firm for a real price q_t. There are two possible uses for her first-period income, $w_t + q_t e_t$: consumption c_t and savings s_t. When old, the individual bequeathes the unextracted resource stock z_t to her heir, invests her savings in productive capital and receives capital income $R_{t+1}s_t$, where $R_{t+1} = 1+r_{t+1}$ is the interest factor and r_{t+1} the interest rate. She consumes all her second-period income (d_{t+1}) and then dies. All this gives the following youth and old-age budget constraints:

$$w_t + q_t e_t = c_t + s_t \tag{2}$$

$$R_{t+1}s_t = d_{t+1} \tag{3}$$

and by the equation of motion of the individual resource stock with harvesting[4]

$$H\left(z_{t-1}\right) z_{t-1} = e_t + z_t \tag{4}$$

2.2.2 Preferences: the "warm glow" or joy-of-giving bequest motive

In the literature on intergenerational altruistic links, several bequest motives have been proposed to explain transfers. According to Barro's (1974) *dynastic altruism* hypothesis, the altruist cares about her direct descendants' utility. In Andreoni's (1989) *joy-of-giving* approach, the altruist gets utility from the bequest flow itself. Finally, under the *family altruism* hypothesis, the altruist values her offspring's adult disposable income (Lambrecht *et al.* (2005) and (2006), and Bréchet and Lambrecht (2009) in a paper with a renewable resource).

When dealing with the issue of the conservation of a natural resource, an infinite-horizon altruism model like Barro's (1974) is ill-suited to address the issue of the sustainable use of the resource in equilibrium. On one side, this approach enables to take into account the actual value of the resource bequest to future generations, because bequests depend on the intertemporal profile of equilibrium price. On the

[4]In models with a market for exchanging the resource as an asset, two alternative timing of decisions are found. In the so-called *beginning-of-period* asset-equilibrium formulation, the market acquisition of the resource (at a spot price p_t) and the harvest (at a one-period forward price q_t) take place when individuals are young (*i.e.* at time t) and the resale of the remaining stock is done when they are old (at time $t+1$). In the *end-of-period* approach, the resource acquisition is done when individuals are young at price p_t and the harvest and re-sale are done at time $t+1$ when individuals are old. It should be noticed that, in our model, such a distinction makes no sense since the "acquisition" of the resource is costless, because motivated by altruism. However, our timing of decision is closer to the beginning-of-period approach.

other side, this approach is quite heroic in assuming that agents have an infinite capacity to foresee the entire future.

What makes the equilibrium analysis interesting in the presence of a natural resource is that private agents precisely *could* exhaust the resource because they do not foresee the future consequences of their present decision. The question to be examined is the following : is there nevertheless a chance to maintain the stock of the resource in the long run? Under a Barro (1974) type of altruism, since agents are assumed to be able to foresee the entire future, the answer to this question is almost always trivially yes. The effective decision unit of Barro's model is actually the whole dynasty of overlapping generations and not the life-cyclers. To the opposite, a finite-horizon form of altruism leaves the answer to the conservation issue open.

The *family altruism* model may be compatible with the finite horizon feature. The main assumption of the family model, as opposed to the dynastic model, is that the decision unit in which intergenerational links are operative is the *family*, as opposed to the *dynasty*. Admittedly, this is a much more realistic and challenging framework than the dynastic model. But the necessity to be able to foresee the entire future can reappear also under this hypothesis if the altruist's bequest depends on her offspring's own bequest decision.[5]

Another finite-horizon bequest motive is Andreoni's (1989) *joy-of-giving*. One interpretation of the joy-of-giving bequest motive is that the individual has the feeling of doing her duty by abstaining from consuming the whole family good. This approach is sometimes also labeled the *warm glow* approach. The individual feels she has to

[5]This would be the case for example in Bréchet and Lambrecht (2009) in the case of perfect foresight.

preserve the resource for the sake of her heir. By doing so, she makes sure that she does not threaten the opportunities of her descendant. It should be emphasized that the bequest motive we assume here is substantially different from a concern for the resource or the environment as a whole. Indeed, not only the individual does not care about the other individuals' resource stocks, but also she gets utility only from her own bequest. We follow this approach in this paper.

The individual's preferences are defined on youth and old-age consumption, c_t and d_{t+1}, and on the level of the unextracted resource stock bequeathed to her heir, z_t. They are represented by the following additively separable utility function

$$U_t = (1-\beta)\log c_t + \beta \log d_{t+1} + \gamma \log z_t \tag{5}$$

The parameter $\beta \in (0,1)$ reflects the weight attached to consuming when old while $\gamma > 0$ is the degree of the joy-of-giving bequest motive. In addition to simplifying the analysis, the additive utility function will allow us to focus more sharply on the role played by the two key preference parameters, γ and β. It is natural to explicitly identify the former, considering the scope of our paper, and we shall see that the latter also plays a key role in our results by shaping savings decisions and resource harvesting.

Two decisions characterize the individual's problem: the saving decision and the harvesting decision. Considering prices as given, the individual chooses s_t and e_t in order to maximize her utility. By substituting c_t, d_{t+1} and z_t by their respective expressions, we get the following maximization problem:

$$\max_{\{s_t, e_t\}} (1-\beta)\log\left(w_t + q_t e_t - s_t\right) + \beta \log\left(R_{t+1} s_t\right) + \gamma \log\left(H\left(z_{t-1}\right) z_{t-1} - e_t\right) \tag{6}$$

and the first-order conditions write

$$\frac{1-\beta}{w_t + q_t e_t - s_t} = \frac{\beta}{s_t} \tag{7}$$

$$\frac{(1-\beta)\, q_t}{w_t + q_t e_t - s_t} \leq \frac{\gamma}{H\left(z_{t-1}\right) z_{t-1} - e_t} \tag{8}$$

with equality if harvesting is positive. Solving the first equation for s_t as a function of e_t yields $s_t = \beta\left(w_t + q_t e_t\right)$. If harvesting is positive, the solution to the maximization problem is given by

$$e_t = \frac{H\left(z_{t-1}\right) z_{t-1}}{1+\gamma} - \frac{\gamma}{1+\gamma}\frac{w_t}{q_t} \tag{9}$$

$$s_t = \frac{\beta}{1+\gamma}\left[w_t + q_t H\left(z_{t-1}\right) z_{t-1}\right] \tag{10}$$

The harvesting decision is driven by two mechanisms. First, it is increasing in the inherited stock. Second, it is decreasing in the relative price of labor with respect to the price of the resource, w_t/q_t. The condition of non-negativity of e_t is given by

$$\gamma \leq \frac{q_t H\left(z_{t-1}\right) z_{t-1}}{w_t} \tag{11}$$

The right-hand side of the non-negativity constraint on e_t is the ratio of the inherited resource stock valued at price q_t on the wage income. This ratio reflects the relative importance of the two individual's sources of income when young. It increases as the individual's dependence on the bequeathed resource increases.

2.3 Firms' behavior

There is a representative firm which produces the consumption/investment good. The technology of production displays constant returns to scale of the three production

factors: capital K, labor L and extracted resource E. It is represented by a linearly homogeneous production function: $F(K_t, L_t, E_t)$. We assume that capital fully depreciates in each period and that there is no acquisition cost.[6] The profit of the representative firm is $\pi_t = F(K_t, L_t, E_t) - R_t K_t - w_t L_t - q_t E_t$. The firm maximizes its profit with respect to K_t, L_t and E_t considering prices as given. The first-order conditions are given by: $F'_K(K_t, L_t, E_t) = R_t$, $F'_L(K_t, L_t, E_t) = w_t$ and $F'_E(K_t, L_t, E_t) = q_t$. We shall consider a CES specification for the production function,

$$F(K_t, L_t, E_t) = A\left(\alpha_K K_t^{-\rho} + \alpha_L L_t^{-\rho} + \alpha_E E_t^{-\rho}\right)^{-1/\rho} \tag{12}$$

with $\alpha_K + \alpha_L + \alpha_E = 1$, $\rho > -1$ and $\rho \neq 0$. In intensive terms the FOCs read as follows

$$\frac{\alpha_K}{A^\rho}\left[\frac{f(k_t, e_t)}{k_t}\right]^{1+\rho} = R_t \tag{13}$$

$$\frac{\alpha_E}{A^\rho}\left[\frac{f(k_t, e_t)}{e_t}\right]^{1+\rho} = q_t \tag{14}$$

$$\frac{\alpha_L}{A^\rho} f(k_t, e_t)^{1+\rho} = w_t \tag{15}$$

where e_t and k_t stand for per capital resource harvesting and capital stock, respectively, and where $f(k_t, e_t) = A\left(\alpha_K k_t^{-\rho} + \alpha_L + \alpha_E e_t^{-\rho}\right)^{-1/\rho}$.

2.4 The competitive equilibrium

We first study the equilibrium of period t. What is given in period t is the inherited resource stock z_{t-1} and the productive capital stock k_t. We determine the following time t variables: the prices w_t, R_t and q_t, the individuals' resource supply, the

[6] In an OLG model where people live for two periods, a period represents roughly 35 years.

bequeathed stock and consumptions: e_t, z_t, c_t and d_t, and the representative firm's factor demands and output supply K_t, L_t, E_t and Y_t. The labor market equilibrium implies $L_t = N$. Hence, $k_t = K_t/N$ and $e_t = E_t/N$ in equilibrium, and the equilibrium expressions of factor prices are given by the marginal productivities valued at these k_t and e_t.

Proposition 1 *(i) In equilibrium, the individual's optimal harvesting is unconstrained;*
(ii) Individual's optimal harvesting does not depend on capital, since $e_t = e(z_{t-1}, \gamma, \rho)$ $\underset{+/-}{}\ \underset{-}{}\ \underset{+/-}{}$ *;*
(iii) An increase in the inherited resource stock z_{t-1} *increases harvesting if the inherited stock is low enough* ($z_{t-1} < \bar{z}$). *Beyond the threshold* $\bar{z}$, *an increase in the inherited resource stock decreases harvesting;*
(iv) An increase in the degree of bequest motive γ *always decreases harvesting.*

Proof. See appendix "Harvesting in equilibrium". ■

The fact that the equilibrium harvesting does not depend on capital is due, at first, to the fact that the relative price w_t/q_t is independent of k_t in equilibrium. Indeed the ratio of the marginal productivities of labor and resource only depends on e_t. Second, the additive separability of the log-linear utility function is also responsible for this feature. It must also be noticed that the sign of the elasticity of e_t *w.r.t.* ρ depends on whether e_t is smaller or larger than 1.

At a steady state equilibrium the economy reproduces itself each period. Harvesting is equal to the natural return which is added each period to steady stock, *i.e.,* per capita, $e(z, \gamma, \rho) = Nh(z)\, z$. The dynamics of the economy is as follows. At each period, we solve for e_t as a function of z_{t-1} and we determine z_t and k_{t+1}. The

dynamics of z_t and k_{t+1} are given by the following two equations:

$$z_t = H(z_{t-1})\, z_{t-1} - e(z_{t-1}, \gamma, \rho) \tag{16}$$

$$k_{t+1} = \frac{\beta}{(1+\gamma)\, A^{\rho}} \left[\alpha_L f\left[k_t, e(z_{t-1}, \gamma, \rho)\right]^{1+\rho} + \alpha_E f\left[k_t, e(z_{t-1}, \gamma, \rho)\right]^{1+\rho} \frac{H(z_{t-1})\, z_{t-1}}{e(z_{t-1}, \gamma, \rho)^{1+\rho}}\right] \tag{17}$$

The dynamics of z_t are independent of capital. Given the initial conditions, *i.e.* given z_{-1} and k_0, we determine the intertemporal equilibrium. By taking more specific functional forms (Cobb-Douglas production function and quadratic natural resource dynamics) one can exhibit explicit solutions for proposition 1 (see Appendix 6.3).

Two issues must be stressed out. First, it may happen that, despite the bequest motive towards the natural resource, this resource collapses, thus compromising the ability of forthcoming generations to fulfill their own needs. Second, the possibility for reaching the maximum steady state consumption level through the competitive equilibrium is not guaranteed. These issues are discussed in the two following sections.

3 The conditions for resource preservation

The possibility to reach a trivial equilibrium where the resource stock is equal to zero cannot be ruled out. In this section we are interested in understanding the conditions under which such outcome may arise. The following proposition gives the conditions for resource extinction not to happen.

Proposition 2 *Let the natural resource dynamics be defined by* (1) *and* λ *be its slope in the absence of harvesting when the stock tends to zero. Then,*

1. *when factors are poor substitutes ($\rho > 0$), and if the concern for the bequeathed resource is higher than the threshold $\underline{\gamma} = \frac{1}{\lambda - 1}$, then resource extinction never occurs;*

2. *when factors are substitutable like in a Cobb-Douglas production function ($\rho = 0$), and if the concern for the bequeathed resource is higher than the threshold $\underline{\gamma} = \frac{\alpha_E}{\alpha_L + \alpha_E} \frac{1}{\lambda - 1}$, then resource extinction never occurs;*

3. *when factors are strong substitutes ($\rho \in (-1, 0)$), then resource extinction never occurs, whatever the value of $\gamma > 0$.*

Proof. See appendix "Dynamics of z_t". ■

The key element of this proposition is the interplay between the characteristics of the technology and the degree of altruism in households preferences. This interplay is made by the interaction between firms and households private decisions in equilibrium. The first result is that the mere existence of a taste for bequeathing the resource is a necessary but not sufficient condition for resource preservation. Even if this result is not that puzzling, we formally provide the threshold values for the degree of altruism such that the bequest motive can prevent resource extinction, related to the degree of substitutability of production factors.[7] The puzzle lies in the following fact. We never go toward extinction when the production factors are high substitutes, but that we can go to extinction when the factors are poor substitutes, like when the resource is essential to the production. This somewhat paradoxical

[7]Let us recall that, in our model, if the degree of altruism were zero then the natural resource would be fully harvested by the first generation.

result is due to the fact that we analyze private agents' behaviors in equilibrium and not the solution of an optimal control problem, as it is done with infinite lived agent models. In our dynamic general equilibrium model, agents (households and firms) follow their private interest and there is no normative analysis about what *should* be done, for example to avoid resource extinction or to maximize intertemporal social welfare. This proposition thus deserves some economic interpretation. The equation that shows the key trade-offs is equation (9). This equation combines all the ingredients: the degree of altruism (γ), the natural resource dynamics and the opportunity cost of resource extraction (w_t/q_t). The last term constitutes indeed an opportunity cost because it combines the two sources of revenue of the agent when young. Depending on the degree of factor substitutability, this ratio will react differently in the equilibrium transition when the resource becomes scarce, *i.e.* when z_t tends to zero. With these preliminary elements we are now equipped to understand Proposition 2.

When production factors are poor substitutes (item (i) of Proposition 2) the taste for bequest must not only be positive, it must be larger than a minimum threshold $\underline{\gamma}$ to guarantee preservation. This threshold value only depends on the natural resource own dynamics. The resource dynamics must be strong enough as the stock becomes very small. Because factors are poor substitutes in the production function, when the resource becomes scarce and harvesting tends to zero, the marginal productivity of the resource tends to a positive finite value. As for capital and labor, their marginal productivity tends to zero. As the harvested resource approaches zero, then it is beneficial for households to bequest less resource, and then to harvest more to sustain production and consumption. It is relatively easy to substitute away z_t in the utility function, to sustain consumption, but substitution between e_t and k_t is relatively more

difficult in the production function, because factors are poor substitutes. This leads households to harvest more and more. The same token is proposed by Smulders (2006). The natural resource will collapse *except if* the bequest motive is strong enough to compensate for the market incentive. This is the reason why there exists a link between the degree of altruism and the resource dynamics. The stronger the resource when it approaches extinction (large λ), the smaller the degree of altruism which is necessary to compensate for the incentive given by the market.

When production factors are strong substitutes (item (*iii*) of Proposition 2) then the market alone gives the right incentive for resource preservation. In other words, extinction will never occur, whatever the degree of households altruism. Here again, altruism in equilibrium plays against market incentives, but as the natural resource becomes scarce its marginal productivity becomes infinite, and so does its price, so the incentive for harvesting becomes very small and the young will always let some positive resource stock to her heir. In the intermediate case where the production function is of a Cobb-Douglas type, there exists a threshold value for the degree of altruism (for resource preservation), and this threshold depends on the share of the natural resource in the production process, α_E. It is increasing with α_E.

Naturally, Proposition 2 says nothing about the level at which the resource will be preserved. This level will be influenced by the degree of altruism, because the *level* of the stock bequested to the heir directly enters the utility function.

4 A preserved but misused resource: the case of stationary paths

Preserving the resource for the next generations is one issue, but it does not guarantee that the resource is used efficiently in the long run. In this section we assume that the economy is at a unique long run *stationary* positive equilibrium level of the resource stock. What ensures that this preserved resource stock maximizes consumption level in the long run?

Productive efficiency in the long run consists in maximizing the net stationary production defined as the difference between production per head and investment in capital per head. The stationary net product is given by $\phi(k,z) = f\left[k, H\left(z\right)z - z\right] - k$. Thus the net product maximization problem writes:

$$\max_{\{k,z\}} \phi(k,z) = f\left[k, H\left(z\right)z - z\right] - k \tag{18}$$

The first-order conditions for an interior maximum are the following:

$$f'_k\left(k^*, H\left(z^*\right)z^* - z^*\right) = 1 \tag{19}$$

$$H'\left(z^*\right)z^* + H\left(z^*\right) = 1 \tag{20}$$

With a CES production function $f\left(k_t, e_t\right) = A\left(\alpha_K k_t^{-\rho} + \alpha_L + \alpha_E e_t^{-\rho}\right)^{-1/\rho}$, there exists an interior solution (k^*, z^*) to this problem if:

$$\lim_{k\to+\infty} f'(k, H(z)z - z) < 1 < \lim_{k\to 0} f'(k, H(z)z - z). \tag{21}$$

This implies that the following two conditions must be satisfied :

$$\lim_{k\to 0} f'(k,.) = A\alpha_K^{-1/\rho} > 1, \quad \text{for} \quad \rho > 0 \quad \text{low substitutability,} \tag{22}$$

$$\lim_{k\to +\infty} f'(k,.) = A\alpha_K^{-1/\rho} < 1, \quad \text{for} \quad \rho \in (0,1) \quad \text{high substitutability.} \tag{23}$$

As far as the resource stock z is concerned, the properties of the resource dynamics imply that z^* is always an interior solution.

In the system of equations (19)-(20), the first equation in k and z is the equivalent of the standard condition defining the Golden rule capital stock. The choice of the golden-rule capital stock k^* is determined by the usual trade-off between the marginal productivity of capital and the growth factor of population (here, 1). The second equation only depends on z. At z^*, the steady harvest e is maximized. The trade-off for the harvested resource is similar to the one for capital. The marginal natural return $(H'(z^*)\, z^* + H(z^*))$ must equal the marginal effort to leave the resource stock unchanged next period (*i.e.* 1).

Let us now explore some properties of inefficient *stationary* paths with regards to resource and capital accumulation. In this purpose we adopt the following terminology.

Definition 1 *An economy is said to be resource-conservationist (resp. resource-wasting) in steady state if its equilibrium resource stock is larger (resp. smaller) than the stationary Golden Rule resource stock z^*.*

A resource-conservationist economy does not maximize net production, and hence consumption. The unharvested resource closely parallels the unconsumed numeraire:

it is invested to restore the next period stock. Thus a resource-conservationist economy is in a state of "over-accumulation" of the resource : restoring the next period stationary stock is too costly in terms of forgone consumption. To the opposite, a resource-wasting economy is in a situation of "under-accumulation" of the resource. The stationary resource stock is too low to achieve the maximum net production and consumptions[8]. Most of the time, the households' preferences, namely the pair(β, γ) of their utility function, will imply a steady state equilibrium in which the capital and the resource stocks differ from the Golden Rule (k^*, z^*). In the simple example of a log-linear utility function and quadratic resource dynamics, we show in appendix 6.4 that there exists a combination of (β^*, γ^*) which decentralizes the Golden Rule stocks. It follows that if households have a taste for bequeathing γ lower (higher) than γ^*, they will under-accumulate (over-accumulate) the resource.

Let us now assume that the taste for bequeathing the resource differs from the one leading to the Golden Rule z^*. What is then the best stationary capital intensity which maximizes net production ? Can we expect that capital accumulation should be increased or, to the opposite, that it should be decreased ?

Proposition 3 *Whenever an economy under-accumulate or over-accumulate its natural resource, the level of capital which maximizes net production is always lower than*

[8]The difference between the over-accumulation and the under-accumulation cases lies in the fact that, under over-accumulation of the resource, net production and consumptions can be increased by increasing harvesting, while, in the case of under-accumulation, harvesting must be reduced at least once. Hence in the later case, the generation who would have to forgo consumption to enable the shift would oppose this proposal : this is the base of the statement of *dynamic* efficiency of under-accumulation steady state equilibria.

k^*.

Proof. See appendix 6.5. ■

What explains this result is that, as long as the resource stock is not equal to z^*, extracted resource is not maximized ($e < e^*$). Indeed, only z^* leads to the *maximum sustainable yield*. As a result, e is relatively scarcer than capital and the marginal productivity of capital decreases[9]. As a consequence, a lower capital stock can increase the net product. As a conclusion, from the point of view of stationary efficiency, a "too-low" resource stock (over-consumed resource) must not be compensated by a higher physical capital stock but, instead, by a lower capital stock. On the contrary, a "too-high" resource stock (under-consumed resource) must be compensated by a lower capital stock.

5 Conclusion

In this article we consider an overlapping generations economy in which individuals are privately-endowed with a renewable resource. This resource can be extracted at no cost by the young households and provided to production as a source of revenue. An altruistic (*joy-of-giving*) bequest motive motivates the transfer of the unexploited resource to the heirs so as to let them the opportunity to raise their own revenues from the resource. The firms' technology of production is of the CES type combining the natural resource, physical capital and labor.

[9]The marginal productivity of the capital stock k^* is lower than the marginal cost of reproducing k^* each period.

Our results shed light on the interplay between the resource bequest motive and the substitutability/complementarity relationship between capital and the natural resource in the determination of the equilibrium propensity to use the resource.

The main findings are the following. In the long run, the bequest motive does not systematically guarantee that the resource is preserved. When production factors are high substitutes and thus when extracted resource is inessential to production, any degree of the bequest motive is compatible with a preserved resource. So, both weak (consumption preservation) and strong sustainability (resource stock preservation) are satisfied. On the contrary, when factors are poor substitutes, *i.e.* when the resource is essential to production, strong sustainability (resource preservation) is required in order to have weak sustainability. We derive a condition on the degree of the bequest motive for strong sustainability to hold. In the case of a Cobb-Douglas production function, we show that there exists a system of preferences which decentralizes the target of the consumption-maximizing path in the long run. But in most cases, preferences will differ from this and the economy will converge to a sub-optimal long run equilibrium. Furthermore, resource-conservationist economies, which run a high steady state resource stock, will compensate with a lower capital stock to maximize the second-best consumption level (substitutability result). On the contrary, resource-wasting economies, which run a low level of steady resource stock, will also keep a lower capital stock to maximize second-best consumption per head (complementarity result).

This paper provides an example of insights that can be drawn from an intertemporal general equilibrium analysis for the studying sustainability issues. In particular, it showed the implications for sustainability of a joy-of-giving bequest motive applied to

a privately-owned renewable resource. The mere existence of a degree of altruism for forthcoming generations is not a sufficient condition for resource preservation, even if it helps. Put differently, there is still room for public policy. Analyzing the optimal public policy in the presence of some form and degree of altruism remains an avenue for research.

6 Appendices

6.1 Harvesting in equilibrium

Proof of point (i) - At an unconstrained-harvesting time t equilibrium, there is a unique finite positive quantity e_t which equalizes the prices from the inverted resource supply and demand functions on the factor market and which is inferior to $H(z_{t-1}) z_{t-1}$. By summing up individual harvesting given by (9) we obtain the expression of the aggregate resource supply

$$Ne_t = N(1+\gamma)^{-1} H(z_{t-1}) z_{t-1} - N(1+\gamma)^{-1} \gamma q_t^{-1} w_t$$

and from the equilibrium value of the real wage rate $w_t = (\alpha_L / A^\rho) f(k_t, e_t)^{1+\rho}$, we derive the inverted resource supply

$$q_t = \frac{\gamma (\alpha_L / A^\rho) f(k_t, e_t)^{1+\rho}}{H(z_{t-1}) z_{t-1} - (1+\gamma) e_t} \tag{24}$$

and, from the FOC (14), the inverted resource demand writes

$$q_t = \frac{\alpha_E}{A^\rho} \frac{f(k_t, e_t)^{1+\rho}}{e_t^{1+\rho}} \tag{25}$$

Equating the above two expressions of the price q_t yields:

$$\frac{\gamma\alpha_L}{H(z_{t-1})\,z_{t-1}-(1+\gamma)\,e_t}=\frac{\alpha_E}{e_t^{1+\rho}} \tag{26}$$

The LHS tends to $\alpha_L\gamma/H(z_{t-1})\,z_{t-1}$ as e_t tends to 0, while the RHS tends to $+\infty$ as e_t tends to 0. The LHS is increasing in e_t until the value $(1+\gamma)^{-1}H(z_{t-1})\,z_{t-1}$, which is smaller than $H(z_{t-1})\,z_{t-1}$, at the limit of which it tends to $+\infty$; from the other side, as e_t tends to $(1+\gamma)^{-1}H(z_{t-1})\,z_{t-1}$, the LHS tends to $-\infty$. Beyond $(1+\gamma)^{-1}H(z_{t-1})\,z_{t-1}$, as e_t increases the LHS increases until 0 at the limit; but this is economically meaningless, since harvesting cannot be larger than the stock. The RHS decreases as e_t increases and tends to 0 as e_t tends to $+\infty$. As a result, there always exists a finite positive $e_t\leq H(z_{t-1})\,z_{t-1}$, such that the two curves cross.

Proof of point (ii) - Harvesting, i.e. $e_t=(1+\gamma)^{-1}H(z_{t-1})\,z_{t-1}-\gamma(1+\gamma)^{-1}w_tq_t^{-1}$, in equilibrium, is given by

$$e_t-\frac{H(z_{t-1})\,z_{t-1}}{1+\gamma}+\frac{\gamma}{1+\gamma}\frac{\alpha_L}{\alpha_E}e_t^{1+\rho}=0 \tag{27}$$

which is obtained by substituting $w_tq_t^{-1}$ with its equilibrium value, i.e.

$$\frac{\alpha_LA^{-\rho}f(k_t,e_t)^{1+\rho}}{\alpha_EA^{-\rho}f(k_t,e_t)^{1+\rho}\,e_t^{-(1+\rho)}}=\frac{\alpha_L}{\alpha_E}e_t^{1+\rho} \tag{28}$$

This equation in e_t is independent of capital. Its solution is a function $e(z_{t-1},\gamma,\rho)$.

Proof of point (iii) - The solution of this equation is a function of z_{t-1}, γ and ρ: $e_t=e(z_{t-1},\gamma,\rho)$. Let us study the derivative of this function w.r.t. z_{t-1}:

$$\frac{de_t}{dz_{t-1}}=\frac{(1+\gamma)^{-1}\left[H'(z_{t-1})\,z_{t-1}+H(z_{t-1})\right]}{1+\gamma(1+\gamma)^{-1}\alpha_L\alpha_E^{-1}(1+\rho)\,e_t^{\rho}} \tag{29}$$

or

$$\frac{de_t}{dz_{t-1}}=\varepsilon(z_{t-1},\gamma,\rho)\left[H'(z_{t-1})\,z_{t-1}+H(z_{t-1})\right] \tag{30}$$

where

$$\varepsilon\left(z_{t-1},\gamma,\rho\right)=\frac{\alpha_E}{\alpha_E+\alpha_E\gamma+\alpha_L\gamma\left(1+\rho\right)e\left(z_{t-1},\gamma,\rho\right)^{\rho}} \tag{31}$$

belongs to the interval $(0,1)$. Thus the derivative de_t/dz_{t-1} has the same sign as the derivative of the dynamics with no harvest $z_t = H(z_{t-1})z_{t-1}$, i.e. $H'\left(z_{t-1}\right)z_{t-1}+H\left(z_{t-1}\right)$. It is first increasing for values $z_{t-1}\in(0,\overline{z}]$ and then decreasing for $z_{t-1}\in(\overline{z},H\left(z_{t-1}\right)z_{t-1})$.

Proof of point (iv) - The derivative of $e\left(z_{t-1},\gamma,\rho\right)$ w.r.t. γ is given by:

$$\frac{de_t}{d\gamma}=-\frac{H\left(z_{t-1}\right)z_{t-1}\left(1+\gamma\right)^{-2}+\alpha_L\alpha_E^{-1}e_t^{1+\rho}\left(1+\gamma\right)^{-2}}{1+\gamma\left(1+\gamma\right)^{-1}\alpha_L\alpha_E^{-1}\left(1+\rho\right)e_t^{\rho}}<0 \tag{32}$$

6.2 Dynamics of z_t

The dynamics of the individual resource stock with harvesting in equilibrium is $z_t - H\left(z_{t-1}\right)z_{t-1}+e\left(z_{t-1},\gamma,\rho\right)=0$. Given the proof presented in the previous subsection, it is obvious that these dynamics have a bell shape, increasing on $(0,\overline{z})$ and decreasing on $(\overline{z},\hat{z})$. The slope of these dynamics are given by:

$$\frac{dz_t}{dz_{t-1}}=\left[1-\varepsilon\left(z_{t-1},\gamma,\rho\right)\right]\left[H'\left(z_{t-1}\right)z_{t-1}+H\left(z_{t-1}\right)\right] \tag{33}$$

It is therefore a fraction of $H'\left(z_{t-1}\right)z_{t-1}+H\left(z_{t-1}\right)$. This last expression is the derivative of the function $\phi\left(z_{t-1}\right)$ which is the dynamics of the resource without harvesting. It is positive for $z_{t-1}\in(0,\overline{z})$ and negative for $z_{t-1}\in(\overline{z},\hat{z})$. Remind that the limits of the no-harvest dynamics are:

$$\lim_{z_{t-1}\to 0}H(z_{t-1})z_{t-1}=0 \tag{34}$$

$$\lim_{z_{t-1}\to \hat{z}}H(z_{t-1})z_{t-1}=0 \tag{35}$$

Remind that the slope of the no-harvest dynamics of z_t as z_{t-1} tends to 0 is λ:

$$\lim_{z_{t-1}\to 0}\left[H'\left(z_{t-1}\right)z_{t-1}+H\left(z_{t-1}\right)\right]=\lambda. \tag{36}$$

As far as $\varepsilon\left(z_{t-1},\gamma,\rho\right)$ is concerned, remind that:

$$\varepsilon\left(z_{t-1},\gamma,\rho\right)=\frac{\alpha_E}{\alpha_E+\alpha_E\gamma+\alpha_L\gamma\left(1+\rho\right)e\left(z_{t-1},\gamma,\rho\right)^{\rho}}, \tag{37}$$

which yields, browsing for the range of values of ρ:

$$\lim_{z_{t-1}\to 0}\varepsilon\left(z_{t-1},\gamma,\rho\right)=\begin{cases}\frac{1}{1+\gamma} & \text{if } \rho>0\\ \frac{\alpha_E}{\alpha_E+\gamma(1-\alpha_K)} & \text{if } \rho=0\\ 0 & \text{if } \rho\in(-1,0)\end{cases} \tag{38}$$

Combining these last elements yields that the slope of the equilibrium dynamics (with harvest) as $z_t\to 0$ is given by

$$\lim_{z_{t-1}\to 0}\left[1-\varepsilon\left(z_{t-1},\gamma,\rho\right)\right]\left[H'\left(z_{t-1}\right)z_{t-1}+H\left(z_{t-1}\right)\right]=\begin{cases}\frac{\gamma}{1+\gamma}\lambda & \text{if } \rho>0\\ \frac{\gamma(1-\alpha_K)}{\alpha_E+\gamma(1-\alpha_K)}\lambda & \text{if } \rho=0\\ \lambda & \text{if } \rho\in(-1,0)\end{cases} \tag{39}$$

In the case of $\rho>0$ (case 1) this slope is greater than 1 iff

$$\gamma>\frac{1}{\lambda-1}. \tag{40}$$

In the case of $\rho=0$ (case 2), the slope is greater than 1 iff:

$$\gamma>\frac{\alpha_E}{(\alpha_L+\alpha_E)(\lambda-1)}. \tag{41}$$

thus in these first two cases, since the dynamics are continuous and concave and end up with negative slope, starting with positive slope larger than 1, there exists a non-trivial steady state z. Finally, when we have $\rho\in(-1,0)$ (case 3) the slope (i.e. λ) is greater than 1 independently of γ.

6.3 Example: the Cobb-Douglas-quadratic case

Explicit solutions can be found for Proposition 1 by using a Cobb-Douglas production function and a quadratic resource dynamics. The production function becomes $f(k_t, e_t) = Ak_t^{\alpha_K} e_t^{\alpha_E}$. In this cas equilibrium prices read $R_t = \alpha_K A k_t^{\alpha_K - 1} e_t^{\alpha_E}$, $w_t = \alpha_L A k_t^{\alpha_K} e_t^{\alpha_E}$ and $q_t = \alpha_E A k_t^{\alpha_K} e_t^{\alpha_E - 1}$. The quadratic resource dynamics write

$$e_t + z_t = [1 + N(\mu - \nu z_{t-1})] z_{t-1}, \tag{42}$$

e_t, z_t and k_{t+1} write as follows

$$e_t = \varepsilon(\gamma) [1 + N(\mu - \nu z_{t-1})] z_{t-1} \tag{43}$$

$$z_t = [1 - \varepsilon(\gamma)] [1 + N(\mu - \nu z_{t-1})] z_{t-1}$$

$$k_{t+1} = \frac{\beta}{1-\beta} \varepsilon(\gamma)^{\alpha_E} \left(\alpha_L + \frac{\alpha_E}{\varepsilon(\gamma)} \right) A k_t^{\alpha_K} [(1 + N(\mu - \nu z_{t-1})) z_{t-1}]^{\alpha_E}$$

where $\varepsilon(\gamma) = \alpha_E (\alpha_E + \alpha_E \gamma + \alpha_L \gamma)^{-1} \in (0, 1)$. At a steady state equilibrium we have $N(\mu - \nu z) z = \varepsilon(\gamma) [1 + N(\mu - \nu z)] z$. We can solve for z and deduce e

$$z = \frac{\mu}{\nu} - \frac{1}{N\nu} \frac{\alpha_E}{\gamma (1 - \alpha_k)} \tag{44}$$

$$e = \frac{\alpha_E}{(1 - \alpha_K)} z \tag{45}$$

The steady state equilibrium value of k is the solution of $s = k$ where $s = \beta (1 - \alpha_k) A k^{\alpha_K} e^{\alpha_E}$:

$$k = (\beta (1 - \alpha_K) A e^{\alpha_E})^{\frac{1}{1 - \alpha_K}} \tag{46}$$

6.4 Conditions on preferences

Let us assume that the production function is a Cobb-Douglas, $Ak^{\alpha_K}e^{\alpha_E}$,. Assume further that the resource evolves according to the quadratic function $z = [1 + N(\mu - \nu z)]z - e$, with $\mu > 0$ and $\nu > 0$. Under these assumptions, the Golden rule individual resource stock, z^*, harvesting, e^*, and capital intensity, k^*, are given by $z^* = \frac{\mu}{2\nu}$, $e^* = \frac{N\mu^2}{4\nu}$ and $k^* = \left(\alpha_K A (e^*)^{\alpha_E}\right)^{\frac{1}{1-\alpha_K}}$. We then derive the conditions on preferences to decentralize the golden rule capital and resource stocks as follows. We derive the pair (β^*, γ^*) which maximizes net stationary production. Remind that z, e and k are, respectively, the steady state *equilibrium* resource stock, harvesting and capital intensity.We have $z = z^*$ if and only if $z = \frac{\mu}{\nu} - \frac{1}{N\nu}\frac{\alpha_E}{\gamma^*(1-\alpha_K)} = \frac{\mu}{2\nu} = z^*$, which leads to the following condition : $\gamma^* = \frac{2\alpha_E}{N\mu(1-\alpha_K)}$. This γ^* must be larger than the threshold γ^* to avoid extinction. In the Cobb-douglas-quadratic case, this threshold is $1/N\mu$. Thus the condition for a positive stationary natural stock z is given by $\gamma^* > \underline{\gamma} \Leftrightarrow \alpha_E > \frac{1-\alpha_K}{2}$. Taking $e = e^* = \alpha_E(1-\alpha_K)^{-1}z^*$ we have $k = k^*$ if and only if $k = (\beta^*(1-\alpha_K)A(e^*)^{\alpha_E})^{\frac{1}{1-\alpha_K}} = (\alpha_K A(e^*)^{\alpha_E})^{\frac{1}{1-\alpha_K}} = k^*$, which leads to the following condition : $\beta^* = \frac{\alpha_K}{1-\alpha_K}$. Since we require $\beta \in (0,1)$, we want to have $0 < \frac{\alpha_K}{1-\alpha_K} < 1 \Leftrightarrow 0 < \alpha_K < \frac{1}{2}$.

Thus, summarizing, this system of preferences and technology which decentralizes

the Golden rule is such that

$$\gamma = \gamma^* \equiv \frac{2\alpha_E}{N\mu\left(1-\alpha_K\right)} \tag{47}$$

$$\beta = \beta^* \equiv \frac{\alpha_K}{1-\alpha_K} \tag{48}$$

$$\alpha_K \in (0, \frac{1}{2}) \tag{49}$$

$$\alpha_E > \frac{1-\alpha_K}{2}. \tag{50}$$

6.5 'Resource-Conservationists' vs 'Resource-Wasting'

Let $\tilde{z} \neq z^*$, then by definition $\widetilde{e} < e^*$. With a CES production function $f\left(k_t, e_t\right) = A\left(\alpha_K k_t^{-\rho} + \alpha_L + \alpha_E e_t^{-\rho}\right)^{-1/\rho}$, we have $f'_{ke} > 0$ and so $f'_k\left(k^*, \widetilde{e}\right) < 1$. As a result, $\tilde{k}$ solution of $f'_k\left(\widetilde{k}, \widetilde{e}\right) = 1$ is such that $\tilde{k} < k^*$.

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